CCIT Report #532 May 2005



# Relaying Protocols for Two Co-located Users

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**Abstract**

We consider a wireless network where a remote source wishes to send information to one of two co-located users, and where the second user can serve as a relay. The source's transmission is subjected to quasi-static flat Rayleigh fading, while the transmissions of the users experience a fixed amplitude gain with uniform random phase, capturing their mutual proximity. All communications share the same time/bandwith resources, and channel state information is known only to the receivers. We propose relaying protocols which are based on Wyner-Ziv quantization at the relay, and demonstrate their high efficiency (in terms of expected throughput) with respect to previously reported relaying schemes based on *amplify-and-forward* and *decode-and-forward*. We further incorporate into the cooperative scheme the notion of *successive quantization* with respect to the relay's observation, and match it with a broadcast transmission approach in the relay to destination link, relying on the inverse relation which exists between the amount of side information at the destination, and the capacity of the relay to the destination link. Finally, we consider a hybrid *amplify-quantize-decode-and-forward* scheme which exhibits superior performance in some situations.

*Keywords*— Ad-hoc networks, amplify and forward, broadcast strategy, cooperative diversity, decode and forward, expected throughput, fading channels, outage capacity, quantize and forward, relay channel, sensor networks, successive refinement, wireless networks.

## I. Introduction

Cooperative strategies for wireless networks are drawing a lot of attention in recent years, mainly due to the proliferation of stand alone wireless networks which do not rely on any fixed infrastructure to facilitate the communications between the nodes in the network. Moreover, these networks are typically employed in a fading environment, rendering the transmitted signals susceptible to severe attenuations in

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The work was supported by the Fund of Promotion of Research at the Technion and the ISRC consortium.

their received strength. In such settings, it becomes necessary for the nodes in the network to cooperate at the physical layer in order to increase the achievable information rates between any pair of users and to ensure robustness of the communications to changes in channel conditions.

The study of the relay channel [1], [2] is of fundamental importance to cooperation in wireless networks, since it captures the ability of a user to assist in transferring information from a source to its destination - a situation which is prevalent in wireless networks due to the sharing of the wireless medium among all users. Unfortunately, the capacity of the relay channel is only known for some specific cases (e.g. degraded and reversely degraded relay channel, semi-deterministic relay channel, relay channel with feedback) which do not apply directly to common wireless settings. Recently, however, there has been some extensive work reported concerning the capacity of the relay channel and its implications on cooperation in wireless channels [3]–[14] (and references therein). In fact, the capacity of the relay channel was recently established for some wireless channel models where the relay terminal is in close proximity to the source terminal and where there is phase uncertainty at the transmitters.

One of the key ingredients to cooperation in wireless channels in general and in the wireless relay channel in particular is intimately related to the notion of *cooperative diversity* which is associated with the ability of the users to share their antenna resources and achieve some of the gain promised by using multiple antenna arrays. This idea was introduced in [15], [16] and developed in follow up works, e.g. [17]–[26] (and references therein).

While the optimal strategy for employing relays in wireless networks is not yet understood, several approaches have already been suggested in the literature. One approach is to have the relay first decode the message sent by the source (if it can), and then re-encode it and re-transmit the same message to the destination. While this strategy was used to obtain the capacity of the degraded relay channel [2], recently it was used for wireless outage settings and named *decode-and-forward* [18]. Another approach, called *amplify-and-forward*, is to let the relay simply send a scaled version of its received observation to the destination, releasing itself from the need to actually decode the message. A disadvantage of this approach is the noise amplification which is incurred by the relay's action. Yet a third strategy, *compress-and-forward* [3], [12] suggests that the relay send a quantized version of its received signal to the destination. This strategy is actually based on the achievable rate suggested in [2]. Cooperative schemes which combine several relaying notions are also discussed in [3], [12]. Due to the surging interest in relays and cooperative approaches in wireless channels, we are unable to include and discuss all of the relevant and important contributions, and we refer the reader to the (far from complete) list of references [27]–[48] (and references therein).

In a recent work [49], [50] a form of decode-and-forward strategy was proposed and analyzed for a network consisting of a remotely located source, sending information to one of $K$ physically co-located users. As opposed to standard half duplex schemes which allocate two equal and orthogonal time/bandwidth slots for the transmission of the source and that of the relay, a variable length coding scheme was suggested where the relay takes just the time needed to decode the message before re-encoding and re-transmitting the message anew, and where the relay's transmission is concurrent with that of the source.

In this paper we consider the same setting as in [50] when specialized to two co-located users. The source wishes to send information to one of two users, where the second user (if present) can serve as a relay. The transmission of the source is affected by quasi-static flat Rayleigh fading, while the transmission between the users is subject to a fixed amplitude gain with uniformly distributed random phase, capturing the close proximity enjoyed by the users with respect to the remote source. Under stringent delay constraints, which prohibit the rescheduling of the message to a later time when channel conditions improve, we propose several cooperative transmission strategies tailored for such a setting and asses them by their average rate performance.

The relaying protocols we suggest are based on the notion of Wyner-Ziv quantization [51], which refers to the relay quantizing its received observation of the source's symbol, while relying on side information which is available at the destination receiver. This side information is due to the correlation which exists between the relay's observed signal and that of the destination. The "amount" of correlation (or side information) depends on the fading gains of the relay and the destination. High gains imply large correlation, while small fading gains result in reduced correlation.

We note that this approach is not new. In fact, it was used to establish one of the achievable rates for the general relay channel [2]. Recently, this scheme was proposed for wireless relay channels [3], [12] showing good behavior for relays which are in close proximity to the destination. A key feature of the Wyner-Ziv quantization scheme is that the relay, while ignorant of the actual side information available to the destination receiver, must know its statistics. For fixed channels, where the channel gains are known to everyone (as in [3]), or in the case of pure phase fading [12] which does not change the statistics of the destination's received signal, the scheme can be employed successfully. However, when the signal from the source to the destination undergoes a more general fading (unknown to the relay), the relay has trouble implementing this quantization approach. We suggest to circumvent this problem by letting the relay assume some minimal level of side information available at the destination, and quantize its received observation based on this level. When the actual side information at the destination is larger, the scheme works. Otherwise, we have an outage event, since the side information is not sufficient to solve the ambiguity incurred by the relay in the binning process associated with the Wyner-Ziv quantization.

