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Wave Propagation in Metamaterial Multi-Layered Structures

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Abstract

Expressions have been given for the overall reflection and transmission coefficients of a multi-layered structure consisting of metamaterial and dielectric slabs. Small-reflection approximations have been discussed. Two applications have been proposed: anti-reflection coatings and high-reflection coatings. For the first application, a structure consisting of two slabs of the same width and opposite permittivities has been studied. If the structure is placed between two semi-infinite media of the same kind, the reflection vanish, while if these two media are different, the reflection depends only on their characteristics and on the angle of incidence. This device could be advantageously used as an antenna radome. For the second application, a structure consisting of a large number of identical pairs of slabs of high and low opposite permittivities and of the same widths, embedded in the same medium, has been studied. The transmittance variation with frequency of this structure shows no ripples, it has a large passband and a monotonous quasi-symmetric rise to the right and to the left of the central frequency. The distinctive feature of these devices is the weakening of the influence of the frequency, the angle of incidence or the polarization on the propagation processes.

1. Introduction

Veselago [1] has shown that the Poynting vector of a plane wave is anti-parallel to its phase-velocity vector in materials whose permittivity and permeability are both negative. These materials have been consequently termed metamaterials, backward-wave materials, left-handed materials, etc. Lindell et al. [2] and Lakhtakia et al. [3] have reviewed these materials. Pendry [4] has shown that a metamaterial slab could focus evanescent modes and resolve objects only a few nanometers wide in the optical domain. Ramakrishna et al. [5] have shown that a metamaterial slab bounded by different dielectric slabs also amplifies evanescent waves. Ziolkowsky et al. [6] have studied metamaterials both analytically and numerically. Engheta [7] has made a theoretical analysis on thin subwavelength cavity resonators containing metamaterials. Zhang et al. [8] have studied electromagnetic fields propagating through metamaterial slabs. Kong [9] has provided a general formulation for the electromagnetic wave interaction with stratified metamaterial structures. Gerardin et al. [10] have shown that the Bragg regime shifts when conventional materials in a multilayer distributed Bragg reflector are replaced by metamaterials.

In this paper, a recursive method [9], [11], [12], [13], is used to calculate the reflection and transmission coefficients of multi-layered structures including both metamaterial and dielectric layers. Two applications are proposed: anti-reflection coatings and high-reflection coatings. It is found that the frequency, the angle of incidence and the polarization are much less dominant in these devices than in their all-dielectric counterparts.

2. Propagation in metamaterial slabs

2.1. Propagation in a single slab

We will first study the propagation of an s-polarized wave in a single slab. A metamaterial slab of thickness d_2, permeability $\mu_2 = -\mu_0$, and permittivity $\varepsilon_2 = -|\kappa|\varepsilon_0$ is embedded between two semi-infinite dielectric media of permeability μ_0 and permittivities ε_1 and ε_3 respectively (see Fig. 1).

The tangential components of the electric and magnetic fields are given as follows:

$$\mathbf{E}_t = \begin{cases} \hat{y}\left[Ae^{-jk_3 z\cos\theta_3} + Be^{+jk_3 z\cos\theta_3}\right] e^{-jk_3 x\sin\theta_3} & (z<0) \quad \text{(1a)} \\ \hat{y}\left[Ce^{+jk_2 z\cos\theta_2} + De^{-jk_2 z\cos\theta_2}\right] e^{-jk_2 x\sin\theta_2} & (0<z<d_2) \quad \text{(1b)} \\ \hat{y}\left[Fe^{-jk_1(z-d_2)\cos\theta_1}\right] e^{-jk_1 x\sin\theta_1} & (d_2<z) \quad \text{(1c)} \end{cases}$$

