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# Separation of semi reflected scenes using secondary reflections

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## 1. Abstract

An image acquired through a transparent medium (such as a glass window) is a superposition of two images: the scene behind the medium, and a reflection of a scene reflected by the medium. The ability to separate the image from the reflection is important and attaches great interest in the research community. Previous studies focused on the use of multiple images. This research will focus on separating the image from the reflection using a single image, in an automatic way, and with minimum prior knowledge about the scenes. The work will exploit the secondary reflections which occur inside the reflecting medium. The research will include theoretical and numeric aspects as well as experimental validation.

## 2. Problem definition

Light rays of scene I reach transparent medium. Each time light ray reaches either edge of the medium, part of it is reflected towards the medium, and part of it passes out of the medium. The power of the reflected ray is the power of the original ray multiplied by a factor R ($R<1$). The power of the ray that passes out of the medium is the power of the original ray multiplied by a factor T ($T=1-R$). From the other side of the medium, there will be a series of rays with degrading power. The scene $S_1$received in the other side of the original scene $I_1$ is:

$S_1(x) = I_1 ** F_1(x)$

Where

$F_1(x)=(\delta(x)+ R^2*\delta(x-d)+(R^2)^2*\delta(x-2d)+\ldots)$

And d is the distance between two successive rays going out of the medium.

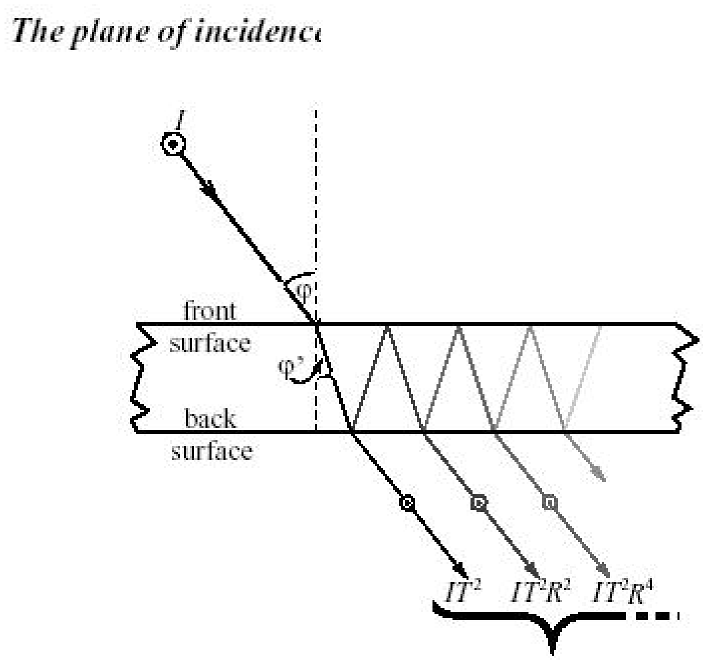


**Figure 1 – light rays passing through the medium**

Similarly, there will be a series of rays reflected from the medium and the image we get is:

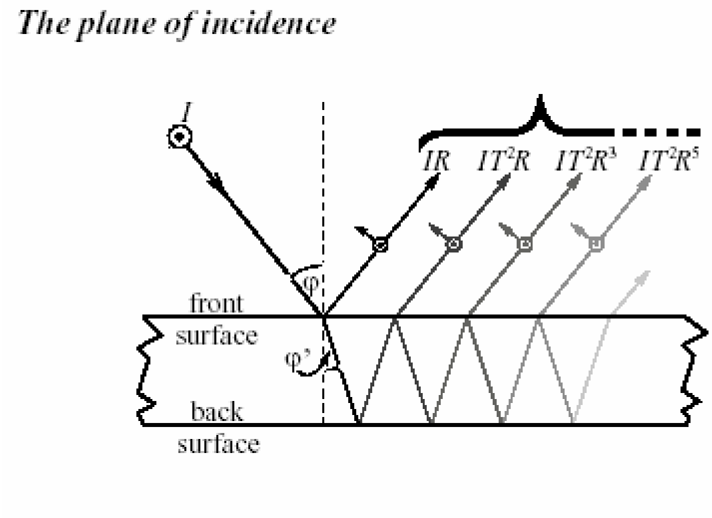


**Figure 2 – light rays reflected from the medium**

$S_2(x)=I^{**}F_2(x)$

where

$F_2(x)=R^*\delta(x)+ T^2R^*\delta(x\text{-}d)+ T^2R^{3*} \delta(x\text{-}2d)+\ldots$

$F_2(x)=R^*( \delta(x)+ T^{2*}\delta(x\text{-}d)+ T^2 R^{2*} \delta(x\text{-}2d)+\ldots)$

When we take a photo of a scene through a medium, we will get a superposition of two scenes: one scene behind the medium ($I_1$) and one scene reflected in the medium ($I_2$). Moreover, the photo we take will not include $I_1$ and $I_2$, but $S_1$ and $S_2$ – the original scenes and their reflections from the medium.

