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Coupling between dissimilar rectangular dielectric waveguides

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Abstract

The coupling between two identically excited, dissimilar rectangular dielectric waveguides, whose axes are parallel, is studied as a function of frequency and of the geometrical and the electrical properties of the structure. It is found that, for a given permittivity ratio, the power transfer between the guides is maximal, but not complete, at some particular frequency. It is also found that, for a given frequency, the power transfer between the guides is complete for the appropriate combination of permittivity and size of the guides which achieves the same propagation coefficient in both.

1 Introduction

Dielectric waveguides have been studied for their potential applications in the transmission of data in millimeter, submillimeter and optical waves. Parallel coupled guides, which can be used as directional couplers, have also been studied [1]–[2]. But while the case of identical guides has been extensively investigated, the case of non-identical guides has not, as far as the authors are aware. The purpose of this paper is to study the coupling between two identically excited rectangular dissimilar dielectric waveguides whose axes are parallel, as a function of frequency, and of the geometrical and electrical properties of the structure.

2 Coupling evaluation

The cross-sections of the guides a and b are rectangular ($a_1 \times a_2$ and $b_1 \times b_2$) and the distance between their centers is $d_0 + (a_2 + b_2)/2$. Their dielectric constants are ε_a and ε_b respectively and the surrounding medium is air (see Fig. 1).

We have assumed that the field components of the electromagnetic wave have a time-dependence $e^{j\omega t}$ and that they propagate in the z-direction with an $e^{-j\beta z}$ variation. We have used the well-known dielectric constant method [3]–[6] to calculate the longitudinal propagation coefficient β in the z-direction as well as the transverse propagation coefficients k_x and k_y in the x- and y- directions respectively, for the dominant TE_{11} mode. Explicit expressions for the field components of this mode can be found in [3].

It can be shown [7] that for weak coupling, the total electric field is given as follows

$$\mathbf{E} = A(z)\mathbf{e}_a(x,y)e^{-i\beta_a z} + B(Z)\mathbf{e}_b(x,y)e^{-i\beta_b z} \tag{1}$$

where $\mathbf{e}_a(x,y)$ and $\mathbf{e}_b(x,y)$ describe the field variations with the transverse coordinates (x,y) for guides a and b, while β_a and β_b are the propagation coefficients of guides a and b respectively, when they are separated. It is assumed that TE_{11} is the only mode of propagation along the guides. The differential equation of the (z-dependent) amplitude $A(z)$ is given as follows [7]

$$\frac{d^2A(z)}{dz^2} + i2\zeta\frac{dA(z)}{dz} + K^2A(z) = 0 \tag{2}$$

where $\zeta = \frac{1}{2}(\beta_b - \beta_a)$ and K is the coupling coefficient of the guides. The amplitude $A(z)$ is given as follows, when only guide b is excited at $z = 0$

$$A(z) = Ge^{-i\zeta z} 2i \sin(Sz) \tag{3}$$

where $S = (K^2 + \zeta^2)^{\frac{1}{2}}$ and G is a constant. Similarly, for the same boundary conditions, the (z-dependent) amplitude $B(z)$ is given as follows

$$B(z) = -\frac{2G}{K} e^{i\zeta z} \Big[S\cos(Sz) - i\zeta \sin(Sz) \Big] \,. \tag{4}$$

It is possible to define various coupling criteria based on the above expressions of amplitudes $A(z)$ and $B(z)$.

The coupling coefficient K which appears in (2) depends on $\mathbf{e}_a(x,y)$ and $\mathbf{e}_b(x,y)$ and is not a function of z. Numerous methods have been devised to evaluate this quantity and the method due to Arnaud [1] has been adopted in this paper. It has been used successfully to find the transverse coupling between adjacent guides of various shapes and compositions [8]–[11]. It is given by

$$K^2 = \frac{\left[\oint_{C_a} (\mathbf{E}_a \times \mathbf{H}_b^+ - \mathbf{E}_b^+ \times \mathbf{H}_a) \cdot d\mathbf{L} \right]^2}{4 \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbf{E}_a \times \mathbf{H}_a) \cdot \hat{\mathbf{z}} \, dx\, dy \right] \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbf{E}_b \times \mathbf{H}_b) \cdot \hat{\mathbf{z}} \, dx\, dy \right]} = \frac{\mathcal{C}^2}{\mathcal{P}_a \mathcal{P}_b} \tag{5}$$

where the indices a and b refer to the appropriate guides, the adjoint fields $(\mathbf{E}^+, \mathbf{H}^+)$ are given by $\mathbf{E}^+ = (E_x, E_y - E_z)$ and $\mathbf{H}^+ = (-H_x, -H_y, H_z)$, and the contour C_a encircles guide a. It is taken along the x-axis and closes at infinity. Since at infinity the evanescent waves vanish, only the x-axis contribution is to be taken into consideration so that $d\mathbf{L} = -\hat{\mathbf{y}} dx$ and

$$\mathcal{C} = \frac{1}{2} \int_{-\infty}^{\infty} (E_{bx} H_{az} + E_{bz} H_{ax} - E_{ax} H_{bz} - E_{az} H_{bx}) \, dx \,. \tag{6}$$