$$\mathbf{H}_t = \begin{cases} -\hat{x}\left[\frac{j}{w\mu_0}\left\{A(-jk_3\cos\theta_3)\, e^{-jk_3 z\cos\theta_3} + B(jk_3\cos\theta_3)\, e^{+jk_3 z\cos\theta_3}\right\}\right] e^{-jk_3 x\sin\theta_3} \\ \qquad (z<0) \quad \text{(2a)} \\ -\hat{x}\left[\frac{-j}{w\mu_0}\left\{C(jk_2\cos\theta_2)\, e^{+jk_2 z\cos\theta_2} + D(-jk_2\cos\theta_2)\, e^{-jk_2 z\cos\theta_2}\right\}\right] e^{-jk_2 x\sin\theta_2} \\ \qquad (0<z<d_2) \quad \text{(2b)} \\ -\hat{x}\left[\frac{j}{w\mu_0}\left\{F(-jk_1\cos\theta_1)\, e^{-jk_1(z-d_2)\cos\theta_1}\right\}\right] e^{-jk_1 x\sin\theta_1} \\ \qquad (d_2<z) \quad \text{(2c)} \end{cases}$$

where $k_i = \omega\sqrt{\varepsilon_i\mu_i} = \omega n_i/c$ and θ_i is the acute angle between the normal and the wave-normal.

Continuity of E_y at $z=0$ and $z=d_2$ requires:

$$A + B = C + D \qquad (z=0) \tag{3a}$$

$$Ce^{+jk_{z2}d_2} + De^{-jk_{z2}d_2} = F \qquad (z=d_2) \tag{3b}$$

while continuity of H_x at $z=0$ and $z=d_2$ requires:

$$-Ak_{z3} + Bk_{z3} = -Ck_{z2} + Dk_{z2} \qquad (z=0) \tag{4a}$$

$$-Ck_{z2}\, e^{+jk_{z2}d_2} + Dk_{z2}\, e^{-jk_{z2}d_2} = -Fk_{z1} \qquad (z=d_2) \tag{4b}$$

where $k_{zi} = k_i\cos\theta_i$.

The slab reflection and transmission coefficients are retrieved from Eqs. (3) and (4). They are given as follows:

$$R_2^s = \frac{B}{A} = \frac{r_{32}^s + r_{21}^s\ e^{+j2\varphi_2}}{1 + r_{32}^s r_{21}^s\ e^{+j2\varphi_2}} \tag{5a}$$

$$T_2^s = \frac{F}{A} = \frac{t_{32}^s t_{21}^s\ e^{+j\varphi_2}}{1 + r_{32}^s r_{21}^s\ e^{+j2\varphi_2}} \tag{5b}$$

where the interface reflection and transmission coefficients are given by:

$$r_{ij}^s = \frac{k_{zi} - k_{zj}}{k_{zi} + k_{zj}} \tag{6a}$$

$$t_{ij}^s = \frac{2k_{zi}}{k_{zi} + k_{zj}} \tag{6b}$$

and $\varphi_i \triangleq k_{zi} d_i$. The interface reflection and transmission coefficients are identical for dielectric and metamaterials. On the other hand the phase φ_2 in Eqs. (5a) and (5b) is preceded by a $+$ sign for a metamaterial slab and by a $-$ sign for a dielectric slab.

For a p-polarized wave propagating in a single metamaterial slab, the slab reflection and transmission coefficients are given as follows:

$$R_2^p = \frac{r_{32}^p + r_{21}^p\ e^{+j2\varphi_2}}{1 + r_{32}^p r_{21}^p\ e^{+j2\varphi_2}} \tag{7a}$$

$$T_2^p = \frac{t_{32}^p t_{21}^p\ e^{+j\varphi_2}}{1 + r_{32}^p r_{21}^p\ e^{+j2\varphi_2}} \tag{7b}$$

where the interface reflection and transmission coefficients are given by:

$$r_{ij}^p = \frac{k_j \cos\theta_i - k_i \cos\theta_j}{k_j \cos\theta_i + k_i \cos\theta_j} \tag{8a}$$

$$t_{ij}^p = \frac{2k_i \cos\theta_i}{k_j \cos\theta_i + k_i \cos\theta_j}\,. \tag{8b}$$

In the p-polarization case too, the interface reflection and transmission coefficients are identical for dielectric and for metamaterials, while the phase φ_2 is preceded by a $+$ sign for a metamaterial slab and by a $-$ sign for a dielectric slab.