The quantization approaches discussed above generally use the error free link, which is available between the relay and the destination, in order to convey the quantized and binned information. This information transfer is done using a code which assumes that the destination considers the transmission coming from the source as noise. Therefore, the capacity of this link is governed by the received power of the transmission coming from the source, which in turn is determined by the fading gain experienced by the destination. For high channel gains, this capacity is reduced, and for small channel gains it is increased. It follows that there is an inverse relation between the capacity of the error free link between the relay and destination on the one hand, and the amount of side information available at the destination on the other. When the channel gain of the destination is high, it has a high level of side information, but limited capacity in the relay to destination link. Contrarily, when the destination's gain is low, this implies small correlation with the relay's observed signal but a high capacity of the link coming from the relay.

This interesting relation suggests that when the destination enjoys a high quality signal coming from the source (corresponding to a high fading gain), the relay should quantize its received observation using fewer bits. This is both because the capacity from the relay to the destination is now limited, and also because the destination has enough side information to reconstruct the quantized information from the relay. On the other hand, when the destination's observation is of poor quality (corresponding to a low fading gain), the relay should perform the quantization using more bits, since in this case, the destination has only limited side information, and hopefully the link from the relay to the destination (which now has higher capacity) can accommodate the higher rate associated with the relay's transmission. In other words, the relay would want to send less information in the error free link when its capacity is low, and more information when its capacity is high.

A way for automatically adapting the transmission rate to the actual instantaneous capacity of a quasi-static flat Rayleigh fading channel (with channel state information known to the receiver only) was suggested in [52], [53] (see also [46], [54], [55]). We incorporate this approach in the following way. The relay produces two information streams. The first stream is the quantized and binned information associated with the Wyner-Ziv quantization which assumes a high level of side information at the destination (corresponding to a high channel gain of the destination, or small capacity of the error free link from relay to destination). The second stream is associated with the Wyner-Ziv coding for the case of a low level of side information. The relay then matches these rates with a broadcast approach, such that when the capacity of the error free link is low (i.e. when the channel gain of the destination is relatively high) the first stream gets decoded, and when the capacity of the error free link is high, both streams get decoded. In this way, when the destination enjoys a high channel gain, it only gets the first stream, which is enough since it has a lot of side information, and when the channel gain of the destination is low, it gets both information streams which it now needs in order to fully recover the quantized information because in this case it has little side information.

We note that the idea of coding the relay's observation for two possible levels of side information available to the decoder is closely related to the notion of successive refinement in the Wyner-Ziv context (see [56] and references therein). Note, however, that in [56], there is an increasing relation between the amount of side information available at the decoder and the code rate used to describe the desired observation. The more side information available, the more rate is used to give a better description. In our setting, the situation is reversed. Coding is done first for the case of high side information (which requires little bits for description), and more code is needed to describe the relay's observation if there is only little side information available.

Finally, we propose a cooperative scheme which combines several of the notions associated with the operation of relays. In this strategy, relying on limited feedback from the destination, the relay tries to decode the message transmitted by the source, and while doing so, it either quantizes and forwards if some minimal side information is available to the destination's disposal, or just amplifies and forwards. Once it decodes the message it re-encodes it and re-transmits the message using a new codeword as was done in [50].

A few words regarding notation. We use the notation $(x)^1$ to designate $\min(1, x)$. Throughout the paper, all logarithms are taken with respect to the base $e$. The notation $1_{\mathcal{A}}(x)$ designates the indicator function, which equals 1 whenever $x \in \mathcal{A}$ and zero otherwise. The notation $\mathcal{CN}\left(0, \sigma^2, \sigma^2\right)$ designates a complex Gaussian random variable whose real and imaginary components are independent Gaussian random variables each with mean zero and variance $\sigma^2$. $X^*$ denotes complex conjugation. The notations $X_n \xrightarrow{P} X$ and $X_n \xrightarrow{D} X$ stand, respectively, for convergence of random variables in probability and in distribution as defined in [57]. Finally, we use $\mathbb{E}(\cdot)$ to denote the expectation operator.

The rest of the paper is organized as follows. Section II describes our considered model; Section III presents preliminary results concerning the amplify-and-forward and decode-and-forward schemes and variations thereof; Section IV presents a cooperative scheme based on quantization and forwarding; Section V extends the quantization scheme using the ideas of successive quantization combined with a broadcast approach; Section VI presents numerical results demonstrating the performance of our proposed schemes and concluding remarks are given in Section VII. Proofs and derivations are deferred to Appendixes A–D.

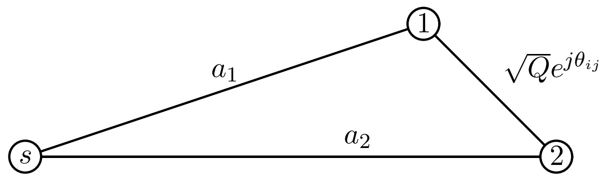



Fig. 1. A network with two co-located users

## II. System Model

We consider the setting discussed in [50] when specialized to the case of two users. For completeness we reiterate the main features of the model. The system consists of a wireless network in which a remotely located source $s$ wishes to send information to one of two co-located users (as depicted in Fig. 1). The information is transmitted over a shared wireless medium where all transmissions coming from the source are subjected to flat Rayleigh fading. The fading coefficients between the source and the users are denoted by $a_1, a_2$ and are modelled by two independent zero mean unit variance circularly symmetric complex Gaussian random variables. Assuming a quasi-static fading dynamics, the fading coefficients are taken to be constant over a coherence time equivalent of $N$ symbols[1], and independent from one block of $N$ symbols to the next. The co-location of the two users is captured by assuming that the gain governing the transmission between the users is $\sqrt{Q}e^{j\theta_{ij}}$, $(i,j) \in \{(1,2),(2,1)\}$, where $\sqrt{Q}$ is a fixed amplitude and $\theta_{ij}$ is a random phase uniformly distributed over $[-\pi,\pi)$ also assumed fixed during one block of symbols and independent from one block to the next. The fading coefficients are assumed to be known to the receivers only. Specifically, receiver 1 knows $a_1$ and the random phase $\theta_{21}$ associated with the transmission from user 2, and similarly user 2 has full knowledge of the fading coefficient $a_2$ and of the random phase $\theta_{12}$ associated with the transmission of user 1. Since the source is not informed of