In our case $E_z = 0$ since we are dealing with the TE_{11} mode. The term $\mathcal{P}_{a,b}$ is given as follows

$$\mathcal{P}_{a,b} = \int_{-\infty}^{+\infty} \int_{-\infty}^{\infty} (\mathbf{E}_{a,b} \times \mathbf{H}_{a,b}) \cdot \hat{\mathbf{z}} dx\, dy \,, \tag{7}$$

where the surface integrals encompass all the $x - y$ plane.

According to eqs. (3) and (4), we can see that if β_a and β_b are different, ζ does not vanish and the power in one guide is not wholly transferred to the other guide. The ratio of the maximum power which can be transferred to guide a to the power excited in guide b at $z=0$ is given by

$$T^2 = \frac{|A_{\max}|^2}{|B(0)|^2} = \frac{K^2}{S^2} = \frac{K^2}{K^2+\zeta^2} \tag{8}$$

and is smaller than one. Only when $\beta_a = \beta_b$, i.e. when $\zeta = 0$, is there a complete power transfer between the guides and $T^2 = 1$. According to eqs. (3) and (4), we can also see that the power transfer is maximal after a distance z_m from the origin (where guide b is excited) given by $\sin(Sz_m) = 1$. Therefore

$$z_m = \frac{\pi}{2}\frac{1}{(K^2+\zeta^2)^{\frac{1}{2}}} = \frac{\pi}{2}\frac{1}{S}\,. \tag{9}$$

T^2 and z_m can be used to evaluate the coupling between the guides.

It is possible to write down the electric field (eq. (1) together with eqs. (3) and (4)) as a superposition of two supermodes [7] whose propagation coefficients are given by

$$\beta'_{a,b} = \frac{\beta_a+\beta_b}{2} \pm \left[K^2 + \left(\frac{\beta_b-\beta_a}{2}\right)^2\right]^{\frac{1}{2}} = \beta_{a,b} \pm \Delta\beta\,, \tag{10}$$

where $\Delta\beta = (K^2+\zeta^2)^{\frac{1}{2}} + \zeta$, from which it is easy to show that $K^2 = \Delta\beta(\Delta\beta - 2\zeta)$. The coupling can also be evaluated by calculating the quantities $\beta'_a - \beta'_b$ or $\beta'_a - \beta_a$ or $\beta'_b - \beta_b$.

3 Numerical Results

In all the calculations, the distance between sides a_1 and b_1 has been kept constant ($d_0 = 8\,\mathrm{mm}$).

Two square guides of 4 mm side have been first analyzed. In Fig. 2a, the maximum power transfer ratio T^2 is given as a function of the frequency f for six different values of the permittivity ratio $\tau = \varepsilon_b/\varepsilon_a$ (0.8, 0.9, 0.95, 1, 1.1, 1.2). It can be seen that $T^2 = 1$ for $\tau = 1$ (i.e. $\varepsilon_a = \varepsilon_b$) at any frequency, while for any other value of τ, each of the curves representing T^2 reaches its maximum at some particular frequency f_m and decreases rapidly to zero on both sides of this maximum. The reason is that ζ^2 grows monotonically while K^2 goes through a maximum so that T^2 itself (see eq. (8)) goes through a maximum at the frequency f_m (see Fig. 2b). The maximum power transfer ratio T^2 is given in Fig. 3 as a function of the permittivity ratio τ for five different frequencies f (10, 16, 20, 25, 30 GHz). It can be seen that T^2 attains its maximum value ($T^2 = 1$) for $\tau = 1$ at all frequencies. The waist of the curves representing T^2 is largest around 16 GHz and narrows as the frequency grows (or diminishes), so that at very high (or very low) frequencies, the device could be used for calibration purposes ($T^2 = 1$ for $\tau = 1$) with a good degree of accuracy.

We now turn to square guides of different sizes ($a_1 = a_2 = 4\,\text{mm}$, $b_1 = b_2 = b$). T^2 is given in Fig. 4 as a function of b at a given frequency ($f = 20\,\text{GHz}$) for three different values of τ (0.8, 1.0, 1.2). It can be seen that T^2 reaches its highest value ($T^2 = 1$) at this frequency for a value of $b\,(b_m)$ which depends on τ and which corresponds to $\beta_a = \beta_b$. As expected, T^2 is highest ($T^2 = 1$) for $\tau = 1$ ($\varepsilon_a = \varepsilon_b$) when $b_m = a = 4\,\text{mm}$. For a given value of ε_a, lower values of ε_b (τ diminishes) must be compensated by larger values of b_m, and vice-versa, in order to achieve this maximum value of T^2, as can be seen in Fig. 5 for three different frequencies f $(10, 30, 50\,\text{GHz})$. The variation of b_m itself with f is shown in Fig. 6. For the two identical square guides, b_m is constant and equal to 4 mm at any frequency. On the other hand, it grows with frequency for $\tau < 1$ and decreases with frequency for $\tau > 1$.

