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Iterative LDPC Coded MIMO Multiple Access with MMSE

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Abstract

An efficient scheme for the multiple-access MIMO channel is proposed, which operates well also in the single user regime, as well as in a DS-CDMA regime. The design features scalability and is of limited complexity. The system employs optimized LDPC codes and an efficient iterative (belief propagation) detection which combines Linear Minimum Mean Square Error (LMMSE) and iterative interference cancellation. Asymptotic density evolution is used to optimize the degree polynomials of the underlining LDPC code, and thresholds as close as 0.77 dB to the channel capacity is evident for a system load of 2. Replacing the LMMSE with the complex individually optimal multiuser detector (IO-MUD) further improves the performance up to 0.14 dB from the capacity. Comparing the thresholds of good single-user LDPC code to the multiuser optimized LDPC code both on the above multiuser channel reveals surprising 8 dB difference. The asymptotic analysis of the proposed scheme is verified by simulations of finite systems, which reveal meaningful differences between performances of MIMO systems with single and multiple users and demonstrate similar performance to previously reported techniques, but with higher system loads, and significantly lower receiver complexity.

Index Terms

MIMO, LDPC, multiuser, CDMA, iterative decoding.

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I. Introduction

Many schemes are designed to approach the overwhelming Multiple Input Multiple Output (MIMO) channel capacity [1], in particular, the case of CSI known at the receiver but not at the transmitter is often assumed. A detailed description of the popular MIMO settings and their solved or unsolved capacity limits is given in [2]. Reference to the multi access MIMO capacity is given in [1], where it is shown that since the single user capacity is achieved with uncoordinated transmissions which corresponds to multi access channel, the sum rate multi access capacity is equal to the single user capacity. This conclusion supports the use of well studied CDMA based multi access techniques in MIMO channels (either single or multiple access) exploiting the mathematical similarities of CDMA and MIMO. The asymptotic capacity of the CDMA channel is explicitly calculated in [3] with unconstrained signalling, where both the number of users and chips is taken to infinity, while their ratio remains fixed. The *Replica method* is used by [4] extend the results to the case of BPSK signalling. Since the asymptotic analysis of both assumes randomly generated spreading sequences and since it relies on the eigenvalue distribution, which is robust to the statistics, their analysis remains valid for the similar MIMO channel (that is flat fading MIMO channel with independent fading coefficients). Many proposed communication schemes are based on the theoretical diversity of the MIMO channel. A nice overview of such systems is given in [5]. We distinguish between systems which achieve good spectral efficiency: [6], [7] and [8] and which maximize the diversity: [9], [10] and [11]. Iterative receivers, in which the detector and the decoder exchange extrinsic information, are known to perform well in a variety of communication systems, specifically in multiple access systems such as CDMA [12], [13]. The asymptotic information theoretic loss due to separated decoding and detection is given in [14] and strongly justifies the use of iterative schemes. A CDMA multiuser detection technique used over MIMO channels is presented in [7]. Asymptotic analysis of the linear LMMSE multiuser detector (MUD) is given in [15] and [16] and is used by Boutros and Caire in [12], who present density evolution (DE) analysis for asymptotic iterative CDMA LMMSE multiuser detector and trellis decoder. This density evolution analysis is used in [17] also for regular LDPC codes over CDMA. The use of LDPC-BP decoder with LMMSE multiuser detector enables elegant density evolution analysis of the decoder in the asymptotic realm. The Density evolution technique was originally described by [18] for the analysis of the BP decoder of the LDPC codes. These rediscovered codes are known to perform well and many reported excellent performance such as 0.06 dB from AWGN channel capacity [19], [20] and [21]. In this paper we combine these techniques to construct a multiuser MIMO communication system employing LDPC codes and LMMSE multiuser receiver. Furthermore, we optimize the degree distribution pairs (DDP) [18] of the LDPC code when iteratively decoded with the LMMSE detector. Asymptotic techniques [22], [12] are used for the code optimization and for the capacity evaluation. The LDPC Codes are optimized for both linear MMSE detectors and individually optimal multiuser detector (IO-MUD). Comparing the thresholds to the capacity reveals excellent performance, with gap to capacity of 0.77 dB for the LMMSE and 0.14 dB for the IO-MUD, which is realized with prohibitive complexity. The LDPC codes are optimized with a global search algorithm (DE-Differential Evolution) that finds good LDPC polynomials that achieve low bit error rate. Simulations of finite systems verify the asymptotic expectations on both CDMA and MIMO for both single and multiple user systems with block lengths of 1.5×10^5 and 10^4 respectively.

II. SYSTEM DESCRIPTION

In this paper we deal with the multi access communication channel where each user transmits independent information by a single antenna over independent Rayleigh fading channel to a receiver equipped with M antennas. We assume full synchronization, perfect power control scheme and a single class of K users. Generalization of this work to systems with several classes and/or received powers follows the same lines. The channel load α is defined by $\frac{K}{M}$ in parallel to an equivalent DS-CDMA setting [12]. The receiver, in the proposed scheme, has full knowledge about the channel state information (CSI), while the transmitters have no CSI available. The MIMO channel coefficients $\{\boldsymbol{h}_{k,t}\}_{k=1}^{K}$ are assumed to remain constant along entire transmitted block and are then randomly and independently chosen again for the next transmitted block, so we drop the time index t. Such statistics approximates quasi-static block fading MIMO channel, while the analysis applies also to the case of fast fading channel. The model also suits more generalized multi access MIMO system of $K_u \neq K$ users, where each transmitter can use number of antennas $\frac{K}{K_u}$, not necessarily a single one. We can also consider the case of single user MIMO system, as a special case of the generalized system above, where there is one user $K_u = 1$ transmitting out of $\frac{K}{1} = K$ antennas. Thanks to the known mathematical equivalence between MIMO and CDMA, evident also in the following sections, the designed scheme also suits non-orthogonal CDMA or any other multi access system that uses some randomly generated vector as the common channel and when this vector is generated by some i.i.d. elements. For example, in multiuser DS-CDMA systems M, in the following equations, stands for the processing gain, and the random channel attenuations coefficients $\{\mathbf{h_k}\}_{k=1}^{K}$ stands for the K-users signatures sequences.

A. Transmitters and the channel

The channel (1) is defined along the lines of [12]. The received signal vector $\boldsymbol{y}_t$ of length M, at time t, consists of linear superposition of the K transmitted symbols $\{x_{k,t}\}_{k=1}^{K}$ multiplied by the channel coefficients vectors $\{\boldsymbol{h}_k\}_{k=1}^{K}$, by scalar random phases $\{e^{j\theta_{k,t}}\}_{k=1}^{K}$ and by received amplitude $\sqrt{\frac{\gamma}{M}}$. It also suffers additive Gaussian complex noise $\boldsymbol{\nu}_t \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I})$ [1].

$$\boldsymbol{y}_t = \sqrt{\frac{\gamma}{M}} \sum_{k=1}^{K} \boldsymbol{h}_k e^{j\theta_{k,t}} x_{k,t} + \boldsymbol{\nu}_t. \tag{1}$$

Here, $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I})$. We use QPSK signals $x \in \{\pm\frac{1}{\sqrt{2}}, \pm\frac{j}{\sqrt{2}}\}$. $e^{j\theta_{k,t}}$ is randomly, independently and uniformly generated from $\{\pm\frac{1}{\sqrt{2}}, \pm\frac{j}{\sqrt{2}}\}$ to render the multiple access interference independent of the transmitted codewords

[1] $\mathcal{N}_{\mathbb{C}}(0, I)$ represents the vector complex Gaussian density with covariance equals to identity matrix I and zero mean and the symbol $\sim$ means ”distributed as”.

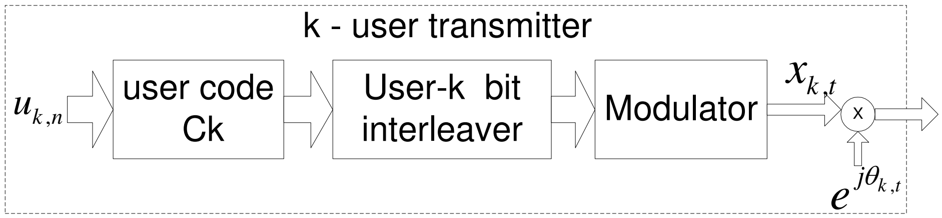

Fig. 1. Transmitter scheme, where the receiver has several antenna.

