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Surface wave propagation along a metamaterial slab

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Abstract

Surface wave propagation along a metamaterial slab having negative permittivity and permeability is analyzed. The dispersion relations are determined for the TM and for the TE (odd and even) modes. The Brillouin diagrams are drawn and show that, for odd modes, after cutoff, there is an initial frequency range for which there are two possible propagation coefficients. All the even modes exhibit the same behaviour except the first one. In this case, a band-pass region appears, suggesting that the device could be used as a filter.

1. Introduction

Wave propagation in materials whose permittivity and permeability are both negative has been investigated by Veselago [1] who has shown that in media of this kind, the Poynting vector is anti-parallel to the phase-velocity vector for a plane wave. These materials have been variously termed as metamaterials, backward wave materials, left-handed materials, etc. . . . . Lindell et al. [2] have reviewed isotropic and uniaxially isotropic metamaterials, and Lakhtakia et al. [3] have made an overview covering developments in the microwave range. Pendry [4] has shown that a slab possessing the above-mentioned characteristics could resolve objects only a few nanometers wide in the optical domain. Ziolkowsky et al. [5] have studied metamaterials both analytically and numerically. Engheta [6] has made a theoretical analysis on thin subwavelength cavity resonators using metamaterials. Shelby et al. [7] have presented experimental data and numerical simulations for a two-dimensional isotropic material at X-band microwave frequencies. Eleftheriades et al. [8] have presented experimental verification of focusing using an implementation of artificial transmission line media in planar form. Abu et al. [9] have studied the radiation from a traveling wave current sheet at the interface between a conventional material and a metamaterial. Feise et al. [10] have investigated the effects of surface waves on the behaviour of perfect lenses and studied metamaterial slabs with transition layers on either side.

The present work analyzes the propagation of surface waves along a metamaterial slab having negative permittivity and permeability. The Brillouin diagrams show interesting characteristics such as the appearance of a band-pass region in the even mode, suggesting that the device could be used as a filter.

2. The Electromagnetic Fields

The TM mode

The tangential electric and magnetic field components are given as follows (see Fig. 1) for the TM odd and even modes (the $e^{-j\beta z}$ factor has been omitted)

$$E_z^{\text{ODD}} = \begin{cases} \dfrac{-B}{j\omega\varepsilon_0}\, h^2\, e^{-hx} \\ \dfrac{-A}{j\omega|\kappa|\varepsilon_0}\, \ell^2 \,\sin \ell x \\ \dfrac{B}{j\omega\varepsilon_0}\, h^2\, e^{+hx} \end{cases} \qquad E_z^{\text{EVEN}} = \begin{cases} \dfrac{-B}{j\omega\varepsilon_0}\, h^2\, e^{-hx} & (x \geq t) \quad (1a) \\ \dfrac{-A}{j\omega|\kappa|\varepsilon_0}\, \ell^2 \,\cos \ell x & (-t \leq x \leq t) \quad (1b) \\ \dfrac{-B}{j\omega\varepsilon_0}\, h^2\, e^{+hx} & (x \leq -t) \quad (1c) \end{cases}$$

$$H_y^{\text{ODD}} = \begin{cases} Bh\, e^{-hx} \\ -A\ell\, \cos \ell x \\ Bh\, e^{+hx} \end{cases} \qquad H_y^{\text{EVEN}} = \begin{cases} Bh\, e^{-hx} & (x \geq t) \quad (2a) \\ A\ell\, \sin \ell x & (-t \leq x \leq t) \quad (2b) \\ -Bh\, e^{+hx} & (x \leq -t) \quad (2c) \end{cases}$$

The following notation has been adopted: $k_{x1} = -jh$, $k_{x2} = \ell$, $k_{x3} = jh$, $k_y = 0$, $k_z = \beta$, where h, ℓ and β are real positive numbers. Therefore, since $k_{xi}^2 + k_{yi}^2 + k_{zi}^2 = \omega^2 \varepsilon_i \mu_i = k_i^2$, $(i = 1, 2, 3)$, we obtain:

$$\ell^2 + \beta^2 = \omega^2 \varepsilon_r \mu_r = \omega^2(-|\kappa|\varepsilon_0)(-\mu_0) = \omega^2 |\kappa| \varepsilon_0 \mu_0 = |\kappa| k_0^2 \qquad (-t \leq x \leq t) \qquad (3a)$$

$$-h^2 + \beta^2 = \omega^2 \varepsilon_1 \mu_1 = \omega^2 \varepsilon_3 \mu_3 = \omega^2 \varepsilon_0 \mu_0 = k_0^2 \qquad (x \leq -t, x \geq t) \qquad (3b)$$

Subtracting Eq. (3b) from Eq. (3a) we obtain:

$$(\ell t)^2 + (ht)^2 = (k_0 t)^2 (|\kappa| - 1)\,, \quad (|\kappa| > 1)\,. \qquad (4)$$

We could retrieve β from Eq. (3a) or Eq. (3b).