[1] $N$ is assumed large enough to allow Shannon theoretic arguments to hold.

the phases $\theta_{ij}$ and since user 1 (resp. user 2) is ignorant of $\theta_{12}$ (resp. $\theta_{21}$), it follows that the signals transmitted by the source and user 1 (resp. user 2) can not be coherently combined at the receiver of user 2 (resp. user 1). The situation would remain the same even if user 1 (resp. user 2) knew the phase $\theta_{12}$ (resp. $\theta_{21}$). Therefore, without loss of generality we assume that $\theta_{ij} = 0$, $(i,j) \in \{(1,2),(2,1)\}$. Finally, we assume that each user is equipped with a transmitter and a receiver capable of working in full duplex.

It is assumed that at the beginning of each fading block, a message arrives at the source intended for one of the two users, which we call the *destination*, the identity of which is assumed to be chosen at random with equal probability. Stringent delay constraints stipulate that the message be delivered from the source to the destination within a period of one fading block. In particular, scheduling the transmission of the message to a later time is not permitted. During this period, the other user, which we call the *relay*, can assist the source in relaying the message to the destination.

By definition of the network it is implicitly assumed that the destination is not transmitting throughout the fading block[2]. It is therefore more convenient to denote the two users as user $d$ (the destination) and user $r$ (the relay), where either $(d,r) = (1,2)$ or $(d,r) = (2,1)$. Assuming an AWGN channel, the received signals at the relay and the destination at time $n$, $n = 1,2,\ldots,N$, are modelled by

$$\begin{aligned} Y_r(n) &= a_r X(n) + Z_r(n) \\ Y_d(n) &= a_d X(n) + \sqrt{Q} X_r(n) + Z_d(n) \end{aligned} \tag{1}$$

where $X(n)$ and $X_r(n)$ are the symbols transmitted during the $n$-th symbol interval by the source and the relay, respectively, $a_r$ and $a_d$ are the fading coefficients of the relay and destination, respectively, and $Z_r(n)$ and $Z_d(n)$ are the AWGNs at the relay and destination respectively, both modelled by i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. It will be convenient to denote the squared magnitude of the fading coefficients by $\nu_d = |a_d|^2$ and $\nu_r = |a_r|^2$ each of which is exponentially distributed with unit mean.

## III. Preliminary Results

In this section we examine several basic (and by now standard) cooperative schemes suited for the network described in section II. The problem here falls within the outage regime, and we use the *expected*

[2]Such an assumption can be easily accommodated by assuming that the identity of the recipient is announced by the source at the beginning of the transmission at a negligible cost of rate.

*throughput* (see [50]) as a figure of merit when comparing between the performance of these schemes. This measure, which is defined as the probability of successful decoding when attempting to communicate at some rate $R$ multiplied by this rate, can be interpreted as the long term average rate which can be delivered by the network to any particular user.

### A. Decode-and-Forward (DF)

We begin by restating the cooperative strategy which was suggested in [50] for this type of network. This scheme can be considered as a variant of the relaying approach where the relay first decodes the message transmitted by the source, and only then re-encodes and retransmits the same message to the destination [18]. Some previous implementations of the decode-and-forward scheme assumed some form of orthogonality between the resources used to deliver the information from the source to the relay, and those which are used to transport the information from the relay to the destination, e.g. [18], [58]. In our setting the protocol assumes that all transmissions are carried out using the same channel resources, i.e. time and bandwidth.

The strategy is based on the observation that users enjoying favorable channel conditions are able to decode the message by observing only the first $m$ out of $N$ ($m \leq N$) symbols of the codeword. We briefly describe this strategy (for more details see [50]). The source starts transmitting a code word describing a message intended for one of the users. If this user is the strongest among the two in the sense that its fading coefficient is greater, then no cooperation takes place. If on the other hand, the destination is the weaker user among the two, then once the strong user decodes the message (typically before the end of the block), it starts transmitting the same message using another predetermined code book acting together with the source as a double transmit antenna array. The expected throughput in this case is given by [50]

$$\begin{aligned} R_{\text{av}}^{\text{DF}}(R,P,Q) &\triangleq R\Pr\{I_{\text{DF}} > R\} \\ &= Re^{-\frac{e^R-1}{P}} + \frac{Re^{\frac{2}{P}}}{P}\int_{\max(1,e^R-Q)}^{e^R} e^{-\frac{1}{P}\exp\left\{\frac{R\log\left(1+\frac{Q}{u}\right)}{\log(u+Q)-R}\right\}-\frac{u}{P}}\,\mathrm{d}u \end{aligned} \tag{2}$$

where $I_{\text{DF}}$ is a random variable designating the average mutual information conveyed to the destination in some fading block.