Similar expressions have been given in [5] and [9].

In both polarizations, the law of conservation of energy is given as follows:

$$|R_2^{s,p}|^2 + |\overline{T}_2^{s,p}|^2 = 1 \tag{9}$$

where $|\overline{T}_2^{s,p}|^2 = (k_{z1}/k_{z3})|T_2^{s,p}|^2$.

For quarter-wavelength slabs, in the s-polarization case, $r_{32}^s + r_{21}^s \ e^{+j2\varphi_2} = 0$, $\varphi_2 = \frac{\pi}{2}$, so that

$$\frac{k_2^2}{k_1 k_3} = \frac{\cos\theta_1 \cos\theta_3}{\cos^2\theta_2} \qquad (s\text{-polarization}) \tag{10a}$$

while in the p-polarization case, $r_{32}^p + r_{21}^p \ e^{+j2\varphi_2} = 0$, $\varphi_2 = \frac{\pi}{2}$, so that

$$\frac{k_2^2}{k_1 k_3} = \frac{\cos^2\theta_2}{\cos\theta_1 \cos\theta_3} \qquad (p\text{-polarization})\,. \tag{10b}$$

2.2. Propagation in a multi-layered structure

In a multi-layered structure, for either the s- or the p-polarization, the total reflection and transmission coefficients are given as follows for layer i:

$$R_i = \frac{r_{i+1,i} + R_{i-1} \ e^{\pm j2\varphi_i}}{1 + r_{i+1,i} R_{i-1} \ e^{\pm j2\varphi_i}} \tag{11a}$$

$$T_i = \frac{t_{i+1,i} T_{i-1} \ e^{\pm j\varphi_i}}{1 + r_{i+1,i} R_{i-1} \ e^{\pm j2\varphi_i}} \tag{11b}$$

where the phase φ_i connected with slab i is preceded by a $+$ sign for a metamaterial and by a $-$ sign for a dielectric material. We shall generally calculate R/T because it is this quantity we wish to minimize or to maximize in order to obtain anti-reflection or high reflection coatings, and because the denominators in Eqs. (11a) and (11b) cancel out in this quantity, making calculations easier.

According to Eqs. (5a) and (5b), or (7a) and (7b), layer 2 has been reduced to an equivalent interface between media 3 and 1. Then Eqs. (11a) and (11b) form the basis of an iteration procedure which, when repeated layer after layer down to the last layer, yield the overall reflection and transmission coefficients of the structure as a whole (see Figs. 2(a), 2(b), ... 2(e)). Snell's law is evidently satisfied at each interface, i.e., $n_1 \sin\theta_1 = n_2 \sin\theta_2 = \cdots = n_N \sin\theta_N$ and provides the angles $\theta_1, \theta_2, \ldots \theta_N$ used in the Fresnel interface reflection and transmission coefficients.

When $N = 4$ (two slabs), we obtain:

$$\frac{R_3}{T_3} = \frac{r_{43} + r_{32}\, e^{j(\pm 2\varphi_3)} + r_{21}\, e^{j(\pm 2\varphi_3 \pm 2\varphi_2)} + r_{43} r_{32} r_{21} e^{j(\pm 2\varphi_2)}}{t_{43} t_{32} t_{21}\, e^{j(\pm\varphi_3 \pm \varphi_2)}} . \tag{12}$$

Explicit expressions for R_{N-1} and T_{N-1} (any N) have been given in [9] and [13]. An alternative matrix method has been given in [9].