Fig. 2. Iterative receiver scheme

[12]. The receiver has full knowledge of the received powers $\frac{\gamma}{M}$, the channel coefficients $\boldsymbol{h}_k$ and the random phases $e^{j\theta_{k,t}}$. Notice that we normalized the received power with M so that each transmitting antenna is received with total received power of $\frac{E_s}{N_0} = \gamma$, regardless of the number of receiving antennas, E_s is the total energy as received from one transmitted symbol and $N_0/2$ is the one sided power spectral density. All the users encode their information bits $u_{k,n}$ with the same Low density Parity Check (LDPC) code and then apply independent random bit interleavering before modulating QPSK symbols. We use LDPC block code with length N and rate R and therefore each transmitted block consists of $t = 1, \ldots, \frac{N}{2}$ QPSK symbols and $\frac{E_b}{N_0} = \frac{E_s}{N_0}\frac{1}{\log_2(4)R} = \frac{\gamma}{2R}$. The MIMO transmitter is depicted in figure 1.

B. The Receiver

The received signal is detected by a multiuser detector (MUD) which produces soft information about the individually coded bits of the K users. These are passed to the K (or K_u) LDPC decoders. The soft outputs of the single user decoders are fed back to the MUD, which improves its outputs along the iterations, see figure 2.

1) The multi user detector: Recall from equation (1) that the inputs to the receiver are $\boldsymbol{y_t}, \{\boldsymbol{h}_k\}_{k=1}^K, \{\theta_{k,t}\}_{k=1}^K, \gamma$ and the inputs to the multiuser detector in the iterative scheme are $\boldsymbol{y_t}, \{\boldsymbol{h}_k\}_{k=1}^K, \{\theta_{k,t}\}_{k=1}^K, \gamma, \hat{\boldsymbol{x}}$, where $\hat{\boldsymbol{x}}$ stands for the soft estimations of the transmitted symbols as obtained from the SISO decoders at the previous iteration. The LLR for the n-th bit of the k-th user codeword can be calculated by the non-linear and highly complex individually optimal multi user detector (IO-MUD) ([22] and [23]), that ignores any code structure. If $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$ and

$\Theta = \mathrm{diag}(\theta_{1,n}, \ldots, \theta_{K,n})$, the IO-MUD is calculated by:

$$\mathrm{LLR_{k,n}} = \log \frac{P\{u_{k,n}=0|\mathbf{y_t}, \overline{\mathrm{LLR}}_{[k,n]}\}}{P\{u_{k,n}=1|\mathbf{y_t}, \overline{\mathrm{LLR}}_{[k,n]}\}} = \log \frac{\sum_{\mathbf{v}\in\mathcal{U}^0_{k,n}} \exp\left(-\frac{1}{2}|\mathbf{y_t}-\sqrt{\frac{\gamma}{M}}\mathbf{H}\Theta\mathbf{x}(\mathbf{v})|^2+\frac{1}{2}\mathbf{v}_{[k,n]}\overline{\mathrm{LLR}}^T_{[k,n]}\right)}{\sum_{\mathbf{v}\in\mathcal{U}^1_{k,n}} \exp\left(-\frac{1}{2}|\mathbf{y_t}-\sqrt{\frac{\gamma}{M}}\mathbf{H}\Theta\mathbf{x}(\mathbf{v})|^2+\frac{1}{2}\mathbf{v}_{[k,n]}\overline{\mathrm{LLR}}^T_{[k,n]}\right)}. \quad (2)$$

Where $\mathbf{v}_{[k,n]} \in \{+1,-1\}^{2K-1}$ and $\overline{\mathrm{LLR}}_{[k,n]} \in \mathbb{R}^{2K-1}$ represent the $2K-1$ bits that compose the symbols at time t, without the n-th bit of the k-th user, and their log likelihood ratios, respectively. $\mathcal{U}^0_{k,n}, \mathcal{U}^1_{k,n}$ are two vector spaces over $\{0,1\}^{2K}$, spanned by all the $2K-1$ bits relating to the symbols in time index t, where the n-th bit of the k-th user is 0 and 1, respectively, so that $\mathbf{v} \in \{0,1\}^{2K}$, and $\mathbf{x}(\mathbf{v})$ is the transmitted vector of the K symbols that corresponds to $\mathbf{v}$. We define the function $s(k,n) : s(k,n) \in \{0,\ldots,\frac{N}{2}-1\}$, such that $s(k,n)$ represents the interleaving and partitioning of N coded bits into $\frac{N}{2}$ symbols, performed on every transmitter, so that $t = s(k,n)$. The optimal MUD can be approximated by linear filters combined with interference cancellation (IC), which ignore any constellations constraints, are widely used and can be implemented with polynomial complexity with the number of users [12]. Known effective linear multiuser detector is the linear minimum mean square error (LMMSE). For LMMSE filter in the iterative interference cancellation scheme (LMMSE-IC) we define ξ_j to be the estimated power of the cancelled j user:

$$\xi_j = \mathrm{E}|x_j - \hat{x}_j|^2 \quad (3)$$

where $\mathrm{E}(x_j - \hat{x}_j) = 0$[2]. Also $\mathrm{E}(x_k-\hat{x}_k)^*(x_j-\hat{x}_j) = 0,\ \forall\, j \neq k$, since the users are uncoordinated and uncorrelated as long as the cycle-free assumption holds. If we define $\boldsymbol{\Sigma}_k = \boldsymbol{I} + \gamma\sum_{i\neq k}\xi_i \boldsymbol{h}_i\boldsymbol{h}_i^H$ (the covariance matrix of the multiple access interference (MAI) plus the noise) and $\hat{\boldsymbol{y}}_k = \sqrt{\frac{\gamma}{M}}\sum_{i\neq k}\boldsymbol{h}_i e^{j\theta_i}\hat{x}_i$, the estimation of the symbol of the k-user from the LMMSE-IC MUD is [24]:

$$z_k = \frac{\sqrt{\gamma}\boldsymbol{h}_k^H\boldsymbol{\Sigma}_k^{-1}(\boldsymbol{y}-\hat{\boldsymbol{y}}_k)}{\gamma\boldsymbol{h}_k^H\boldsymbol{\Sigma}_k^{-1}\boldsymbol{h}_k}. \quad (4)$$

The symbol estimation is converted to LLRs of the code bits with some simple QPSK de-mapper χ for the gray mapping:

$$(l_{1,k}, l_{2,k}) = \chi(z_k) \triangleq 2\sqrt{2}\mathrm{SIR}(\Re\{z_k\}, \Im\{z_k\}),$$

where $l_{i,k}, i = 1,2$ are the LLRs of the bits that map the symbol which is estimated by z_k and $\mathrm{SIR} = \frac{\gamma}{M}h_k^H\Sigma_k^{-1}h_k$ is the signal to noise ratio in the output of the LMMSE [24]. In this iterative scheme, both the conditional and unconditional LMMSE filters [12], [16] can be applied. The conditional LMMSE filter minimizes the MSE $\mathrm{E}|x_{k,t} - z_{k,t}|^2$ with the per symbol estimated powers of the interferers $\{\xi_{j,t}\}_{j=1,j\neq k}^K$ conditioned on $\{\hat{x}_{j,t}\}_{j=1,j\neq k}^K$, namely $\xi_{j,t} = 1 - |\hat{x}_{j,t}|^2$, while the unconditional LMMSE assumes this estimated power remains approximately constant

[2]E is the expectation operator.

along the transmitted block and estimates it for entire block of the interferer as $\xi_{j,t} = \xi_j = 1 - \frac{1}{N}\sum_{t=1}^{\frac{N}{2}} |\hat{x}_{j,t}|^2$. Thus the unconditional LMMSE requires recalculation of the filter for every user every interferer and every iteration, but not for every symbol, like the conditional LMMSE, which suffer prohibitive complexity even for modest block lengths N. It is seen in the following sections that in terms of complexity performance tradeoff, the conditional LMMSE offers only minor performance improvement over the unconditional LMMSE, for increase of approximately N-fold in the MUD complexity.