### B. Amplify-and-Forward (AF)

Another standard approach for operating the relay in such a setting is the amplify-and-forward approach [18]. Here the relay, adhering to a causality restriction, transmits in every symbol interval a scaled version of its received observation from the previous symbol interval. In our setting this means that the signals received by the relay and destination are now given by

$$\begin{aligned}Y_r(n) &= a_r X(n) + Z_r(n)\\ Y_d(n) &= a_d X(n) + \beta Y_r(n-1) + Z_d(n)\\ &= a_d X(n) + \beta\big[a_r X(n-1) + Z_r(n-1)\big] + Z_d(n)\\ &= a_d X(n) + \beta a_r X(n-1) + \beta Z_r(n-1) + Z_d(n), \quad n = 1, 2, \ldots, N \end{aligned} \tag{3}$$

where $\beta = \sqrt{\frac{q(|a_r|^2)}{1+|a_r|^2 P}}$ is the amplifying gain employed by the relay $r$ which results in a reception power of $q(|a_r|^2)$ at the receiver of the destination $d$, where $q(\cdot)$ is some decentralized power policy satisfying

$$\begin{aligned} q(0) &= 0\\ 0 \le q(u) &\le Q\\ \lim_{u\to\infty} q(u) &= Q\\ q'(u) &\ge 0. \end{aligned} \tag{4}$$

The rational behind these constraints is based on the intuitive assumption that as the observation of the relay gets better and better, it is desirable to transmit this observation to the destination with greater and greater power. To clarify this idea, consider the two extreme cases. If $a_r = 0$, the relay receives just noise, and therefore it does not pay to amplify this noise and only make it worse for the destination to try and decode the message. On the other hand, if the relay receives the source's symbol with absolute certainty (i.e., when $|a_r| \to \infty$) then it pays to use the maximum available power at the relay's disposal in order to transmit this observation to the destination.

Effectively, the channel viewed by the destination is a one tap ISI channel. Since the channel coefficients are unknown to the transmitter, we assume that i.i.d. signaling is employed, in which case the average mutual information between the source and the destination in some fading block is shown in Appendix A to be

$$\begin{aligned} I_{\mathrm{AF}}(q) = \log &\left(1 + P\frac{\nu_d(1+\nu_r P) + \nu_r q(\nu_r)}{1 + \nu_r P + q(\nu_r)}\right)\\ &+ \log\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \left(\frac{2P\sqrt{q(\nu_r)\nu_r\nu_d}}{(1+\nu_d P + q(\nu_r))\sqrt{1+\nu_r P}}\right)^2}\right) \end{aligned} \tag{5}$$

where the second term on the right hand side (RHS) of (5) lies in the interval $[-\log 2, 0]$. It follows that a trivial upper bound to the mutual information (5) is given by discarding the second term in (5), namely

$$I_{\text{AF}}(q) \leq I_{\text{AF-UB}}(q) \triangleq \log\left(1 + P\frac{\nu_d(1+\nu_r P) + \nu_r q(\nu_r)}{1 + \nu_r P + q(\nu_r)}\right). \tag{6}$$

It would be desirable to find which decentralized power control policy at the relay maximizes the expected throughput in the amplify-and-forward strategy, and also what this maximal throughput is. A partial answer to these questions is given by the following propositions. Recall from [50] that the performance of the best user is given (irrespective of the co-location gain) by

$$R_{\text{av}}^{\text{BU}}(R, P) = 2Re^{-\frac{e^R-1}{P}} - Re^{-2\frac{e^R-1}{P}}.$$

*Proposition 1:* For any power control policy satisfying (4), the expected throughput of the AF protocol is upper bounded by the performance of the best user, i.e.

$$R_{\text{av}}^{\text{AF}}(R, P, Q, q(\cdot)) \triangleq R\Pr\{I_{\text{AF}}(q) > R\} \leq R_{\text{av}}^{\text{BU}}(R, P).$$

*Proof:* See Appendix B.

*Proposition 2:* The power control policy given by

$$q^*(u) = \begin{cases} 0, & u < \frac{e^R-1}{P} \\ Q, & u \geq \frac{e^R-1}{P} \end{cases}. \tag{7}$$

is asymptotically optimal for $Q \to \infty$ in the sense that

$$\lim_{Q\to\infty} R_{\text{av}}^{\text{AF}}(R, P, Q, q^*(\cdot)) = R_{\text{av}}^{\text{BU}}(R, P).$$

*Proof:* See Appendix C.

Note that if the relay knew whether it is the strongest or the weakest user, it could either transmit with full power or stay silent, respectively. In the former case, assuming $Q \to \infty$, the relay effectively transforms the destination to the best user by overlaying its “good” observation over the “bad” version of the destination. In the latter case, by staying silent, the performance is also that of the best user (which is the destination in this case).

However, when the relay does not have this information (as in our model), and must decide which power to use based only on its own fading gain, it is not immediately clear if such a policy exists which can attain the optimal (best user) performance even asymptotically. Proposition 2 establishes that this in fact is possible.

While this power control policy does not necessarily maximize the actual expected throughput of the amplify-and-forward scheme for arbitrary $Q$, the intuition behind it is clear. Let the relay amplify and forward when it has some confidence in the quality of its received observation. The policy $q^*(u)$ ensures that the relay amplifies and forwards when it receives the source's codeword with a quality which enables it to decode the message with high probability. The asymptotic optimality of this policy is confirmed by numerical results in the sequel (see section VI).

Using this power control policy we get the following expected throughput

$$
\begin{aligned}
&R_{\text{av}}^{\text{AF}}(R,P,Q,q^*(\cdot)) = \\
&= R\int_0^\infty \Pr\Bigg\{\log\left(1+P\frac{\nu_d(1+\nu_rP)+\nu_r q^*(\nu_r)}{1+\nu_rP+q^*(\nu_r)}\right)+ \\
&\qquad + \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{q^*(\nu_r)\nu_r\nu_d}}{(1+\nu_dP+q^*(\nu_r))\sqrt{1+\nu_rP}}\right)^2}\right) > R\Bigg|\nu_r\Bigg\} e^{-\nu_r}\,\mathrm{d}\nu_r \\
&= Re^{-\frac{e^R-1}{P}}\left(1-e^{-\frac{e^R-1}{P}}\right)+ \\
&\quad + R\int_{\frac{e^R-1}{P}}^\infty \Pr\Bigg\{\log\left(1+P\frac{\nu_d(1+\nu_rP)+\nu_rQ}{1+\nu_rP+Q}\right)+ \\
&\qquad + \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{Q\nu_r\nu_d}}{(1+\nu_dP+Q)\sqrt{1+\nu_rP}}\right)^2}\right) > R\Bigg|\nu_r\Bigg\} e^{-\nu_r}\,\mathrm{d}\nu_r \qquad (8) \\
&= Re^{-\frac{e^R-1}{P}}\left(1-e^{-\frac{e^R-1}{P}}\right)+R\int_{\frac{e^R-1}{P}}^\infty \mathrm{d}\nu_r\int_0^\infty \mathrm{d}\nu_d\ 1_{\{\mathcal{A}(R,P,Q)\}}(\nu_d,\nu_r)e^{-\nu_d-\nu_r}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{A}(R,P,Q) \triangleq \Bigg\{\nu_d,\nu_r\geq 0 : \log\left(1+P\frac{\nu_d(1+\nu_rP)+\nu_rQ}{1+\nu_rP+Q}\right)+ \\
+ \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{Q\nu_r\nu_d}}{(1+\nu_dP+Q)\sqrt{1+\nu_rP}}\right)^2}\right) > R\Bigg\}
\end{aligned}
$$