Our goal is to restore $I_1$and $I_2$ (the original scenes) from the photos we take.

## 3. Simplifying the problem

First we will simplify the problem – assume $I_2=0$, i.e. there is no scene reflected in the medium. Furthermore, assume the parameters d (distance between reflecting rays) and R (reflection coefficient) of the problem are known. If this is the case, the photo S we take will be:

$S(x) = I{*}{*}F(x)$

where

$F(x) = (\delta(x)+ R^2*\delta(x-d)+( R^2)^2*\delta(x-2d)+\ldots)$

We look for a function H where

$F{*}{*}H= \delta(x)$.

In Z-plane we get:

$F(z)=1+ R^2*Z\text{^}{-1}+( R^2)\text{^}2*Z\text{^}{-2}+\ldots$

$$F(z)=\sum_{i=0}^{\infty}\left(\frac{R^2}{z}\right)^i=\frac{1}{1-\frac{R^2}{z}}=\frac{z}{z-R^2}$$

$$H(z)=\frac{1}{F(z)}=\frac{z-R^2}{z}=1-R^2z^{-1}$$

$I(z)=S(z){*}{*}H(z)$

And back to spatial plane:

$I=S(x)- R^2*S(x-d)$

Doing so will restore I perfectly from S.

## 4. Evaluating the unknown parameters

In order to restore I we need to know (or to estimate) the parameters d and R.

The following procedure will estimate d with great accuracy. Because the reflections are distance d from each other, there will be relatively large correlation between parts in the image which are d apart from each other. Therefore, we need to calculate the correlation between different parts of the image with different distances from each other, and the maximum correlation will occur where this distance is d.

Tests done in this method show that the best way is to calculate several restorations in the same process mentioned earlier (for the case there is no reflected scene, or $I_2=0$). Then we take derivative of this restoration. We measure the correlation between such restorations for different distances between the parts in the image and different values of d which were taken for the restoration itself. The results will be similar to the plot below (in the plot, "movement" is the distance for the correlation, and "tau" is the value of d taken for the restoration):