The first three terms in the numerator of Eq. (12) represent the three first-order reflections from the three interfaces $4-3, 3-2$ and $2-1$ respectively, while its fourth term contains the product of three interface reflection coefficients and is, evidently, smaller in absolute value than each of the other three terms. The denominator of Eq. (12) contains the product of the three interface transmission coefficients. Then, if the interface reflection coefficients are small, multiple reflections could be neglected, and we obtain:

$$\frac{R_3}{T_3} \cong \frac{r_{43} + r_{32} e^{j(\pm 2\varphi_3)} + r_{21} e^{j(\pm 2\varphi_3 \pm 2\varphi_2)}}{t_{43} t_{32} t_{21} e^{j(\pm\varphi_3 \pm \varphi_2)}} . \tag{13}$$

It can be shown [13] that the terms representing reflections other than the first-order ones in the general expression of R_{N-1}/T_{N-1} contain the products of at least three interface reflection coefficients, indicating that the first-order approximation could be adequate if one wishes to obtain some guidelines in the design of multi-layered structures. Moreover, a large number of terms representing multiple reflections are set equal to zero if one uses pairs of slabs with identical widths and opposite permittivities whose interface reflection coefficients vanish.

2.3. Small interface reflection coefficients

The total reflection coefficient is given as follows, for a structure consisting of $N-2$ slabs, in both polarizations, if the interface reflection coefficients are small:

$$R_{N-1} \cong \sum_{i=2}^{N-1} r_{i,i-1}\, e^{j\sum_{k=i}^{N-1}(\pm 2\varphi_k)} + r_{N,N-1} . \tag{14a}$$

The total transmission coefficient is given as follows under the same conditions:

$$T_{N-1} \cong \left[\prod_{i=2}^{N} t_{i,i-1}\right] \cdot \left[e^{j\sum_{k=2}^{N-1}(\pm\varphi_k)}\right] . \tag{14b}$$

Moreover, we obtain, for alternating dielectric and metamaterial slabs whose phases φ_k are all equal ($\varphi_k = \varphi$), and for small interface reflection coefficients:

$$R_{N-1} \cong \sum_{i=1}^{(N-1)/2} \left[r_{2i+1,2i} + r_{2i,2i-1}\ e^{\pm j2\varphi}\right]$$

$$= (r_{32} + r_{54} + \cdots + r_{N,N-1}) + (r_{21} + r_{43} + \cdots + r_{N-1,N-2})e^{\pm j2\varphi} \quad \text{(N odd)} \tag{15a}$$

$$R_{N-1} \cong \sum_{i=1}^{N/2} r_{2i,2i-1} + \sum_{i=1}^{(N-2)/2} r_{2i+1,2i}\ e^{\pm j2\varphi}$$

$$= (r_{21} + r_{43} + \cdots + r_{N,N-1}) + (r_{32} + r_{54} + \cdots + r_{N-1,N-2})e^{\pm j2\varphi} \quad \text{(N even)} \tag{15b}$$

$$T_{N-1} \cong \prod_{i=2}^{N} t_{i,i-1}\ e^{\pm j\varphi} = t_{21}t_{32}\cdots t_{N,N-1}e^{\pm j\varphi} \quad \text{(N odd)} \tag{15c}$$

$$T_{N-1} \cong \prod_{i=2}^{N} t_{i,i-1} = t_{21}t_{32}\cdots t_{N,N-1}\,. \quad \text{(N even)} \tag{15d}$$

It should be noted that the phases φ_k ($\varphi_k = \frac{\omega}{c} n_k d_k \cos\theta_k$) of the alternating dielectric and metamaterial slabs could be all equal only if $\dfrac{\omega}{c}\sqrt{n_k^2 - n_N^2 \sin^2\theta_N}\,d_k$ is constant, θ_N being the angle of incidence. The original structure behaves then as a single slab embedded between two semi-infinite media. The characteristics of this structure could be inferred from those of the original structure.