2) The Decoder: to decode the LDPC codes, the receiver utilizes the well known BP decoder. Such decoders are often described using a Tanner graph, which is composed of variable nodes and check nodes and of interconnecting edges. The variable nodes for the standard single antenna single user channel are initialized by the channel outputs (which remain constant along the iterations). The multiuser receiver graph, however, is composed of three classes of nodes: the $N \times K$ LDPC variable nodes and $N \times (1-R) \times K$ check nodes that represent the individual codes' parity check equations, including any interleaving and the $\frac{N}{2}$ multiuser detector nodes that iteratively improve the LLRs of the codewords' bits. This is illustrated in figure 3, where two users with regular (2,4) LDPC user code of length 8, are decoded and detected using four multiuser detectors nodes for two users with QPSK modulation, and two LDPC tanner graphs.

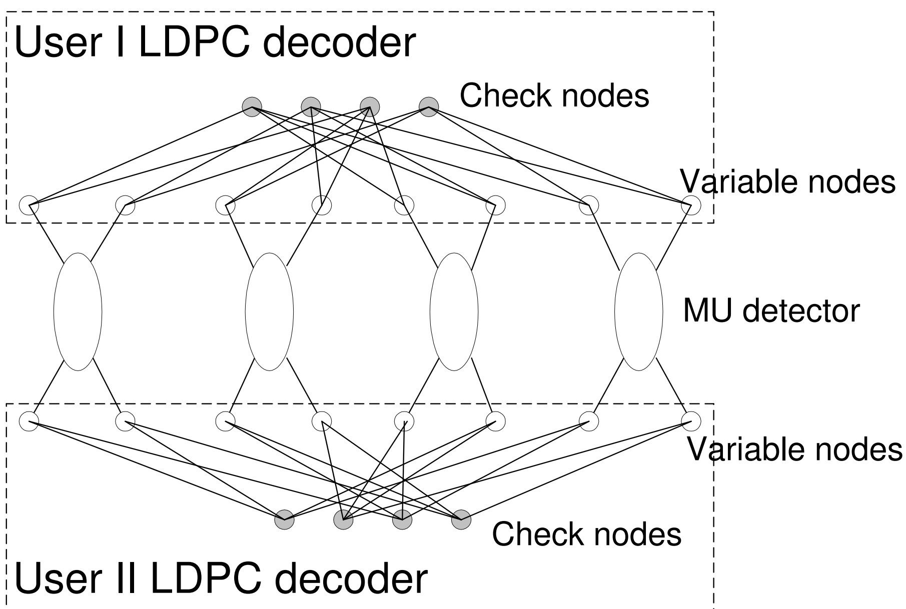

Fig. 3. LDPC BP decoder with multiuser detector scheme

3) Iterative detection and decoding: The receiver steps through 4 phases in each iteration, as shown in figure 4. In the figure, the user interleavers and the LDPC code interleavers are denoted as π_u and π_c, respectively, χ represents the symbol to bit estimations conversion and the plus signs represent the check nodes, when they are enclosed by squares and variable nodes when they are enclosed by circles. At first iteration the multi user detector estimates $K \times \frac{N}{2}$ symbols without any a priori information from the SISO decoders, which means that it acts as detector in separate detection-decoding scheme. In phase (1), all the likelihood ratios, calculated from these symbol estimations are simultaneously forwarded to all the respective variable nodes of all the users' decoders. These variable nodes, then send messages to the check nodes, which replay back (phases (2) and (3)). The messages from variable to check and from check to variable nodes are denoted as VC and CV respectively. The replied CV messages are added at each variable node, to provide the multiuser detector with independent estimations of the symbols for the next iteration (phase (4)). These CV messages represent the present extrinsic decoder estimations of the corresponding bits. The CV messages are stored during the first and fourth phases, so the variable nodes can sum the extrinsic information both from the MUD and the check nodes.

The proposed receiver uses parallel scheduling scheme, so that all the users are simultaneously detected, decoded and then subtracted in every iteration. It is different than e.g. BLAST techniques which sequentially detect, decode and subtract user after user. We search for good codes, so that the LDPC decoder would iteratively improve the multiuser detector's a priori inputs ($\hat{x}$) and reach low bit error rate.

System with IO-MUD (equation (2)) does not require the symbol to bit χ and bit to symbol estimation conversion, since the IO-MUD incorporates the constellation information and outputs the corresponding LLRs of the bits.

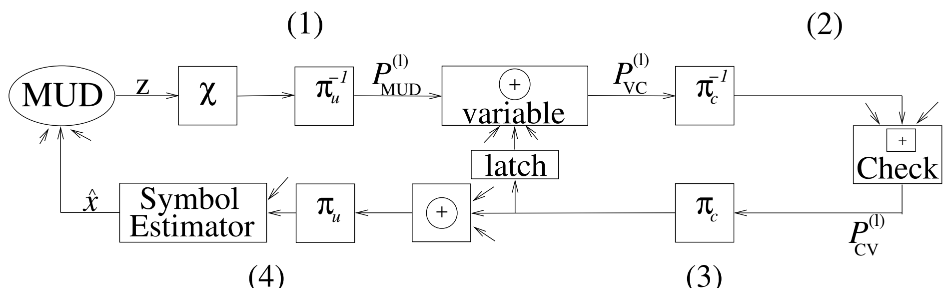

Fig. 4. Illustration of the four phases in each receiver's iteration

III. Asymptotic Analysis

We follow [12] and [22] and use asymptotic systems analysis, where the code length, number of users and number of antennas is taken to infinity, while the channel load remains fixed. Such analysis, which considers the statistics of the channel, gives rise to a cycle free decoder graph for trellis code, to concentration theorems and also to elegant analytical expressions [12] when using the linear MUD. It is known from [22] that in such realm, the outputs of both the linear filters and the IO-MUDs converge to Gaussian random variables. Thus the multiuser efficiency η [3], defined as the output SINR from the multiuser detector divided by the SNR of the user, without the interferers, is sufficient to describe the density.

In order to verify convergence of the MUD's outputs, so the concentration and convergence theorems of [12] hold, in system that uses LDPC code, first we have to ratify that the empirical distribution of the estimated interfering symbols $\hat{x}$, calculated from the SISO outputs $\mathbf{v}$, converges to some limiting distribution for finite number of iterations. Unlike trellis codes, for which [12] proved convergence of the SISO decoders' estimations, the outputs $\mathbf{v}$ of LDPC BP decoder are time dependent, so one might suspect that since $N \to \infty$, and $K \to \infty$, a situation where the SISO's symbol estimation has no limiting density can happen. Since we consider random LDPC graph with random user bit interleavers π_u (if we independently generate random LDPC code for each user, convergence is assured even without π_u) this is not the case. To see that, randomly choose time index t and examine the connected SISO outputs $\{v_{k,n} : s(k,n) = t\}$ over the LDPC graph ensemble. The density of these outputs is actually mixture of the densities of some function of the CV messages [18] (the asymptotic random LDPC graph assure convergence of the CV messages' densities to the ensemble average, as shown by the concentration theorems proved in [18] and [12]). These CV messages, transmitted on the randomly picked LDPC graph edges, are assumed to be i.i.d. due to the cycle free assumption and their probability is assumed to converge to some limiting probability law $P(v)$ [18]. Specifically, if v is the SISO extrinsic output originating from the v variable node, v_{max} is the maximum variable degree and CV_j is the message from the check node which is connected in the j-th variable node's socket, it's probability averaged over the graph ensemble, when the graph is taken to infinity, given the all zero word is transmitted [12] can be written as:

$$\begin{aligned}\Pr(v \le V) = \sum_{dv=1}^{v_{max}} & \Pr(\text{v is with degree } dv) \cdot \\ & \Pr\left(\tanh\left(\frac{\sum_{j=1}^{dv} \mathrm{CV}_j}{2}\right) \le V \right),\end{aligned} \tag{5}$$

and we recall that $v_{max} << N$, so that in the asymptotic realm, the density of the estimation of the bit converges to some limiting density. Note that this conclusion remains true as long as v_{max} is finite, so we assume LDPC codes, with finite connectivity.

Since the estimations of the interferers $\hat{x}_k$ at time t are calculated from $\{v_{k,n} : s(k,n) = t\}$, its density converges to some density and the cumulative distribution of residues of the interferers ξ_k also converges to some limiting cumulative density $F_\xi(\xi)$, which can be directly expressed using the probability density of the SISO decoder extrinsic output $p(v)$. The cycle-free assumption, which is necessary for the variable to MUD messages v i.i.d assumption, should also be verified, to ensure the convergence of densities of the output of the MUDs. For that we can use the single-user decoder cycle-free proof of [18] and prove that the random multiuser LDPC graph converges to cycle free graph as $N \to \infty$ as long as K is bounded. To see that, one just needs to assume (finite) K and variable connectivity v_{max}. Since each MUD is connected to all $2K$ variable nodes, the proof is complete by considering the respective single user LDPC-BP graph (which is proved by [18]) with maximum variable degree of $2Kv_{max}$ and rate of $1 - \frac{cK}{N}$ (c is the number of check nodes in each block). Notice that this indicates systems with $K << N$, which is the case in many of the communication systems.