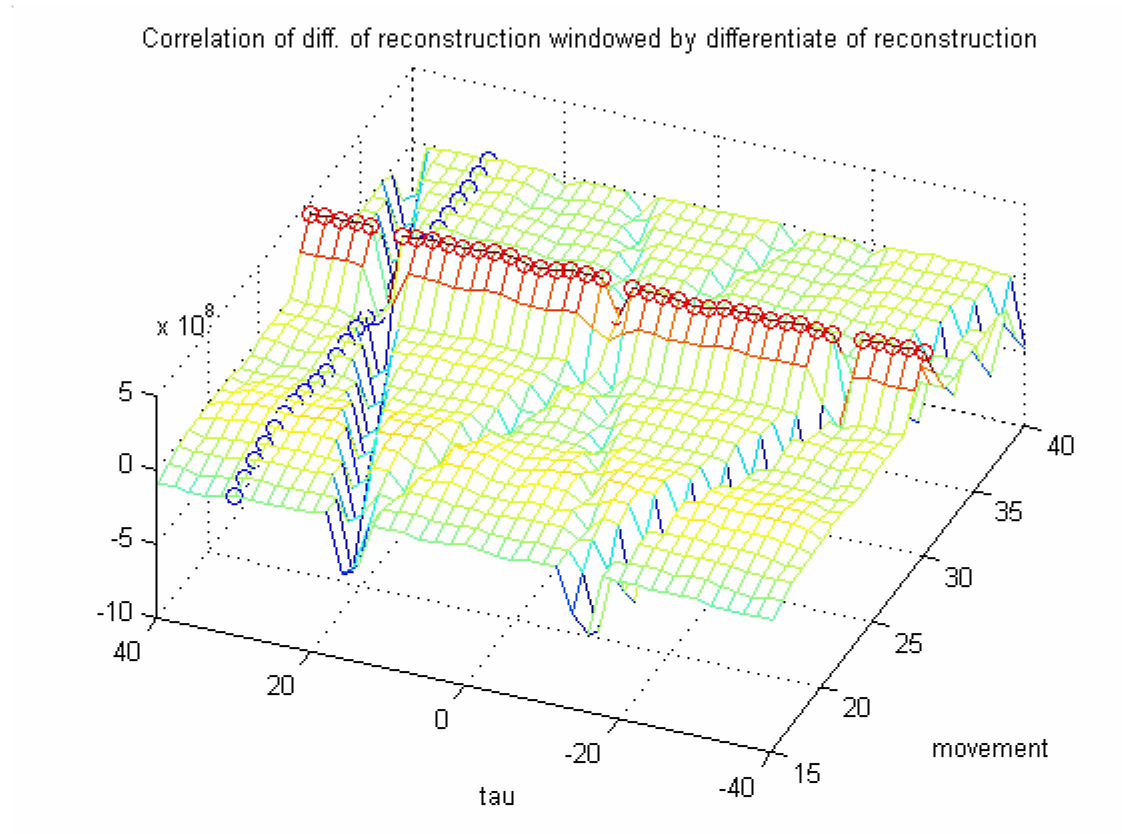


**Figure 3 – correlation of diff. of restored (reconstructed) images**

In the plot, it is clearly seen that the maximum correlation is when "movement"=30, which is the correct d for the case.
It is important to specify that tests show that in order to estimate d correctly we do not need to know the exact value of R. The plot will look similar even if we use different values of R (even values very far from the correct one). This is the reason why we can't estimate R by similar method – the plot will look the same for a wide range of values of R, and accurate estimation of R will be impossible.

For the time, we will leave the problem of estimating R and continue when we assume we know the exact value of R. we will return to this problem later on.

## 5. Using polarizers

In order to solve the original problem of restoring the original scenes, we will take two photos, each with different polarization of light. One photo will be with polarization parallel to plane of incidence, and the other with polarization that is perpendicular to plane of incidence. We will use the indices $_{\amalg}$ and $_{\perp}$ respectively. Polarizers are widely used, for example in [1].

### *5.1. Mathematical model*

We will describe mathematically the relations between the original scenes and the photos we get.

$$S_{\perp}(x) = I_t **F_{t\perp}(x) + I_r **F_{r\perp}(x)$$
$$S_{\amalg}(x) = I_t **F_{t\amalg}(x) + I_r **F_{r\amalg}(x)$$

$S_{\perp}$ - Photo taken with perpendicular polarization.
$S_{\amalg}$ - Photo taken with perpendicular polarization.
$I_t$ – Scene transmitted through medium.
$I_r$– Scene reflected in medium.

$F_{r\perp}$ - Transformation function of $I_r$ with $R_{\perp}$.
$F_{r\amalg}$ - Transformation function of $I_r$ with $R_{\amalg}$.
$F_{t\perp}$ - Transformation function of $I_t$ with $R_t$.
$F_{t\amalg}$ - Transformation function of $I_t$ with $R_{\amalg}$.

### *5.2. Solving the problem mathematically*

All we need to do is to find $I_t$ and $I_r$ from these two equations, given the photos and values of d, $R_{\perp}$ and $R_{\amalg}$.

Suppose we find two functions G which hold:

$$G_{\perp} ** F_{t\perp} = \delta(x)$$
$$G_{II} ** F_{tII} = \delta(x)$$

We convolve each equation with its corresponding G function and get

$$S_{\perp}(x) ** G_{\perp} = I_t ** F_{t\perp}(x) ** G_{\perp} + I_r ** F_{r\perp}(x) ** G_{\perp}$$
$$S_{II}(x) ** G_{II} = I_t ** F_{tII}(x) ** G_{II} + I_r ** F_{rII}(x) ** G_{II}$$

From definition of G, this can be simplified to

$$S_{\perp}(x) ** G_{\perp} = I_t + I_r ** F_{r\perp}(x) ** G_{\perp}$$
$$S_{II}(x) ** G_{II} = I_t + I_r ** F_{rII}(x) ** G_{II}$$

Subtracting the equations one from the other gets

$$S_{\perp}(x) ** G_{\perp} - S_{II}(x) ** G_{II} = I_r ** (F_{r\perp}(x) ** G_{\perp} - F_{rII}(x) ** G_{II})$$