3. Applications

3.1. Antireflection coatings

We shall study first a structure consisting of a pair of metamaterial and dielectric slabs ($N = 4$), the two slabs having the same width d, and opposite permittivities $n_3 = -|\kappa|\varepsilon_0$ and $n_2 = |\kappa|\varepsilon_0$ (see Fig. 3(a)). In this case $\varphi_3 = \varphi_2$, $\theta_3 = \theta_2$, $r_{32}^{s,p} = 0$ and $t_{32}^{s,p} = 1$. If $n_1 = n_4$, for any frequency and for any angle of incidence, $R_3^{s,p} = 0$ and $T_3^{s,p} = 1$. If $n_1 \neq n_4$, for any frequency and for normal incidence, $R_3^{s,p}/T_3^{s,p} = r_{41}^{s,p}/t_{41}^{s,p}$, this case being akin to the case of the initial semi-infinite medium being adjacent to the final one as though the pair of slabs was missing. If $n_1 \neq n_4$, for any frequency and for any angle of incidence, $R_3^{s,p}/T_3^{s,p} = (r_{43}^{s,p} + r_{21}^{s,p})/t_{43}^{s,p}t_{21}^{s,p}$. For s-polarization, $R_3^s/T_3^s = (n_4\cos\theta_4 - n_1\cos\theta_1)/(2n_4\cos\theta_4)$ and for p-polarization, $R_3^p/T_3^p = (n_1\cos\theta_4 - n_4\cos\theta_1)/(2n_4\cos\theta_4)$, i.e., in both cases, R_3/T_3 depends only on the values of the refractive indices of the initial and final media, n_4 and n_1, and on the angle of incidence θ_4. It is evident that any number of additional pairs of alternating slabs, all identical to the first pair, would not change the results. Neither would any number of additional pairs of alternating slabs, all different from the first pair, provided in each pair the width and the permittivity absolute value of each of the two slabs composing the pair are identical. The physical reason of this feature is that, at the interface separating the two slabs of each pair, the reflection coefficient vanishes and the transmission coefficient is unity, while any phase a wave may have after its propagation through one of the slabs is completely cancelled after its propagation through the other slab, making each pair of slabs a transparent structure. The advantage of structures composed of pairs of alternating slabs is that, since the phases φ disappear, the reflection and transmission processes are frequency independent, while they are independent of the angle of incidence too only if the initial and final semi-infinite media are identical. These structures could be advantageously used as antenna radomes.

We shall study afterwards a structure consisting of a pair of metamaterial and dielectric slabs with an additional metamaterial slab only ($N = 5$), the three slabs having the same width d and permittivities $n_4 = -|\kappa|\varepsilon_0$, $n_3 = |\kappa|\varepsilon_0$ and $n_2 = -|\kappa|\varepsilon_0$ (see Fig. 3(b)). In this case $R_4^{s,p}/T_4^{s,p} \cong (r_{54}^{s,p} + r_{21}^{s,p}e^{+j2\varphi_2})/t_{54}^{s,p}t_{21}^{s,p}e^{+j2\varphi_2}$ where $\varphi_2 = \frac{\omega}{c}n_2 d\cos\theta_2$ ($n = n_4 = n_3 = n_2$, $\theta = \theta_4 = \theta_3 = \theta_2 = \arcsin(n_5\sin\theta_5/n)$). If the additional metamaterial slab has a different width d' and a different permittivity $-|K'|\varepsilon_0$, then $R_4^{s,p}/T_4^{s,p} \cong (r_{54}^{s,p} + r_{32}^{s,p} + r_{21}^{s,p}e^{+j2\varphi_2})/t_{54}^{s,p}t_{32}^{s,p}t_{21}^{s,p}e^{+j2\varphi_2}$. The disadvantage of structures containing an odd number of slabs is that, since the phases φ do not disappear, the reflection and transmission processes are dependent on frequency and on the angle of incidence.