It is very easy to show that, in the slab, the wave propagates in the z-direction, while the z-component of Poynting's vector points in the opposite direction.

Continuity of the E_z component at $x = t$ gives for the TM mode:

$$A\ell^2 \cdot \sin \ell t = |\kappa| B h^2\, e^{-ht} \qquad \text{(TM ODD)} \qquad (5a)$$

$$A\ell^2 \cdot \cos \ell t = |\kappa| B h^2\, e^{-ht} \qquad \text{(TM EVEN)}\,, \qquad (5b)$$

while continuity of the H_y components at $x = t$ gives for the same mode

$$-A\ell \cdot \cos \ell t = Bh\, e^{-ht} \qquad \text{(TM ODD)} \tag{6a}$$

$$A\ell \cdot \sin \ell t = Bh\, e^{-ht} \qquad \text{(TM EVEN)}\,, \tag{6b}$$

so that the dispersion relations are given as follows:

$$(\ell t) \cdot \tan(\ell t) = -|\kappa|(ht) \qquad \text{(TM ODD)} \tag{7a}$$

$$(\ell t) \cdot \cot(\ell t) = |\kappa|(ht) \qquad \text{(TM EVEN)} \tag{7b}$$

The TE mode

The analysis is similar for the TE mode. In this case, the dispersion relations are given as follows:

$$(\ell t) \cdot \tan(\ell t) = -(ht) \qquad \text{(TE ODD)} \tag{8a}$$

$$(\ell t) \cdot \cot(\ell t) = (ht) \qquad \text{(TE EVEN)} \tag{8b}$$

We observe that the dispersion relations for the TE mode are the same as for the TM mode except for a $|\kappa|$ factor appearing in the TM mode. The right-hand side of Eqs. (7a),(7b),(8a),(8b) which apply to metamaterials has a sign opposite to the right-hand side of the corresponding equations which apply to dielectric materials.

3. Numerical Results

The slab geometry is shown in Fig. 1. The following data have been used in our analysis: $2t = 0.75$ cm, $\varepsilon_r = -2$, $\mu_r = -1$. The TM odd mode has been studied first. The graphs of ht versus ℓt derived from Eq. (7a) are shown in Fig. 2. They will be called "quasi-trigonometric curves" (QTC). The circles with radii $k_0 t\sqrt{|\kappa| - 1}$ given by Eq. (4) have been superimposed upon them. The values of ht and ℓt defining the various TM odd modes are given by the intersection of these two sets of graphs. It is worth noting that the QTC derived from Eq. (7a) are the mirror images of the corresponding curves which apply to dielectric materials (see Fig. 3). It follows that, for any mode, for frequencies larger than its cutoff frequency, while there is always only one intersection point between the circles and the QTC for dielectric materials, there are two such points for metamaterials for an initial frequency range, and only one for subsequent frequencies, as could be seen in Fig. 2. The limits of this initial frequency range are given by the mode cutoff frequency (when the circles are tangent to the QTC) and by $mc/(2t\sqrt{|\kappa|-1})$, $m = 1, 2, 3, \ldots$, (obtained through Eqs. (4) and (7a) for $ht = 0$). The corresponding curves in the Brillouin βt versus $k_0 t$ diagram could be seen in Fig. 4.

The TM even mode has been studied next. The ht versus lt graphs show that the first mode exhibits a peculiar behaviour. Indeed, the curve describing the first mode possesses a curvature opposite to that of all the other curves, and ranges from $(lt = 0,\, ht = 1/|\kappa|)$ to $\left(lt = \pi/2,\, ht = 0\right)$, as could be seen in Fig. 5. Therefore, circles with radii smaller than $1/|\kappa|$ would not intersect this curve, and circles with radii larger than $\pi/2$ would not intersect it either. Consequently, from $f_1 = 0$ to $f_2 = c/(2\pi t\,|\kappa|\sqrt{|\kappa|-1})$ there is no propagation. From $f_2 = c/(2\pi t\,|\kappa|\sqrt{|\kappa|-1})$ to $f_3 = c/(4t\sqrt{|\kappa|-1})$ there is a surface wave propagation along the metamaterial in a frequency range $\Delta f = c\left(\frac{1}{2} - \frac{1}{\pi|\kappa|}\right)\Big/(2t\sqrt{|\kappa|-1})$, which is large for small $|\kappa|$ and small for large $|\kappa|$. Again, there is no propagation from $f_3 = c/(4t\sqrt{|\kappa|-1})$ to the cutoff frequency f_4 of the second mode (circle tangent to the QTC). Thereafter the behaviour of the slab in the even mode resembles its behaviour in the odd mode which has been studied previously. The corresponding curves in the Brillouin βt versus $k_0 t$ diagrams could be seen in Fig. 6.