Now, we will look for function V which holds

$$V ** (F_{r\perp}(x) ** G_{\perp} - F_{rII}(x) ** G_{II}) = \delta(x)$$

If we find such function V, it is obvious that

$$I_r = (S_{\perp}(x) ** G_{\perp} - S_{II}(x) ** G_{II}) ** V$$

And $I_t$ can be calculated easily by

$$S_{\perp}(x) ** G_{\perp} = I_t + I_r ** F_{r\perp}(x) ** G_{\perp}$$
$$I_t = S_{\perp}(x) ** G_{\perp} - I_r ** F_{r\perp}(x) ** G_{\perp}$$

All we have to do now is to find the functions G and V.

The function G was actually found by us earlier in the section dealing with restoring the original scene when $I_r = 0$. The function G is

$$G_{\perp,II} = \frac{\delta(x) - R_{\perp,II}\delta(x-d)}{T_{\perp,II}^{\;2}}$$

We can find the function V by:

$$F_r(x) = R*\delta(x) + T^2R\delta(x-d) + T^2R^3\delta(x-2d) + ...$$

$$F_r(x)**G = \frac{1}{T^2}(Fr(x) - R^2Fr(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (T^2R - R^3)\delta(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (R(T^2 - R^2)\delta(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (R((T+R)(T-R))\delta(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (R(T-R)\delta(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (R(1-2R)\delta(x-d))$$

$$F_r(x)**G = \frac{1}{T^2}(R\delta(x) + (R-2R^2)\delta(x-d))$$

(All above calculations are the same for $G_\perp$ and $G_{II}$.)

Now we have

$$F_{r\perp}(x)**G_\perp - F_{rII}(x)**G_{II} = \frac{1}{T_\perp{}^2}(R_\perp\delta(x) + (R_\perp - 2R_\perp{}^2)\delta(x-d)) - \frac{1}{T_{II}{}^2}(R_{II}\delta(x) + (R_{II} - 2R_{II}{}^2)\delta(x-d))$$

$$F_{r\perp}(x)**G_\perp - F_{rII}(x)**G_{II} = \left(\frac{R_\perp}{T_\perp{}^2} - \frac{R_{II}}{T_{II}{}^2}\right)\delta(x) + \left(\frac{R_\perp - 2R_\perp{}^2}{T_\perp{}^2} - \frac{R_{II} - 2R_{II}{}^2}{T_{II}{}^2}\right)\delta(x-d)$$

For convenience, we will notify the coefficients in the last equation by a and b, and will rewrite the last equation:

$$F_{r\perp}(x)**G_\perp - F_{rII}(x)**G_{II} = a\delta(x) + b\delta(x-d)$$

Now the function V needs to hold

$$V**(a\delta(x) + b\delta(x-d)) = \delta(x)$$

By transforming to Z-plane:

$$V(z)*(a+bz^{-1})=1$$

$$V(z)=\frac{1}{a+bz^{-1}}=\frac{z}{az+b}$$

$$V(z)=\frac{1}{a}\left(\frac{z}{z-\left(\frac{-b}{a}\right)}\right)=\frac{1}{a}\left(1+\frac{\left(\frac{-b}{a}\right)}{z-\left(\frac{-b}{a}\right)}\right)$$

$$V(x)=\frac{1}{a}\left(\delta(x)+\sum_{n=0}^{\infty}\left(\left(\frac{-b}{a}\right)^{n}\delta(x-nd)\right)\right)$$

We have found explicit expressions for G and V, so $I_t$ and $I_r$ can be restored.

Tests show that this algorithm gives excellent results for images with various angles of incidence (which means various values for $R_{\perp}$ and $R_{\text{II}}$), and various values of d.

## 6. Estimating values of R

We go back now to the problem we earlier left unsolved – estimating R. actually, we have to estimate to values of R - $R_{\perp}$ and $R_{\text{II}}$, but we can replace this problem by the problem of estimating a single variable – the angle of incidence. Both values of R can be derived directly from the angle of incidence, by using Fresnel equations.

For estimating the AOI (angle of incidence), we use the following assumption. The algorithm we presented actually takes two mixtures of the original scenes (those mixtures are the photos we take) and separates them so we get the original scenes back. By using this algorithm with erroneous parameters (d or AOI), the separation will not be perfect.
Thus, if we estimate the AOI erroneously, the two restored images will still each contain a mixture of the two scenes. If this is the case, we can expect that the mutual information between the restored images will be greater than the same mutual information with correct separation of the mixtures.