3.2. High-reflection coatings

A careful examination of Eq. (12) shows that in order to maximize R_3/T_3, one has to chose a pair of adjacent dielectric and metamaterial slabs with highly contrasted refractive indices ($n_4 < n_3$, $n_3 > n_2$, $n_2 < n_1$), and, as usual, $\varphi_2 = \varphi_3 = \pi/2$ at the central frequency for normal incidence (see Fig. 3(c)). In this case all the terms in the numerator of Eq. (12) are negative, i.e., add in phase, the first one because $n_4 < n_3$ ($r_{43} < 0$), the second one because $n_3 > n_2$ ($r_{32} > 0$) and $2\varphi_3 = \pi$, the third one because $n_2 < n_1$ ($r_{21} < 0$) and $-2\varphi_2 + 2\varphi_3 = 0$, and the fourth one because $r_{43}r_{32}r_{21} < 0$

and $-2\varphi_2 = -\pi$, while the denominator is positive, $t_{43} > 0, t_{32} > 0, t_{21} > 0$ and $-2\varphi_2 + 2\varphi_3 = 0$.

At the central frequency $\omega = \omega_0$, then, for $\theta_4 = 0$, $\varphi_2 = \frac{\omega_0}{c} n_2 d_2 = \varphi_3 = \frac{\omega_0}{c} n_3 d_3 = \frac{\pi}{2}$ so that $n_2 d_2 = n_3 d_3 = c\pi/2\omega_0$. Therefore, for a metamaterial slab adjacent to a dielectric slab, the phase $-2\varphi_2 + 2\varphi_3 = 0$, and this phase is the same as for a dielectric slab adjacent to another dielectric slab: $-2\varphi_2 - 2\varphi_3 = -2\pi$. But for a different frequency $\omega = \omega'$, also for $\theta_4 = 0$, $\varphi_2 = \frac{\omega'}{c} n_2 d_2 = \varphi_3 = \frac{\omega'}{c} n_3 d_3 = \frac{\omega'}{\omega_0}\frac{\pi}{2}$, so that for a metamaterial-dielectric pair of slabs, the phase $-2\varphi_2 + 2\varphi_3$ remains equal to zero, while for a dielectric-dielectric pair of slabs, the phase $-2\varphi_2 - 2\varphi_3 = -2\pi\frac{\omega'}{\omega_0}$, which is different from -2π by a factor $\frac{\omega'}{\omega_0}$.

At the central frequency $\omega = \omega_0$, and for oblique incidence $\theta_4 = \theta' \neq 0$ ($\sin\theta_4 \ll 1$), $\varphi_2 = \frac{\omega_0}{c} n_2 d_2 \cos\theta_2 = \frac{\pi}{2}\cos\theta_2$ ($\sin\theta_2 \ll 1$) and $\varphi_3 = \frac{\omega_0}{c} n_3 d_3 \cos\theta_3 = \frac{\pi}{2}\cos\theta_3$ ($\sin\theta_3 \ll 1$). Therefore, for a metamaterial-dielectric pair of slabs $-2\varphi_2 + 2\varphi_3 = -\pi(\cos\theta_2 - \cos\theta_3) = \pi\left(\frac{\theta_2^2}{2} - \frac{\theta_3^2}{2}\right)$ (since $\cos\theta = 1 - \frac{\theta^2}{2!} + \ldots$ for $\theta \sim 0$) while for a dielectric-dielectric pair of slabs $-2\varphi_2 - 2\varphi_3 = -\pi(\cos\theta_2 + \cos\theta_3)$ $= -2\pi + \pi\left(\frac{\theta_2^2}{2} + \frac{\theta_3^2}{2}\right)$. It is evident that $(\theta_2^2 - \theta_3^2)/2$ is smaller than $(\theta_2^2 + \theta_3^2)/2$. Similar arguments, applied to $\omega = \omega' \neq \omega_0$ together with $\theta_4 = \theta' \neq 0$, yield the same conclusions. Therefore, by a judicious combination of metamaterial and dielectric slabs, a high-reflection coating is achieved, for which the dependence of R_3/T_3 on the frequency and on the angle of incidence is consequently diminished.