Another interesting configuration consists of a square upper guide ($a_1 = a_2 = 4\,\text{mm}$) near a rectangular lower one ($b_1 = 4\,\text{mm}$, $b_2 = b_2$). A graph of T^2 versus b_2 is given in Fig. 7 for various values of τ (0.8, 1.0, 1.2) at a frequency $f = 20\,\text{GHz}$. A second configuration ($b_1 = b_1, b_2 = 4\,\text{mm}$) has also been studied. The general variation of the curves is similar in both cases (the second one is not shown), the maximum in T^2 occurring as usual when $\beta_a = \beta_b$, and particularly when $b_2 = 4\,\text{mm}$ (in the first case) or $b_1 = 4\,\text{mm}$ (in the second case).

Lastly, the variation of z_m (the distance from the origin at which the power transfer is maximal) is given in Fig. 8 for two square guides of different sizes ($a_1 = a_2 = 4\,\text{mm}$, $b_1 = b_2 = b$) as a function of b , at a given frequency ($f = 20\,\text{GHz}$), for three different values of τ (0.8, 1.0, 1.2). It can be seen that in order to obtain the highest possible power transfer, a lower distance z_m is needed for a lower permittivity ratio τ at a given frequency f. A similar behavior (not shown) is observed when the lower guide is rectangular and b_2 is varied.

4 Conclusion

From the above results, we conclude that the maximum power transfer T^2 is equal to one at any frequency when the two guides are identical in every respect (in shape, size and composition) because in this case $\beta_a = \beta_b$ so that $\zeta = 0$. If the two guides are identical in shape and size only, we note that, for a given frequency, their composition has a great influence on the coupling, a slight deviation in the permittivity ratio τ from the value $\tau = 1$ causing a large decrease in T^2, and that, for a given τ, T^2 reaches its highest value at some particular frequency f_m. If the two guides have a square shape but differ in their size and composition, we note that T^2 attains a maximum value of one (at a given frequency) for a side $b_{\max}$ of one of the guides which depends on τ and which corresponds to $\beta_a = \beta_b$, that $b_{\max}$ increases as τ decreases (at a given frequency), and that it increases or decreases with frequency according to whether $\tau < 1$ or $\tau > 1$. If the two guides are different in every respect, we note that the variation of T^2 with either the height or the width of one of the guides is similar to its variation with the side of a guide of square shape.

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Figure Captions

Figure 1: The geometry of the two rectangular dielectric waveguides and the path of integration.

Figure 2a: The maximum power transfer ratio T^2 as a function of the frequency f for different permittivity ratios τ.

Figure 2b: The coupling coefficient squared K^2, and ζ^2, as a function of the frequency f for the permittivity ratio $\tau = 0.9$.
− · − · − · K^2, − ·· − ·· − ·· ζ^2, − − − − − − − $K^2 + \zeta^2$, ———————— T^2.

Figure 3: The maximum power transfer ratio T^2 as a function of the permittivity ratio τ for different frequencies f.

Figure 4: The maximum power transfer ratio T^2 as a function of the side b for different permittivity ratios τ at the frequency $f = 20\,\mathrm{GHz}$:
———————— $\tau = 0.8$, - - - - - - - - - - - $\tau = 1$, − − − − − − − $\tau = 1.2$.

Figure 5: The side b_m at which the maximum power transfer ratio T^2 is highest as a function of the permittivity ratio τ for different frequencies f:
———————— $f = 10\,\mathrm{GHz}$, − − − − − − − $f = 30\,\mathrm{GHz}$, - - - - - - - - - - - $f = 50\,\mathrm{GHz}$.

Figure 6: The side b_m at which the maximum power transfer ratio T^2 is highest as a function of the frequency f for different permittivity ratios τ:
———————— $\tau = 0.8$, - - - - - - - - - - - $\tau = 1$, − − − − − − − $\tau = 1.2$.

Figure 7: The maximum power transfer ratio T^2 as a function of the side b_2 for different permittivity ratios τ at the frequency $f = 20\,\mathrm{GHz}$:
- - - - - - - - - - - $\tau = 0.8$, − − − − − − − $\tau = 1$, ———————— $\tau = 1.2$.

Figure 8: The distance z_m from the origin at which the power transfer is maximal as a function of the side b for different permittivity ratios τ at the frequency $f = 20\,\mathrm{GHz}$:
———————— $\tau = 0.8$, - - - - - - - - - - - $\tau = 1$, − − − − − − − $\tau = 1.2$.