and (8) follows because when $\nu_r < \frac{1}{P}\left(e^R-1\right)$ the integrand reduces to $\Pr\{\log(1+\nu_dP)>R\}$.

### C. *Amplify-Decode-and-Forward (ADF)*

The first two schemes, namely decode-and-forward and amplify-and-forward can be combined to form a hybrid scheme which we call *amplify-decode-and-forward*. In this approach, assuming that the relay has sufficient channel conditions to decode the message before the block ends (i.e. $\log(1+\nu_rP)>R$), the relay amplifies-and-forwards its received observations until it is able to decode the message, and then it re-encodes the message and starts emitting a fresh codeword describing the same message, just as if it were another transmitting antenna. For the amplify part, we use the same power control policy which was used for the amplify-and-forward scheme. That is if channel conditions do not allow the relay to decode the message, the relay remains silent throughout the fading block.

Denote by $\varepsilon \triangleq \left(\frac{R}{\log(1+\nu_r P)}\right)^1$ the fractional time within a codeword when the relay decodes the message. Then, the expected throughput for the amplify-decode-and-forward cooperative scheme is given by

$$R_{\text{av}}^{\text{AF}} = Re^{-\frac{e^R-1}{P}}\left(1-e^{-\frac{e^R-1}{P}}\right) + R\int_{\frac{e^R-1}{P}}^{\infty} \mathrm{d}\nu_r \int_0^{\infty} \mathrm{d}\nu_d \; 1_{\{\mathcal{B}(R,P,Q)\}}(\nu_d,\nu_r)e^{-\nu_d-\nu_r} \tag{9}$$

where

$$\mathcal{B}(R,P,Q) \triangleq \Bigg\{\nu_d,\nu_r \geq 0 : \varepsilon \Bigg[\log\left(1+P\frac{\nu_d(1+\nu_r P)+\nu_r Q}{1+\nu_r P+Q}\right) + \\ + \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{Q\nu_r\nu_d}}{(1+\nu_d P+Q)\sqrt{1+\nu_r P}}\right)^2}\right)\Bigg] + (1-\varepsilon)\log(1+\nu_d P+Q) > R\Bigg\}.$$

## IV. Quantize-and-Forward (QF)

We now examine an entirely different approach for employing the relay in our co-located setting. In this approach the relay terminal who receives the noisy observation of the source's transmitted symbol $Y_r$, sends a quantized version of this observation, $U$, to the destination. The quantization is done in the Wyner-Ziv [51] spirit, making use of the correlation between $Y_r$ and $Y_d$ which is available at the destination receiver. The relay uses its own code book in order to convey reliably the quantized information to the destination. The destination first decodes this information while considering the signal coming from the source as noise, and then cancels out the relay's codeword. Finally, the destination completes the reconstruction of the quantized information, $U$, and combines it together with its own received signal in an optimal manner in order to decode the message. This is a modification of techniques employed in general relay channels [2, Thm. 6] and in wireless relay channels [3], [12]. The channel model in this case can be written as

$$\begin{aligned} Y_r(n) &= a_r X(n) + Z_r(n) \\ Y_d(n) &= a_d X(n) + Z_d(n) \end{aligned} \tag{10}$$

where the relay can send information to the destination on a separate channel the capacity of which is governed by $\nu_d$ and given by

$$C_{\text{rd}}(\nu_d) = \log\left(1+\frac{Q}{1+\nu_d P}\right).$$

Note that the capacity of the link between the relay and the destination is a *monotonically decreasing* function of the gain $\nu_d$. That is when the received source's power at the destination is high, it interferes more severely with the transmission coming from the relay and vice verse.

Lets focus on the quantization procedure which is to be carried out by the relay. The relay wishes to send a quantized version of its received signal, $Y_r$, while relying on the statistical dependence which exists between $Y_r$ and the signal received by the destination, $Y_d$. Note that this statistical dependence is also governed by the fading coefficient of the destination, namely $\nu_d$. Large values of $\nu_d$ imply higher correlation between $Y_r$ and $Y_d$, whereas small values of $\nu_d$ imply that $Y_r$ and $Y_d$ have less correlation between them. In other words, the correlation between $Y_r$ and $Y_d$ is a *monotonically increasing* function of $\nu_d$.

### *A. An Upper Bound - The relay knows $\nu_d$*

We first assume that the relay has full knowledge of the fading gain experienced by the destination, $\nu_d$. The results in this sub section will later be used for the actual strategy where the relay does not have such knowledge, and in any case can be considered as a performance upper bound for the actual scheme. We begin by stating the following proposition which is a direct consequence of [2, Thm. 6] (see also [3], [12]).

*Proposition 3:* For the network described in section II in which the fading gains $a_r$ and $a_d$ are held fixed, where $a_r$ is known to the relay, and $a_d$ is known to both the relay and the destination, the rate $R$ is achievable[3], where

$$R = \sup I(X; Y_d, U) \tag{11}$$

subject to the constraint

$$I(U; Y_r) - I(U; Y_d) \leq C_{\text{rd}}(\nu_d), \tag{12}$$

and where the supremum is over all random variables $U$ satisfying the Markov property $U \Longleftrightarrow Y_r \Longleftrightarrow Y_d$.