A. Analysis of the LDPC SISO decoder

The limiting density of the asymptotic LDPC decoder's output $p(v)$ can be determined by the density evolution procedure [18].

In order to get numerical results, we assume without any loss of generality that the transmitted codeword is the all zero codeword and use the technique that was suggested by [19] to calculate densities that are defined over discrete and finite space. This way, the calculation of the densities of the CV and VC messages, along the iterations, can be efficiently performed by FFT for the VC and by using some lookup table for the CV densities. The calculated densities are, in fact, the limiting densities of the messages, while $N \to \infty$ and while the cycle free assumption holds.

B. Optimal multiuser detector analysis

Asymptotic analysis of the MMSE detector, and the cost of using such detector in separated scheme, is nicely presented by Verdú and Guo in [22], where it is shown that the asymptotic optimal MUD's output is gaussian RV with variance and mean that can be calculated via the replica method. However, [22] refers to the case where no a priori information is available to the detector, assumption that is no longer true in iterative schemes. Tanaka, Caire and Müller in [23], extend the analysis to the case where the MMSE detectors use a priori information. We use their analysis in our iterative scheme to calculate the limiting density of the IO-MUD's output (that is gaussian). From [23], the asymptotic achievable multiuser efficiency η, of IO-MUD for real valued system (such as real valued CDMA) is the solution of the following fixed point equation:

$$\frac{1}{\eta} = 1 + \alpha\gamma \mathrm{E}_t[(1-t^2)\int \frac{1-\tanh(z\sqrt{\eta\gamma}+\eta\gamma)}{1-t^2\tanh^2(z\sqrt{\eta\gamma}+\eta\gamma)} Dz], \tag{6}$$

where $t = \tanh(\frac{\mathrm{LLR}}{2})$, and $Dz = \frac{\exp(-\frac{z^2}{2})}{\sqrt{2\pi}} dz$. Following the Gaussian symbol case, we do assume that the result here, which originally developed for the real case, extends to the complex channel.

C. Linear multiuser detector analysis

For the linear MMSE in IC schemes, we can use the results of Boutros and Caire in [12] which include a closed form expressions for the filter's output descriptive statistics, as a functional of the density of the estimated symbol. Although the results of [12] were done for multi access system where the users use trellis codes it carries over for LDPC codes as well (unisotropy degree). These elegant and useful results rely on the fact [15] that when dealing with random matrices in the asymptotic regime, the resulting SIR converges to a deterministic limit. If the conditions defined in [15] are fulfilled, the resulting asymptotic multiuser efficiencies η of LMMSE filter converges (weakly) to a deterministic value, as $K \to \infty$. This value is the solution of the following fixed point equation:

$$\eta = \frac{1}{1 + \alpha \int_0^\infty \frac{\xi}{1+\xi\eta} dF_\xi(\xi)}, \tag{7}$$

where $F_\xi(\xi)$ is the limiting cumulative probability function of mean power of the users: $F_\xi(\xi) = \lim_{K\to\infty} \frac{1}{K}\sum_{j=1}^{K} \mathbf{u}(\xi - \xi_j)$[3]. This asymptotic result stems from the asymptotic distribution of the eigenvalues of the channel matrices (specifically, the eigenvalues of $\sum_{k=1}^{K} \boldsymbol{h}_{k,t}^H \boldsymbol{h}_{k,t}$), which is robust to the specific distribution of the matrix elements,

[3] $\mathbf{u}(x)$ is the indicator function, that equals one for $x \geq 0$ and 0 otherwise.

and remains true for complex valued H.

Notice that from the asymptotic analysis point of view, the only difference between the LMMSE-IC and the IO-MUD is in the resulting η. Where for the LMMSE, it is the solution of (7), and for the optimal detector, it is the solution of (6). For the conditional LMMSE $F_\xi(\xi)$ is directly calculated from the output of the SISO decoder, that changes for every iteration, every symbol and every user and for the unconditional LMMSE, $F_\xi(\xi) = \mathbf{u}(\xi - \bar{\xi})$, where $\bar{\xi}$ is the block-wise average of the residues since the unconditional filter remains constant for all the symbols of the same user at the same iteration. Since the integral over ξ reduces to a simple expression, the fixed point of (7) for the unconditional LMMSE is explicitly written in equation (8):

$$\eta = \frac{2}{1 + (\alpha - 1)\bar{\xi} + \sqrt{4\bar{\xi} + (1 + (\alpha - 1)\bar{\xi})^2}}. \tag{8}$$

It is noticed that conditional LMMSE's performance will never be worse than performance of unconditional LMMSE, because of the convexity of $\frac{\xi}{1+\xi\eta}$ in equation (7). The conditional LMMSE complexity is prohibitive since it requires matrix inversion for every symbol of every interferer of every user on every iteration, whereas the unconditional LMMSE require such inversion once per interferer per user per iteration (factor N in the required complexity).

In a similar way, using results of [15] and [12], the SIR at the output of multiuser detector which uses single user matched filter (SUMF) with IC converges to:

$$\eta = \frac{1}{1 + \alpha\bar{\xi}}. \tag{9}$$

Figure 5 demonstrates the differences between these asymptotic multiuser efficiencies, as function of the power γ. It can be seen that the LMMSE's asymptotic multiuser efficiency is significantly better than that of the SUMF and is only slightly degraded compared to that of the IO-MUD.

D. Gaussian approximation of the density evolution

Although the density evolution predicts the actual performance very well, it also requires significant amount of computations, since it uses quantization of real valued continuous functions. Less complex analysis, which assumes that messages are distributed according to Gaussian densities and therefore iterates only single parameter, is the Gaussian approximation technique [25]. The optimization process uses this approximation due to it simplicity, leaving the density evolution only for more accurate evaluations of the optimization products. Although the approximation is motivated by the central limit theorem for the variable nodes, it is close enough, in most cases, to the results of the density evolution. We modified the technique of [25] for the multi access receiver, where the LLRs originate from the MUD instead of from the memoryless channel and therefore change along the iterations. The Gaussian analysis considers the messages from the check nodes to the variable nodes (CV), when the all zero codeword is transmitted. Since both CV and VC are Gaussian, only the mean values m_{CV} and m_{VC} are used by the analysis. If $m_{\mathrm{CV}}^{(l)}$ is the mean of the messages that are sent from the check nodes to the variable node at the l-th iteration then $m_{\mathrm{CV}}^{(l+1)}$ of the next iteration is calculated by equation (10). The asymptotic multiuser efficiency of the

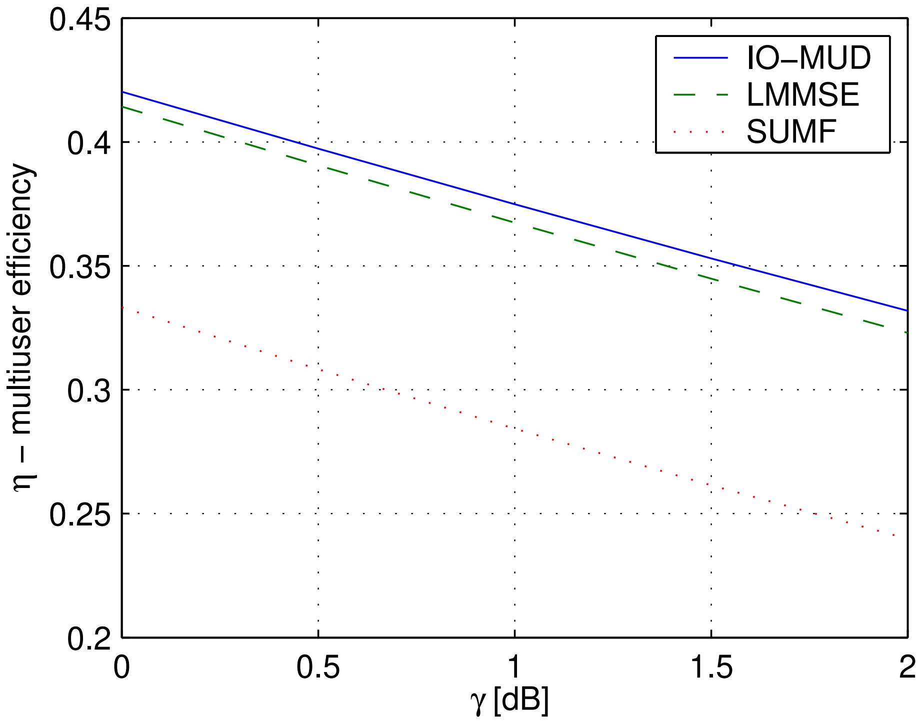

Fig. 5. The asymptotic multiuser efficiencies, for IO-MUD, LMMSE and SUMF detectors, all for channel load of $\alpha = 2$, and various values of γ. No a priori information is assumed.