4. Numerical results

In order to illustrate the ideas exposed in the preceding sections and to compare, for high reflectance coatings, the results obtained by using structures consisting of metamaterial and of dielectric slabs to those obtained by using their all-dielectric counterparts, the transmittance of $(\mathrm{HL})^{10}\mathrm{H}$ quarter-wavelength structures embedded in air has been calculated as a function of frequency in various cases. The refractive index of the H medium is 2.25 and that of the L medium is 1.45. The central frequency has been chosen as $f = 200\,\mathrm{GHz}$.

The transmittance of an all-dielectric structure is shown in Figs. 5a and 5b as a function of frequency between 100 and 300 GHz for s- and p-polarizations and for various angles of incidence ($\theta = 0^o, 20^o, 40^o$). The transmittance variation with frequency of the corresponding metamaterial

(L)-dielectric (H) structure is shown in Figs. 6a and 6b with the same parameters as in Fig. 5. Finally, the transmittance variation with frequency from 0 to 400 GHz for the metamaterial-dielectric structure is shown in Fig. 7 for the s-polarization.

It could be inferred from these graphs that for the metamaterial-dielectric structure, the pass-band is much larger and the influence of the angle of incidence less conspicuous than for its all-dielectric counterpart. The metamaterial-dielectric structure shows no ripples, no sharp shifts in frequency between the s- and p-polarizations, but a monotonous quasi-symmetric rise in the transmittance to the left and to the right of the central frequency. All these features should make it attractive in the design of high-reflectance coatings.

5. Conclusion

Propagation in metamaterial slabs has been studied and expressions for the overall reflection and transmission coefficients have been given for multi-layered structures consisting of metamaterial or metamaterial and dielectric slabs. Small-reflection approximations have been deduced and their validity has been discussed. Two applications have been proposed: anti-reflection coatings and high-reflection coatings. It has been found that for a pair of slabs having the same width and opposite permittivities, the overall reflection coefficient would either vanish, if the initial and final media were identical, or depend only on the refractive indices of these media and on the angle of incidence, if they were not. These structures could be advantageously used as antenna radomes. It has also been found that the metamaterial dielectric structures have very interesting features when used as high-reflection coatings. Their pass-band is larger and they show no ripples but a monotonous quasi-symmetric rise in their transmittance to the left and to the right of the central frequency. The distinctive feature of these devices is the weakening of the influence of the frequency, the angle of incidence or the polarization on the reflection and the transmission processes.

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Figure captions

Figure 1: Wave propagation through a metamaterial slab embedded between two dielectric semi-infinite media.

Figure 2: (a) The overall reflection and transmission coefficients of layer 2.
(b) The overall reflection and transmission coefficients of layers 2 and 3.
(e) The overall reflection and transmission coefficients of the N-2 layers.

Figure 3: (a) A pair of slabs with the same width d and opposite permittivities $(-|\kappa|\varepsilon_0, +|\kappa|\varepsilon_0)$ embedded between two dielectric semi-infinite media.
(b) Three slabs with the same width d and alternating permittivities $(-|\kappa|\varepsilon_0, +|\kappa|\varepsilon_0, -|\kappa|\varepsilon_0)$ embedded between two dielectric semi-infinite media.
(c) A pair of quarter-wavelength dielectric and metamaterial slabs with highly contrasted refractive indices embedded between two dielectric semi-infinite media.

Figure 4: The transmittance as a function of frequency of a $(\mathrm{HL})^{10}\mathrm{H}$ quarter-wavelength dielectric slabs structure embedded in air, for various angles of incidence:
(a) s-polarization
(b) p-polarization.

Figure 5: The transmittance as a function of frequency of a $(\mathrm{HL})^{10}\mathrm{H}$ quarter-wavelength alternating metamaterial (L)-dielectric (H) slabs structure embedded in air, for various angles of incidence:
(a) s-polarization
(b) p-polarization.

Figure 6: The transmittance as a function of frequency of a $(\mathrm{HL})^{10}\mathrm{H}$ quarter-wavelength alternating metamaterial (L)-dielectric (H) slabs structure embedded in air (s-polarization), for various angles of incidence, with a very large frequency range.