Due to the appearance of the very interesting frequency range Δf, the device could be used as a band-pass filter whose width can be determined by the value of $|\kappa|$. Moreover, wave propagation along adjacent slabs would be possible without cross-talking by a judicious choice of their respective $|\kappa|$. In our numerical example, $f_1 = 0$, $f_2 = 6.4\,\text{GHz}$, $f_3 = 20\,\text{GHz}$ and $f_4 = 53.5\,\text{GHz}$. For $|\kappa| = 2$ we obtain $\Delta f = 13.6\,\text{GHz}$ and for $|\kappa| = 10$ we obtain $\Delta f = 6.2\,\text{GHz}$.

There are no significant changes for the TE modes, except that f_2 is larger by a factor $|\kappa|$, f_3 remaining the same. There is moreover a certain change in f_4.

4. Conclusion

Surface wave propagation along a metamaterial slab having negative permittivity and permeability has been analyzed. The dispersion relations have been determined for the TM and for the TE (odd and even) modes. The Brillouin diagrams have been drawn and show that, for odd modes, after cutoff, there is an initial frequency range for which there are two possible propagation coefficients. All the even modes exhibit the same behaviour except the first one. In this case, a band-pass region appears, suggesting that the device could be used as a filter.

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References

[1] Veselago, V.G., "The electrodynamics of substances with simultaneously negative values of ε and μ", *Soviet Phys. Uspekhi*, **10**, 4, pp. 509–514, 1968.

[2] Lindell, I.V., Tretyakov, S.A., Nikoskinen, K.I. and Ilvonen, S., "BW media – media with negative parameters, capable of supporting backward waves", *Microw. and Opt. Techn. Letters*, **31**, 2, pp. 129–133, 2001.

[3] Lakhtakia, A., McCall, M.W. and Weiglhofer, W.S., "Brief overview of recent developments on negative phase-velocity mediums (alias left-handed materials)", *Int. J. Electron. Commun. (AEÜ)*, **56**, 6, pp. 407–410, 2002.

[4] Pendry, J.B., "Negative refraction makes a perfect lens", *Phys. Rev. Letters*, **85**, 18, pp. 3966–3969, 2000.

[5] Ziolkowsky, R.W. and Heyman, E., "Wave propagation in media having negative permittivity and permeability", *Physical Review E*, **64**, 056625, 2001.

[6] Engheta, N., "An idea for thin sub-wavelength cavity resonators using metamaterials with negative permittivity and permeability", *IEEE Ant. and Wireless Prop. Letters*, **1**, 1, pp. 10–13, 2002.

[7] Shelby, R.A., Smith, D.R., Nemat-Nasser, S.C. and Schultz, S., "Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial", *Appl. Phys. Letters*, **78**, 4, pp. 489–491, 2001.

[8] Eleftheriades, G.V., Iyer, A.K. and Kramer, P.C., "Planar negative refractive index media using periodically L-C loaded transmission lines", *IEEE Trans. Microw. Theory and Techn.*, **50**, 12, pp. 2702–2712, 2002.

[9] Alu, A. and Engheta, N., "Radiation from a traveling-wave current sheet at the interface between a conventional material and a metamaterial with negative permittivity and permeability", *Microw. and Opt. Techn. Letters*, **35**, 6, pp. 460–463, 2002.

[10] Feise, M.W., Bevelacqua, P.J. and Schneider, J.B., "Effects of surface waves on the behaviour of perfect lenses", *Physical Review B*, **66**, 035113, 2002.

Figure Captions

Figure 1: The slab geometry.

Figure 2: TM ODD mode: lt versus ht. $\varepsilon_r = -|\kappa|\varepsilon_0 = -2\varepsilon_0$, $\mu_r = -\mu_0$. Circles with constant radii $k_0 t\sqrt{|\kappa| - 1} = k_0 t$ are superimposed upon the graphs.

Figure 3: TM ODD mode: lt versus ht. $\varepsilon_r = |\kappa|\varepsilon_0 = 2\varepsilon_0$, $\mu_r = \mu_0$. Circles with constant radii $k_0 t\sqrt{|\kappa| - 1} = k_0 t$ are superimposed upon the graphs.

Figure 4: TM ODD mode: βt versus $k_0 t$. $\varepsilon_r = -|\kappa|\varepsilon_0 = -2\varepsilon_0$, $\mu_r = -\mu_0$.

Figure 5: TM EVEN mode: lt versus ht. $\varepsilon_r = -|\kappa|\varepsilon_0 = -2\varepsilon_0$, $\mu_r = -\mu_0$. Circles with constant radii $k_0 t\sqrt{|\kappa| - 1} = k_0 t$ are superimposed upon the graphs.

Figure 6: TM EVEN mode: βt versus $k_0 t$. $\varepsilon_r = -|\kappa|\varepsilon_0 = -2\varepsilon_0$, $\mu_r = -\mu_0$.