MUD (η) is represented by ψ_{MUD} in equation (10), where ψ_{MUD} include the various functionals (e.g. equations (5),(6),(8)) for calculating η given the density of VC messages.

$$\begin{aligned} m_{\mathrm{VC}}^{(l)} &= 2\gamma\psi_{\mathrm{MUD}}(m_{\mathrm{CV}}^{(l)}) + (i-1)m_{\mathrm{CV}}^{(l)} \\ m_{\mathrm{CV}}^{(l+1)} &= \sum_{j=2}^{c_{max}} \rho_j \phi^{-1}\left(1 - \left[1 - \sum_{i=2}^{v_{max}} \lambda_i \phi(m_{\mathrm{VC}}^{(l)})\right]^{j-1}\right) \end{aligned} \qquad (10)$$

Where $\phi(x)$ is defined as in [25] as

$$\phi(x) = \begin{cases} 1 - \frac{1}{\sqrt{4pix}} \int_{\mathbb{R}} \tanh(\frac{u}{2}) e^{-\frac{(u-x)^2}{4x}} du & \text{if } x > 0 \\ 1 & \text{if } x = 0. \end{cases}$$

E. Capacity

The capacity for the single user MIMO channel, when CSI is known at the receiver but unknown to the transmitter, is achieved when coded independent information is sent with the same power and the same code rate from all the transmitting antennas ([1]). Thus this single user capacity is equal to the multi access channel's sum rate capacity and if we will find good multi access scheme it can be used for the single user case as well. The asymptotic sum rate capacity of multiuser system constrained to BPSK is calculated in [22]. It was originally derived by Tanaka in [4] assuming the replica method was available and [22] extended the results for the capacity when using linear detectors (non-iteratively). If the transmitted symbols are BPSK with the amplitude of $\sqrt{\gamma}$ and the channel transfer matrix has identically, independently, Gaussian distributed (i.i.d.) entries, then as $M, K \to \infty$ with $\frac{K}{M} = \alpha$, the resulting asymptotic multiuser efficiency η of individual optimal detector is the solution of the following fixed point equation:

$$\frac{1}{\eta} = 1 + \alpha\gamma\Big[1 - \sqrt{\frac{\eta\gamma}{2\pi}}\int_{-\infty}^{\infty}\tanh(\eta\gamma x)\exp\Big(-\frac{\eta\gamma(x-1)^2}{2}\Big)\mathrm{d}x\Big], \tag{11}$$

where the additive white Gaussian noise has variance of 1 and zero mean as before. The capacity is then calculated with:

$$\begin{aligned} C = \eta\gamma + &\frac{\eta - 1 - \log(\eta)}{2\alpha} \\ &- \sqrt{\frac{\eta\gamma}{2\pi}}\int_{-\infty}^{\infty}\log\Big(\cosh(\eta\gamma x)\Big)\exp\Big(-\frac{\eta\gamma(x-1)^2}{2}\Big)\mathrm{d}x. \end{aligned} \tag{12}$$

If (11) has more than single fixed point solution, the correct one is the one that results in the smallest capacity. Note that this capacity is computed when the signaling is assumed to be BPSK. This capacity result and the optimal asymptotic MIMO capacity results obtained by Emre Telatar in [1] are used to assess the information theoretic loss, at least for the asymptotic system, when we constrain ourselves to QPSK signaling. We assume that the asymptotic capacity of complex QPSK system is exactly twice the asymptotic capacity of the BPSK, real valued channel system [22], albeit we do not prove it. Viewing figure 6, which depicts the Gaussian optimal MIMO and the (conjectured) QPSK asymptotic capacities, we conclude that the asymptotic capacity using QPSK signaling is very close to the

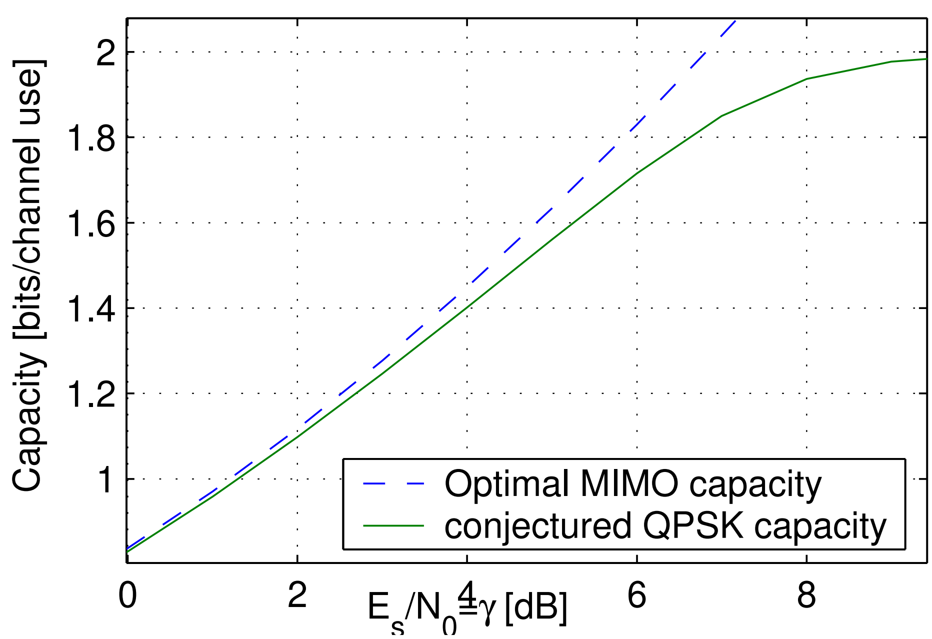

Fig. 6. The (conjectured) Loss due to the use of QPSK instead of optimal constellation in MIMO scheme, for the asymptotic case with channel load of $\alpha = 1$

optimal asymptotic capacity in the lower SNR region. Verdú showed that such QPSK signalling is second order optimal in [26], further substantiating our conjecture that larger constellations do not provide meaningful gain at the lower SNR region. It is also consistent with the intuition presented in [27], where it is expected that as the SNR gets smaller and the channel load α larger, the loss inflicted by the use of small constellation is reduced.

IV. Code search results

In this section we present the search results for good LDPC degree distribution pairs (DDPs) [18]. The problem of finding good codes is global optimization problem, that maximizes the rate of the code R under the constraint of asymptotic error free decoding for given γ and α.

A. Search algorithm

The search for globally good codes is performed by stochastic genetic algorithm that is known as differential evolution. This algorithm is used also by [18] and [21] for optimizing LDPC codes. Briefly speaking, this algorithm starts off with an initialization stage and then iteratively repeats two stages, until convergence is achieved [28]. In the initialization stage, initial DDPs population is randomly chosen according to the uniform distribution over the constrained space. Notice that this leaves the code rate as the objective parameter (unlike [18] and [21]). The following stage consists of randomly altering each DDP in the population by generating random perturbations from randomly selected pairs or quadruplets of DDPs [28]. In the second stage, the population undergo natural selection which leaves either the DDPs or their altered versions, depending on their rate and threshold. The main idea is that the population gets more homogeneous and have higher rates as the iterations progress [28].