[3]A coding scheme which achieves this rate is the block Markov encoding approach [2, Thm. 6], where the message is partitioned into many long blocks, and where in every block the relay sends a quantized version of its previous received block of observations. The destination, after decoding the relay's transmission containing the quantized and binned information, cancels out the relay's codeword, and uses its received symbols from the previous block in order to resolve the ambiguity incurred by the binning process.

We now propose a strategy suited for the case where the network is experiencing slow quasi-static fading as described above, and the relay has full knowledge of the fading gain experienced by the destination, $\nu_d$. The relay chooses the auxiliary random variable $U$ to be a degraded version of $Y_r$, namely

$$U = Y_r + W \tag{13}$$

where $W \sim \mathcal{CN}\left(0, \frac{\sigma^2}{2}, \frac{\sigma^2}{2}\right)$ is independent of $X$, $Z_r$, and $Z_d$ and $\sigma^2$ is a parameter to be optimized. In Appendix D we derive expressions for the mutual information terms in (11) and (12), and these are given by

$$I(X; Y_d, U) = \log\left(1 + \nu_d P + \frac{\nu_r P}{1+\sigma^2}\right) \tag{14}$$

$$I(U; Y_r) - I(U; Y_d) = \log\left(\frac{1 + \sigma^2 + \frac{\nu_r P}{1+\nu_d P}}{\sigma^2}\right) \leq \log\left(1 + \frac{Q}{1+\nu_d P}\right). \tag{15}$$

As is seen in (14), in order to maximize $I(X; Y_d, U)$ the relay must choose[4] the smallest possible $\sigma^2$ such that the constraint (15) is not violated. In other words, the relay must choose $\sigma^2$ to satisfy

$$\frac{1 + \sigma^2 + \frac{\nu_r P}{1+\nu_d P}}{\sigma^2} = 1 + \frac{Q}{1+\nu_d P} \tag{16}$$

which yields

$$\sigma^2(P, Q, \nu_r, \nu_d) = \frac{1}{Q}\Big(1 + (\nu_d + \nu_r)\, P\Big). \tag{17}$$

Therefore, the rate which can be delivered to the destination in each fading block is given by

$$I_{\text{QF-UB}} = \log\left(1 + \nu_d P + \frac{\nu_r P Q}{1 + \nu_d P + \nu_r P + Q}\right). \tag{18}$$

where the subscript QF-UB stands for the quantization and forwarding upper bound where the value of $\nu_d$ is known to the relay. We now calculate the expected throughput associated with this upper bound. To this end we compute the probability of successful decoding. First consider the conditional probability

[4]Note that the relay chooses a new appropriate value of $\sigma^2$ at each fading block depending on the value of $\nu_r$ and $\nu_d$. For each such $\sigma^2$ an appropriate code book must be constructed and delivered to the destination receiver. This can be accommodated by designing multiple code books for a large but finite number of possible values of $\nu_r$ and $\nu_d$. Then, the only thing that must be conveyed to the destination receiver is the value of $\nu_r$ which can be done at a negligible cost of rate. The destination then knows which code book to use. Our results can then be approached as close as desired by choosing a finer and finer grid for $\nu_r$ and $\nu_d$.

of successful decoding given $\nu_d$. For $\nu_d < \frac{e^R-1}{P}$ we have

$$\begin{aligned}\Pr\left\{I_{\text{QF-UB}} > R|\nu_d\right\} &= \Pr\left\{\frac{\nu_r PQ}{1+\nu_d P+\nu_r P+Q} > e^R-1-\nu_d P\middle|\nu_d\right\}\\ &= \Pr\left\{\nu_r > \frac{\left(e^R-1-\nu_d P\right)\left(1+\nu_d P+Q\right)}{PQ-\left(e^R-1-\nu_d P\right)P}\middle|\nu_d\right\}\end{aligned}$$

Note that this probability vanishes for $\nu_d < \frac{1}{P}\left(e^R-1-Q\right)$ and assumes the value of unity whenever $\nu_d > \frac{e^R-1}{P}$. Averaging the conditional probability over $\nu_d$ we get

$$\Pr\left\{I_{\text{QF-UB}} > R\right\} = e^{-\frac{e^R-1}{P}} + \int_{\max\left(0,\frac{1}{P}(e^R-1-Q)\right)}^{\frac{1}{P}(e^R-1)} \exp\left\{-\frac{\left(e^R-1-\nu P\right)\left(1+\nu P+Q\right)}{PQ-\left(e^R-1-\nu P\right)P}\right\} e^{-\nu}\,\mathrm{d}\nu \quad (19)$$

and the expected throughput is thus

$$R_{\text{av}}^{\text{QF-UB}}(R,P,Q) = Re^{-\frac{e^R-1}{P}} + R\int_{\max\left(0,\frac{1}{P}(e^R-1-Q)\right)}^{\frac{1}{P}(e^R-1)} \exp\left\{-\frac{\left(e^R-1-\nu P\right)\left(1+\nu P+Q\right)}{PQ-\left(e^R-1-\nu P\right)P}\right\} e^{-\nu}\,\mathrm{d}\nu. \quad (20)$$

### *B. The relay does not know $\nu_d$*

We now present a strategy which is suited for the setting where the relay only has access to its own fading realization. We propose the following "outage" approach. The relay assumes that the realization of $\nu_d$ is in the interval $[a,b]$, where $a$ and $b$ are parameters to be optimized. The outage event is associated then with $\Pr\left\{\nu_d \notin [a,b]\right\}$. Note then that, under no outage:

$$C_{\text{rd}}(\nu_d) \geq C_{\text{rd}}(b).$$

Furthermore, the side information between $Y_r$ and $Y_d$ is lower bounded by taking the left edge of the interval, $a$. Let this side information be designated by $Y_d^{(a)}$. In other words $Y_d^{(a)} = \sqrt{a}X + Z_d$. The idea is that the relay can perform the quantization assuming that the side information available to the receiver is $Y_d^{(a)}$, and send the quantized information via a channel with capacity $C_{\text{rd}}(b)$. In reality, the side information available to the destination decoder is larger and it will be able to decode as if it really had only $Y_d^{(a)}$. This is so because the destination receiver can mimic this worst case assumption by scaling its channel output and adding independent Gaussian noise. Furthermore, by similar arguments, the codes which the relay uses in order to convey reliably the quantized information via the $r \to d$ link, will be decoded correctly even if $\nu_d < b$. We therefore have the following corollary:

