CCIT Report #915 December 2017

Geometry-based distortion measures for space deformation

Alexander Naitsat[a], Emil Saucan[a,b], Yehoshua Y. Zeevi[a]

[a] *Electrical Engineering Department, Technion, Haifa, Israel*
[b] *Applied Mathematics Department, ORT Braude College, Karmiel, Israel*

ARTICLE INFO

Keywords: Digital Geometry Processing, Shape Deformation, Distortion Minimization, Quasi-Conformal Mappings, Triangle and Tetrahedral meshes.

ABSTRACT

We present a framework for optimizing a rich family of geometry-based energies defined on planes, surfaces and volumetric domains. Our approach is based on the concept of first order distortion measures and on the steepest descent optimization. Specifically, we present an algorithm for inducing optimal deformations for triangular and tetrahedral meshes. The resulting techniques apply to a variety of geometry processing problems, including ones that are highly non-convex. Among these problems are deformation and parametrization of planes, surfaces and volumetric meshes, surface mapping using volumetric textures, generation of triangular and tetrahedral meshes. In particular, the proposed techniques can be employed to devise "as close to being conformal as possible" mappings and other deformations that are nearly optimal with respect to related distortion measures, such as the isometric distortion and the distortion of a local volume. Tests, carried out on 2D and 3D data, show that the optimization process is numerically stable and fast-converging. Our approach is general and it can be run in parallel processes.

1. Introduction

A wide class of problems considered in geometry processing and computer graphics involve computation of mappings between domains in Euclidean space. These problems are often formulated in terms of geometric energies that assess the quality of a map. Let f be a smooth mapping of a domain $\mathcal{S} \in \mathbb{R}^n$ and consider the following energy

$$E(f) \triangleq \int_{\mathcal{S}} \omega(x)\,\mathrm{dist}\,(f,x)dx, \tag{1}$$

where $\omega(x)$ is a normalized cost function defined over domain $\mathcal{S}$, and $\mathrm{dist}(f,x)$ is the energy density at point x that depends on f and varies between different geometric problems. Minimizing $E(f)$ yields the highest quality mapping, and thus it provides a preferable mapping between $\mathcal{S}$ and a given target domain $\mathcal{T}$ (see Fig. 1). Formally, we consider the following fundamental problem:

Problem 1. Let $\mathcal{S}$ and $\mathcal{T}$ be compact domains with non-empty interiors in $\mathbb{R}^n$ ($n \geq 2$). Assume that $\mathcal{S}$ and $\mathcal{T}$ are homeomorphic, i.e., there is a 1:1 continuous map of one domain onto the other, and let $\mathrm{Def}(\mathcal{S},\mathcal{T})$ be a certain subset of smooth deformation functions from $\mathcal{S}$ to $\mathcal{T}$. Construct an optimal mapping

$$f_{\mathrm{opt}} = \underset{f \in \mathrm{Def}(\mathcal{S},\mathcal{T})}{\operatorname{argmin}} \int_{\mathcal{S}} \omega(x)\,\mathrm{dist}\,(f,x)\,dx, \tag{2}$$

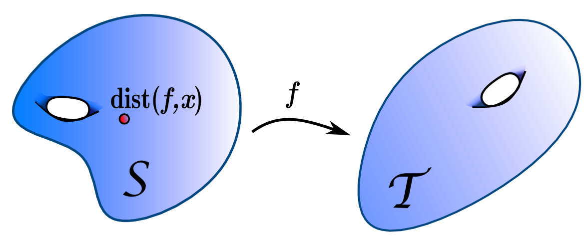

Fig. 1: An energy density $\mathrm{dist}(f,x)$ of a spatial mapping f.

where the fitness of a mapping is measured with respect to an energy density $\mathrm{dist}(f,x)$, called the *distortion* of f at x.

A most common constraints for (2) considered in this work is the fixation of f on a subset $\mathcal{A} \subset \mathcal{S}$, i.e., a spatial constraint

$$f|_{\mathcal{A}} = g\,, \tag{3}$$

where $g : \mathcal{S} \mapsto \mathcal{T}$ is a given continuous function.

We refer to Problem 1 as *the optimal mapping problem*. In practice we are more interested in approximated solutions than in global minimizers of (2), because the proposed problem is too general to be solved under practical restrictions and constraints. We should, therefore, focus on a narrower class of density functions $\mathrm{dist}(df,x)$, which will be referred to as *first order distortion measures*, where "first order" refers to the fact that $\mathrm{dist}(df,x)$ does not depend on the 2^{nd} and higher order derivatives of the mapping. Despite these restrictions, the proposed