B. Search results

Good pairs are found for five channel loads: α =0.2,0.5,1,1.5 and α =2, each with four threshold constraints: E_s/N_0 =1,2,3 and 4 dB. The DDPs were obtained while limiting the search space by constraining the check and variable degrees. The use of QPSK signalling is quite sufficient for the lower powered schemes (figure 6). Notice that the number of antennas at the receiver and the MUD's complexity are reduced by the usage of high channel loads. Table I presents the results of the search process as the gap between the achieved asymptotic thresholds and the asymptotic capacities, as presented in subsection III-E. Figure 7 plots the designed systems as markers, where the markers vary according to the thresholds and are depicted as function of the channel load α. The respective capacities with QPSK signalling are also drawn for comparison. The loss due to the QPSK restriction can be appreciated by comparing this capacity to the optimal MIMO capacity, as shown in figure 6 and we notice a gap of 0.05 dB for load $\alpha = 2$ and $\gamma = 4$ dB. The scheme reaches up to 0.3 dB from the channel capacity for α =0.2 and gets as close as 0.77 dB away from the channel capacity for α =2. We compared the code that was designed for channel load of 2 to LDPC code that was designed for AWGN channel (from [18]). The differences between the thresholds of these codes and the channel capacity are shown in table II for both multi access and single user and single antenna channels. Notice that although the AWGN-code performs well on the AWGN, it's performance

TABLE I

DISTANCE TO CAPACITY IN [dB] OF SEARCH RESULTS FOR UNCONDITIONAL LMMSE, OBTAINED WITH THE DIFFERENTIAL EVOLUTION WITH CONSTRAINT UTILIZING GAUSSIAN APPROXIMATION

E_s/N_0	1 dB	2 dB	3 dB	4 dB
$\alpha = 0.2$	0.33	0.31	0.38	0.40
$\alpha = 0.5$	0.44	0.44	0.55	0.63
$\alpha = 1$	0.66	0.69	0.8	1
$\alpha = 1.5$	0.77	1.06	1.09	1.26
$\alpha = 2$	0.77	1.16	1.35	1.69

TABLE II

DISTANCES TO CAPACITY OF THE CODE DESIGNED FOR MULTI ACCESS AND FOR A CODE OPTIMIZED FOR THE AWGN ON BOTH SCENARIOS

	distance to capacity on AWGN [dB]	distance to capacity on multi access [dB], α =2
multi access code R=0.352	2	0.77
AWGN code R=0.5	0.2	9

on the multi access channel is poor (even when considering the fact that it has higher rate). In contrast, the code, that was designed for the multi access channel, performed reasonably well on the AWGN channel (difference of 2 dB). This emphasizes the importance of including the multiuser analysis in the search for good codes.

The gaps to the channel capacity for high channel loads, as appear in table I, are mainly due to the limitations of the linear MUD. To confirm this conclusion, we searched for good DDPs for iterative schemes which include the conditional LMMSE-IC and the IO-MUD. The distances between the thresholds of these systems to the capacity, with channel load α=2 are presented in table III. We see insignificant improvement of 0.1 dB for the conditional LMMSE over the unconditional LMMSE and much more significant 0.63 dB improvement for the IO-MUD. This exemplifies the linear detectors limitations, especially for high channel loads.

TABLE III

DISTANCE TO CAPACITY, AS ACHIEVED WITH IO-MUD, CONDITIONAL LMMSE AND UNCONDITIONAL LMMSE, FOR α=2 AND THRESHOLD OF $\gamma = 1$ dB.

	unconditional LMMSE	conditional LMMSE	IO-MUD (MMSE)
distance [dB]	0.77	0.67	0.14

V. Simulation results

The system was simulated to verify the analysis and to assess performance with finite number of users. All the simulations were performed with channel load of $\alpha = 2$ and $\alpha = 1.875$. The LDPC code was constructed according to the degree distribution pair that was found by the asymptotic analysis for threshold of $E_s/N_0 = 1$ dB and for channel load of α =2. The resultant code rate is $R = 0.352$.

A. Comparison of the asymptotic analysis to the simulations of finite systems

The asymptotic analysis expectation and simulation results are drawn in figure 8 as function of the iterations. These simulation were done for multiuser MIMO system with K =200 users ($\alpha = 2$) and $\gamma = E_s/N_0$ =1.45 dB (E_b/N_0 =2.98 dB). It can be seen that the density evolution predicts the performance of the system well, until about the 20-th iteration. The simulations will probably agree with the asymptotic density evolution prediction along more iterations if the codeword length N will be increased.

B. Finite LMMSE

Separated simulations of LMMSE filter, designed for CDMA spreading sequences or for random MIMO channel attenuations can assess the difference between the asymptotic and the average MSE (mean square error) at the output of finite filter. It also indicates how many users such system should include to approach it's asymptotic performance. These simulations test transmission of un-coded bits through MIMO or random non-fading CDMA channels. The receiver uses LMMSE filter where both the power of the users and the channel transfer coefficients are known. In the following, we compare the average MSE of the LMMSE output to the asymptotic MSE as calculated from equation (8) for different filter lengths and channel statistics. Notice that according to [29], the asymptotic behavior of these two channels is identical but the convergence rates depend on higher order statistics. Figure 9 plots these differences in dB, for systems with channel load $\alpha = 2$ and $\gamma = 1.2$ dB as function of the filter's size. It is noticed that the difference to the asymptotic MSE is about 0.35 dB, for MIMO system with as few as 10 users (5 receive antennas) and 0.2 dB for non-fading CDMA system with 10 users and 5 chips. This difference decreases further with the number of antennas. As expected, the MSE for both MIMO and non-fading CDMA finite filters converges to the asymptotic MSE with different rates, although these rates are quite close. These results demonstrate the effect of the finite filter on the MSE, excluding any effects of the feedback from the decoders.

C. MIMO versus non-fading CDMA systems

Figure 10 shows the performances of multiuser MIMO system versus non-fading CDMA system, for both K =15 and K =200 users. It draws the average BER from simulations of these systems, for various E_b/N_0, for channel load α =2 (K=200)and α =1.875 (K =15). The receiver iterated the decoding-detection cycle until no further improvements were noticed, so that the graph represents the performance after enough receiver iterations. We can

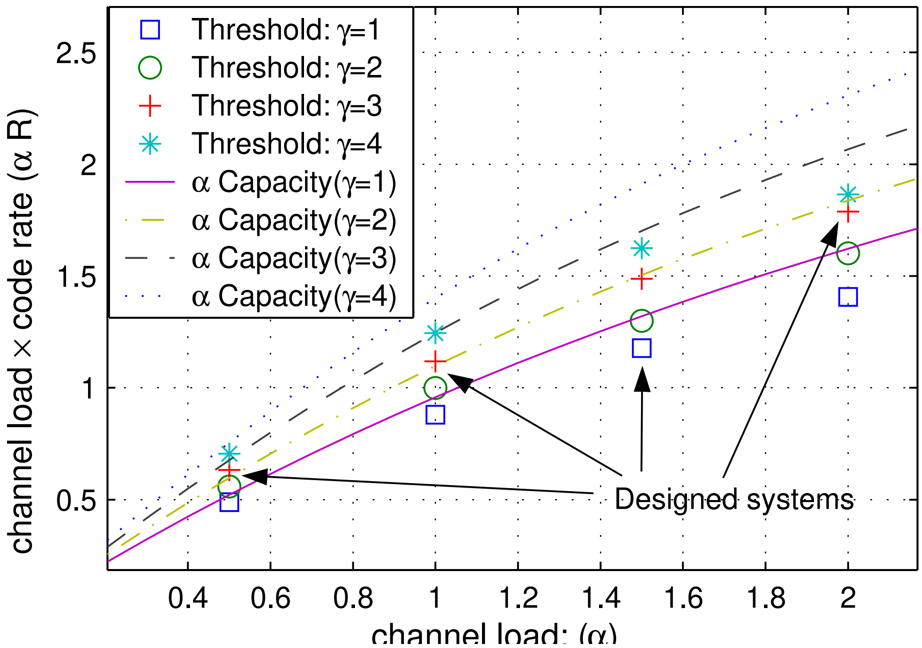

Fig. 7. Thruput per receiver antenna of the designed systems and the respective capacities curves. $E_s/N_0 = \gamma$ are expressed in dB