*Corollary 1:* For the network described in section II in which the fading gains $a_r$ and $a_d$ are held fixed, where $a_r$ is known to the relay and $a_d$ is known to the destination, the rate $R$ is achievable, where

$$R = \sup_{a<b}\sup_{U} I(X;Y_d,U) \quad (21)$$

subject to the constraint

$$I\left(U;Y_r\right) - I\left(U;Y_d^{(a)}\right) \leq C_{\text{rd}}(b), \tag{22}$$

where the internal supremum is over all random variables $U$ satisfying the Markov property $U \Longleftrightarrow Y_r \Longleftrightarrow Y_d^{(a)}$.

Note that if $\nu_d > b$, the destination will not be able to decode the quantized information sent by the relay, because the code which the relay used assumed that the interference from the source is limited by the value of $\nu_d = b$. If this happens, the destination might not be able to cancel out this codeword, and it will be in a worse situation than if the relay had remained silent altogether. It would therefore be desirable if the destination could somehow signal the relay to stop its transmission if such a situation occurs, and try to decode the message on its own.

If on the other hand $\nu_d < a$, then the destination will be able to decode the relay's information, but due to inadequate side information it will probably not be able to decode the auxiliary $U$. In this case, it will be left only with its own observation of the source. Note that in this case, the transmission from the relay does not do much harm, but does not provide any help either.

We will assume that in an outage situation of both kinds, the relay is notified to stay silent and refrain from transmitting. In other words, under no outage, the destination can reconstruct the auxiliary random variable $U$, and when outage does occur, it is on its own.

We now wish to develop a corresponding expression for the expected throughput of this cooperative transmission scheme. We select the random variable $U$ as in (13). Appropriate expressions are obtained similarly to (14) and (15) by replacing $\nu_d \to a$ in $I\left(U;Y_d\right)$ and $\nu_d \to b$ in $C_{\text{rd}}(\nu_d)$, namely

$$I(X;Y_d,U) = \log\left(1+\nu_d P + \frac{\nu_r P}{1+\sigma^2}\right) \tag{23}$$

$$I(U;Y_r) - I\left(U;Y_d^{(a)}\right) = \log\left(\frac{1+\sigma^2+\frac{\nu_r P}{1+aP}}{\sigma^2}\right) \leq \log\left(1+\frac{Q}{1+bP}\right). \tag{24}$$

As in the case of the upper bound in IV-A, the aim is to choose the smallest possible $\sigma^2$. However, this value of $\sigma^2$ can no longer depend on $\nu_d$. In this case the relay chooses the smallest possible $\sigma^2$ such that the constraint (24) is not violated. In other words

$$\frac{1+\sigma^2+\frac{\nu_r P}{1+aP}}{\sigma^2} = 1+\frac{Q}{1+bP} \tag{25}$$

which yields

$$\sigma^2(P,Q,a,b,\nu_r) = \frac{1}{Q}\left(1+bP+\nu_r P\frac{1+bP}{1+aP}\right). \tag{26}$$

So when there is no outage the source can transfer to the destination a rate

$$I_{\text{QF}} = \log\left(1 + \nu_d P + \frac{\nu_r P Q}{1 + bP + \nu_r P \frac{1+bP}{1+aP} + Q}\right). \tag{27}$$

As in the previous case, we assume that if an outage occurs, the relay remains silent.

Next, for any $a < b$ we calculate the expected throughput of the scheme. We begin with the conditional probability of successful decoding given $\nu_d$. For $\nu_d \in [a, b]$ and $\nu_d < \frac{e^R - 1}{P}$ we have

$$\begin{aligned}\Pr\{I_{\text{QF}} > R|\nu_d\} &= \Pr\left\{\frac{\nu_r P Q}{1 + bP + \nu_r P \frac{1+bP}{1+aP} + Q} > e^R - 1 - \nu_d P \middle| \nu_d\right\} \\ &= \Pr\left\{\nu_r > \frac{\left(e^R - 1 - \nu_d P\right)\left(1 + bP + Q\right)}{PQ - \left(e^R - 1 - \nu_d P\right) P \frac{1+bP}{1+aP}} \middle| \nu_d\right\}.\end{aligned}$$

Note that this probability vanishes for $\nu_d < \frac{1}{P}\left(e^R - 1 - Q\frac{1+aP}{1+bP}\right)$ and assumes the value of unity whenever $\nu_d > \frac{e^R-1}{P}$. Furthermore, if $\nu_d \notin [a, b]$ and $\nu_d < \frac{e^R-1}{P}$ then decoding is not possible. Averaging the conditional probability over $\nu_d$ we get

$$\Pr\{I_{\text{QF}} > R\} = e^{-\frac{e^R-1}{P}} + \int_a^b 1_{[\xi_1,\xi_2]}(\nu) \exp\left\{-\frac{\left(e^R - 1 - \nu P\right)\left(1 + bP + Q\right)}{PQ - \left(e^R - 1 - \nu P\right) P \frac{1+bP}{1+aP}}\right\} e^{-\nu}\, \mathrm{d}\nu \tag{28}$$

where

$$\begin{aligned}\xi_1 &= \frac{1}{P}\left(e^R - 1 - Q\frac{1+aP}{1+bP}\right) \\ \xi_2 &= \frac{1}{P}\left(e^R - 1\right)\end{aligned}$$

and the expected throughput is thus

$$R_{\text{av}}^{\text{QF}}(R, P, Q, a, b) = Re^{-\frac{e^R-1}{P}} + R\int_a^b 1_{[\xi_1,\xi_2]}(\nu) \exp\left\{-\frac{\left(e^R - 1 - \nu P\right)\left(1 + bP + Q\right)}{PQ - \left(e^R - 1 - \nu P\right) P \frac{1+bP}{1+aP}}\right\} e^{-\nu}\, \mathrm{d}\nu \tag{29}$$