The process of estimating AOI will involve running the algorithm for various AOIs. The correct AOI will result the least mutual information between the restored images.

Tests of this method show that least mutual information is indeed related to the correct AOI, or an AOI very close to the real one.

In order to improve the estimation of AOI, another criterion can be used. The algorithm presented here not only separates the mixtures of images, but also cancels the effect of the many reflected rays. We can expect that for the correct value of AOI, restored images will not include "reverberations" of themselves distant d from each other. Such restored images should have similar correlation (of the type mentioned earlier), regardless of the distance taken between parts of the restored image.

The following plot shows mutual information between the two restored images vs. the AOI taken. The correct AOI is 45 degrees (shown in plot).

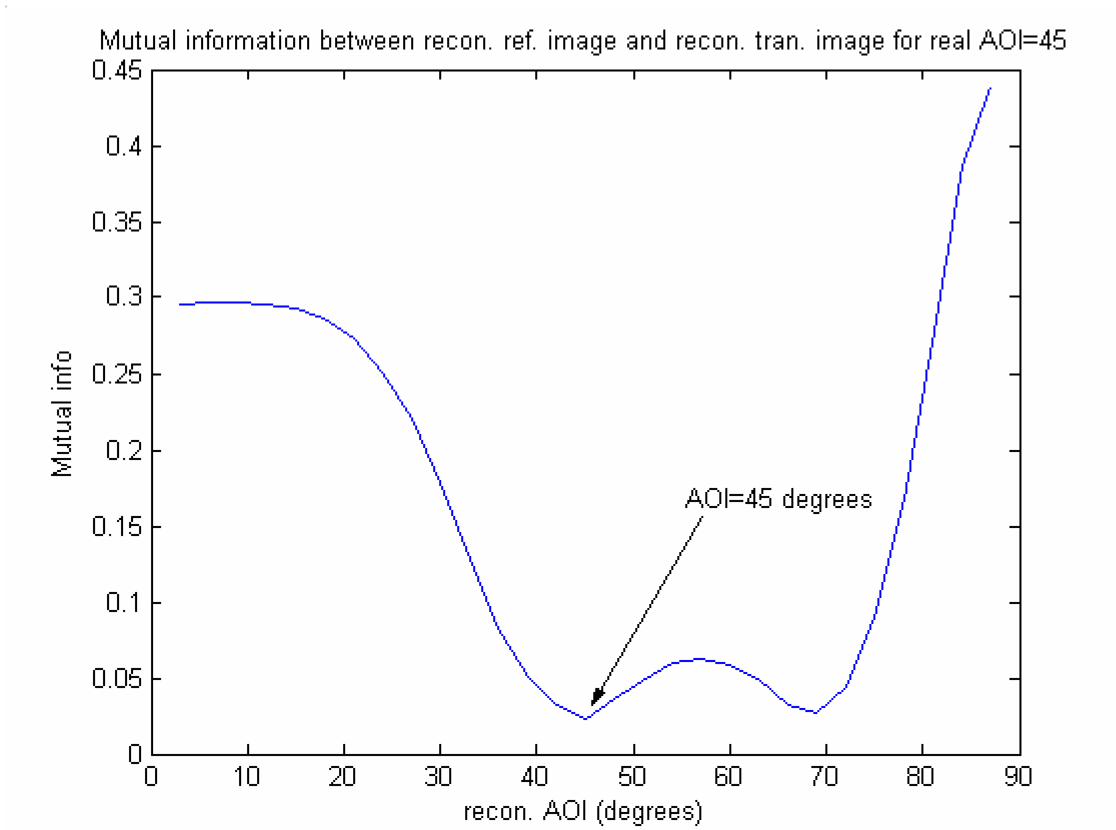


**Figure 4- mutual information vs. AOI**

It is clearly seen that indeed in AOI=45 degrees, the mutual information is one of the least.

## 7. Bibliography

[1] Polarization-based Decorrelation of Transparent Layers: The Inclination Angle of an Invisible Surface, Yoav Shechner, Joseph Shamir and Nahum Kiryati, *Proc. International Conference on Computer Vision, pp. 814-819 (Kerkyra, 1999).*

[2] User assisted separation of reflections from a single image using a sparsity prior, Anat Levin and Yair Weiss, *Proc. of the European Conference on Computer Vision (ECCV), Prague, May 2004.*