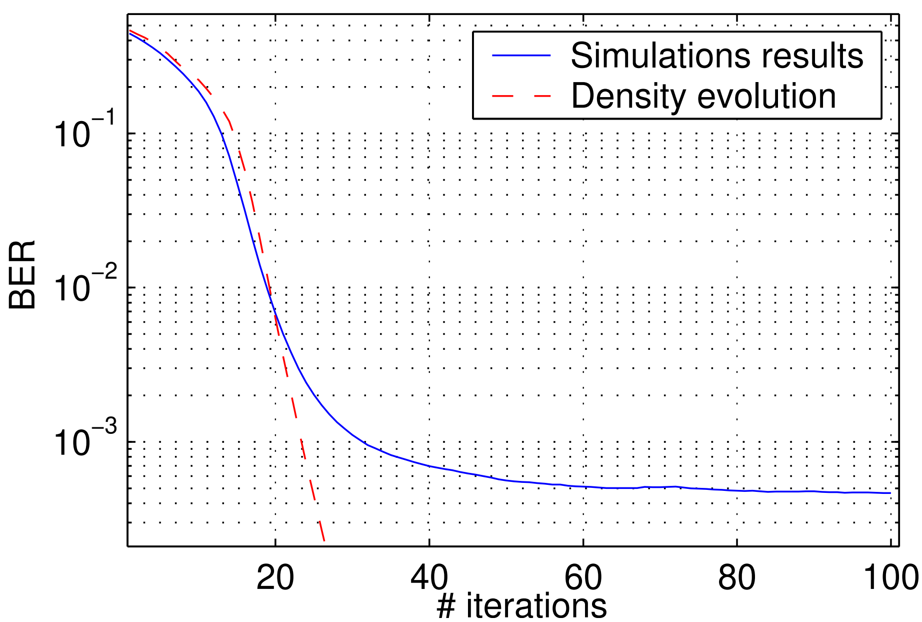

Fig. 8. Simulation results versus density evolution of the check to variable messages, of MIMO system with K =200 users, that use LDPC code with $R = 0.352$, and with a channel load α =2, E_s/N_0 =1.45 dB (E_b/N_0 =2.98 dB), as function of iterations

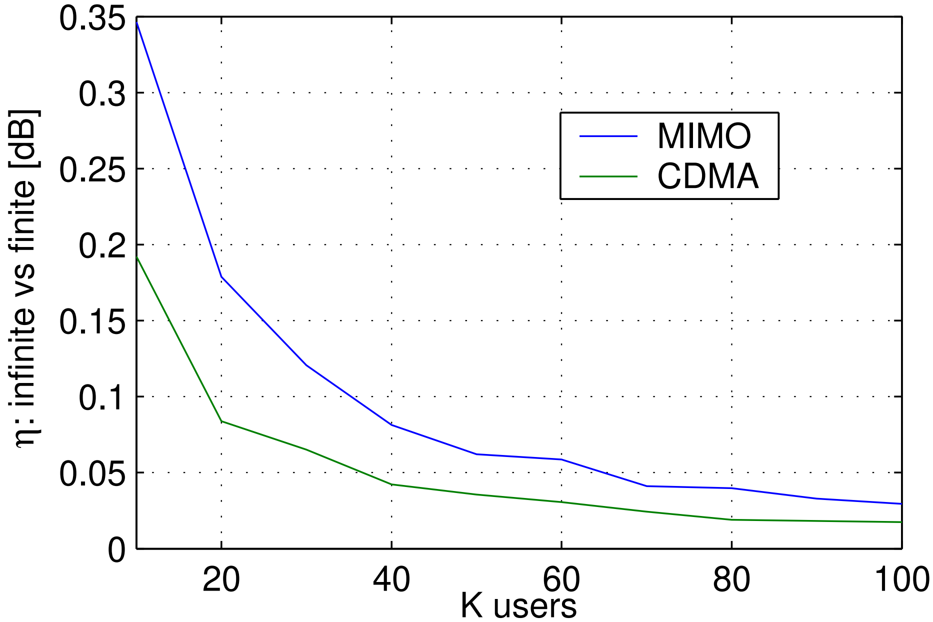

Fig. 9. Differences, in dB, of finite LMMSE filter to the asymptotic MSE, for $E_s/N_0 = 1.2$ dB and $\alpha = 2$, with both non fading CDMA and MIMO systems as function of the number of users K

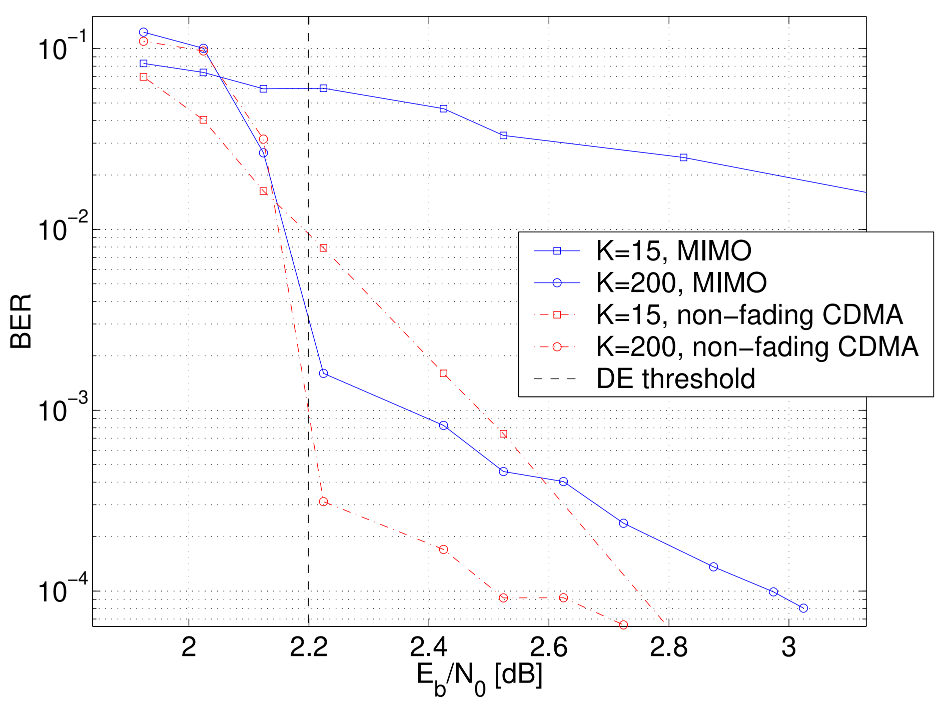

Fig. 10. Simulation results of multiuser MIMO versus non-fading CDMA systems, with K =15 and 200 users, with channel load α =2 and α =1.875, respectively, after sufficient number of iterations, as function of E_b/N_0. Theoretical threshold is presented, for comparison.

see that in both MIMO and CDMA systems, the more dimensions we operate in (diversity), the better the BER performance we get. So that system with only $K = 15$ users is worse than that of $K = 200$ users, in the higher SNR region ($E_b/N_0 > 2.12$ dB). We note that reducing the number of users in the very low SNR region helps to improve the BER performance. Notice, also, that while the non-fading CDMA system performs quite well, even with only K =15 users, the performance of MIMO system with the same number of users and the same channel loading is severely degraded. When increasing the dimensionality to $K = 200$ users, the MIMO and the non-fading CDMA performances are close, especially in the waterfall region. The difference between them is in the higher SNR region, where the non-fading CDMA system performs better. The fading effect, as was seen in subsection V-B, is evident in figure 10, when comparing the multiuser non-fading CDMA system curve to the MIMO curve, both with K =15 and K =200, where with K =200 users, the spacial diversity overcomes the fading effect.
The very poor performance of the multiuser MIMO system with K=15 is attributed to the outage probability of the MIMO channel. Since we deal with independent codes on block fading channel, severely faded block can impair the user's code ability to correct any errors and therefore also deny any possibility for reducing the MAI of other users with, possibly better channels.

D. Single user versus Multiple user MIMO block fading

The multiple access MIMO system simulation approximated the asymptotic analysis prediction with well with $K = 200$ users (although sufficient performance was achieved with only $K = 50$ users). That means using $M = 100$ antennas in the receiver. Significantly less users, such as $K = 15$ with $\alpha = 1.875$ results in severe degradation in the performance. Single user MIMO system, however, can utilize single LDPC code with multiplexer to transmit from all the antennas. The corresponding receiver uses single decoder that will benefit from much more diverse channel (factor of K) than the multiple access decoders, so that the probability of all the entries of the channel matrix being very weak is very low. This can be seen in figure 11, where we simulated a single user and multiple user MIMO systems with code lengths of $N = 15 \times 10^4$ and $N = 10^4$, respectively and with $K = 15$ and $K = 200$ transmitting antennas. It is clear that the single user systems outperform the respective multiuser systems. Single user MIMO system with $K = 200$ reaches BER of 10^{-4} within 0.2 dB from the predicted density evolution threshold, and within 1 dB from the channel capacity. It is comparable to the excellent performance reported in [7] for system which includes sphere decoder, except that our proposed system operates at higher channel load, require less complex receiver but uses more antennas. It is evident that in our setting more users are required to achieve the asymptotic performance than the number required to achieve the ergodic single user capacity [1]. This is a consequence of our block fading model which prevents coding over differently faded symbols, thus causing erroneous blocks for small number of users.

E. Effect of coefficients estimation error

The above simulation results are obtained when the receiver has full knowledge of the channel coefficients. In real communication systems, however, such perfect knowledge is rare and more realistic model should consider some imperfections in the estimations of the coefficients. This imperfection can be modelled by white additive noise vector $\mathbf{e_{t,k}}$, so that $(\mathbf{y_t}, \{\mathbf{h_k} + \sqrt{\epsilon}\mathbf{e_{t,k}}\}_{k=1}^{K})$ is known to the receiver, where $\{\mathbf{e_{t,k}}\}_{k=1}^{K}, \{\mathbf{h_k}\}_{k=1}^{K} \sim \mathcal{N}(0, I)$, ϵ is some constant and where $\mathbf{y}$ still follows equation (1). The simulation results of this scenario are drawn in figure 12. Notice that this system is quite sensitive to these estimation errors and less than -10 dB ($\epsilon|_{\mathrm{dB}}$ <-10) is needed for satisfactory performance. This result is understandable since the scheme is not designed for such estimation errors and by including such imperfections in the asymptotic analysis one might improve its resilience. If the additive estimation error $\sqrt{\frac{\gamma\epsilon}{M}}\mathbf{e_k}$ is modelled just as a decrease in the signal power $\gamma \leftarrow \frac{\gamma}{1+\epsilon}$, equivalent simulations with perfect coefficients knowledge can be done which reflect the resilience of designs made for lower thresholds against these imperfections (The thresholds are expressed in terms of the power γ). These are also plotted in figure 12. It is clear that with lower estimation error, the two models agree and with higher estimation errors there seems to be a difference, due to the use of LMMSE in the receiver.

VI. Conclusion

In this paper we described an efficient MIMO communication scheme for both single and multiple users. The users' LDPC code was optimized with asymptotic analysis for the entire iterative receiver, which is a variation of what was presented by Caire and Boutros in [12] for trellis codes. The resulting low complexity scheme performs well for large number of users, and for channel load of $\alpha = 2$, the asymptotic threshold of the system is only 0.77 dB away from the corresponding capacity (evaluated with the replica method). It is also compared to both conditional LMMSE and IO-MUD systems, for the assessment of the cost of using linear filter, which turns out to be less than 0.6 dB. Comparing the performance of the optimized code to that of LDPC code which was optimized for AWGN reveals substantial merit of the modified search. The asymptotic analysis is verified by simulations of finite systems which indicate differences between single and multiple users systems. It is seen that single user system does well enough with only 15 antennas whereas the multiuser system requires considerably more. These differences stem from the non-ergodic nature of the block fading channel with finite number of antennas and can be reduced by using more than one transmitting antenna for each user, or by adopting faster fading model, so that each decoder experiences several attenuations in single block. Future work may include generalization of the model to the cases where no synchronization can be assumed and where several classes of users exist. Another interesting aspect is the case where some feedback channel exist between the receiver and the transmitter. This feedback can substantially improve the overall achievable spectral efficiency of such system by some power profiling technique. Better complexity-performance tradeoff can be achieved by integrating more then one kind of MUD. Such scheme can incorporate sphere decoder and LMMSE or SUMF.

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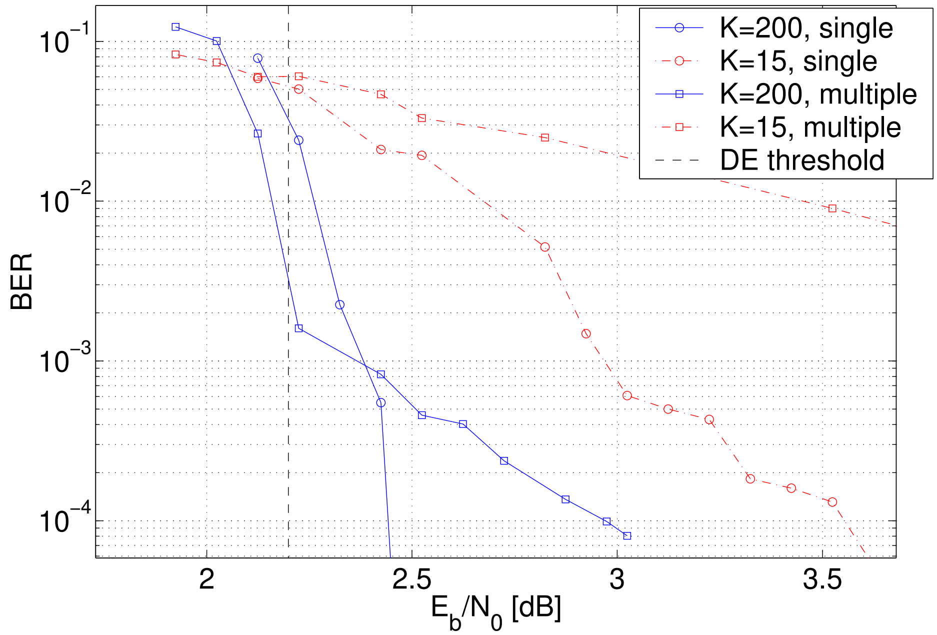

Fig. 11. Simulation results of multiuser vs. single user MIMO systems, with $K = 200$ and $K = 15$ users with channel loads $\alpha = 2$ and $\alpha = 1.875$, where the transmitters use LDPC code with $R = 0.352$ and lengths of $N = 10^4$ and $N = 15 \times 10^4$ respectively. The BER presented was measured after sufficient number of iterations, as function of E_b/N_0. DE threshold is also presented for comparison.

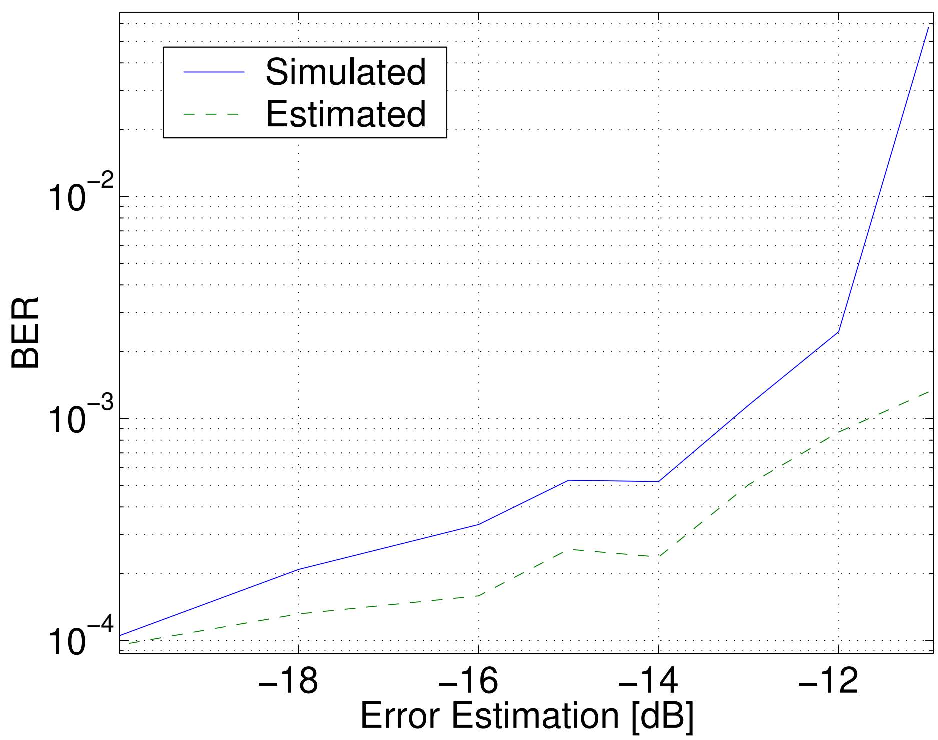

Fig. 12. Simulation results of receiver with estimation errors ϵ. Simulation of perfect estimations system, but with modified signal power $\gamma \leftarrow \frac{\gamma}{1+\epsilon}$ is drawn for comparison.

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