Finally, this throughput can be further optimized (numerically) over $a$ and $b$. Inspection of (29) shows that there is no point in choosing $b > \frac{1}{P}\left(e^R - 1\right)$. This is because for any value of $a$, increasing the value of $b$ beyond $\frac{1}{P}\left(e^R - 1\right)$ only decreases the integrand without enlarging the effective interval of integration. We can therefore assume that $b \leq \frac{1}{P}\left(e^R - 1\right)$. Note further that for any value of $b$, we can restrict the values of $a$ to satisfy $a > \xi_1$ without losing optimality. To see this, assume that $a < \xi_1$. Then by increasing $a$, we are reducing $\xi_1$, thus enlarging the interval of integration, and also increasing the integrand. We can continue this procedure until the point where $\xi_1 = a$. It follows that the expected throughput can be written slightly in simpler form, namely

$$R_{\text{av}}^{\text{QF}}(R, P, Q, a, b) = Re^{-\frac{e^R-1}{P}} + R\int_a^b \exp\left\{-\frac{\left(e^R - 1 - \nu P\right)\left(1 + bP + Q\right)}{PQ - \left(e^R - 1 - \nu P\right) P \frac{1+bP}{1+aP}}\right\} e^{-\nu}\, \mathrm{d}\nu \tag{30}$$

where the rate can be optimized over positive $a$ and $b$ which satisfy

$$\frac{1}{P}\left(\frac{e^R}{1+\frac{Q}{1+bP}}-1\right)\leq a\leq b\leq\frac{1}{P}\left(e^R-1\right).$$

### C. Amplify-Quantize-Decode-and-forward (AQDF)

Note that in the QF protocol, the relay continues to quantize and forward even though it might have been able to decode the message before the end of the block. For instance, if $\nu_r > \frac{1}{P}\left(e^R-1\right)$, then the relay is able to decode the message after only $\lceil\varepsilon N\rceil$ symbols (where $\varepsilon$ is as defined in section III-C). It would therefore be desirable to let the relay stop quantizing and forwarding and start sending a new code word, if and when it decoded the message. This will allow the destination to start enjoying a mutual information of

$$I_{\text{TD}}=\log\left(1+\nu_d P+Q\right)$$

where TD stands for full transmit diversity, instead of only

$$I_{\text{QF}}=\log\left(1+\nu_d P+\frac{\nu_r PQ}{1+bP+\nu_r P\frac{1+bP}{1+aP}+Q}\right).$$

Note further that we assumed in the QF protocol that the relay remains silent in the event that the value of $\nu_d$ either does not provide sufficient side information needed to fully decode the quantized information coming from the relay (the case where $\nu_d < a$), or that it does not provide sufficient capacity in the relay to destination link (the case where $\nu_d > b$). We have noticed by numerical observations, though, that the latter case is less disturbing, because $b$ is optimally chosen such that if $\nu_d > b$, the destination can decode the message by itself. In the former case, however, when $\nu_d < a$ the destination has practically no chance to decode the message. This can be improved by letting the relay amplify and forward whenever $\nu_d < a$ and the relay's observation is strong enough (i.e. $\nu_r > \frac{1}{P}\left(e^R-1\right)$).

In other words we would like to combine the three strategies: AF, QF, and DF. Assuming that the relay knows whether or not $\nu_d < a$, it can either amplify and forward (if $\nu_d < a$) or quantize and forward (if $a \leq \nu_d \leq b$) until it is able, if at all, to decode the message, and then send a new codeword. This scheme requires some feedback from the destination, telling the relay which scheme is best to use in each fading block, however this feedback is limited to one bit of information per fading block, which can be sent from the destination with little impact on the system's performance.

As in the ADF protocol, $\varepsilon$ designates the fractional time within the code word when the relay had succeeded to decode the message. Based on the previous results, the mutual information which is conveyed to the destination conditioned on $\nu_d$ is given by:

$$I_{\text{AQDF}} = \begin{cases} \varepsilon \left[ \log\left(1 + P\frac{\nu_d(1+\nu_r P)+\nu_r q^*(\nu_r)}{1+\nu_r P+q^*(\nu_r)}\right) + \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{q^*(\nu_r)\nu_r\nu_d}}{(1+\nu_d P+q^*(\nu_r))\sqrt{1+\nu_r P}}\right)^2}\right)\right] & \\ +(1-\varepsilon)\log(1+\nu_d P+Q), & \nu_d < a \\ \varepsilon \log\left(1+\nu_d P + \frac{\nu_r PQ}{1+bP+\nu_r P\frac{1+bP}{1+aP}+Q}\right) + (1-\varepsilon)\log\left(1+\nu_d P+Q\right), & \\ & a \le \nu_d \le \frac{1}{P}\left(e^R - 1\right) \\ \log\left(1+\nu_d P\right), & \nu_d > \frac{1}{P}\left(e^R-1\right) \end{cases}$$

where $q^*(\cdot)$ is as defined in (7). For $\nu_d < a$, the conditional probability of successful decoding given $\nu_d$ is given by

$$\begin{aligned} \Pr\left\{I_{\text{AQDF}} > R|\nu_d\right\} &= \int_0^\infty 1_{\{\mathcal{C}(R,P,Q,\nu_d)\}}(\nu_r)e^{-\nu_r}\,\mathrm{d}\nu_r \\ &= \int_{\frac{e^R-1}{P}}^\infty 1_{\{\mathcal{C}(R,P,Q,\nu_d)\}}(\nu_r)e^{-\nu_r}\,\mathrm{d}\nu_r \end{aligned}$$

where

$$\mathcal{C}(R,P,Q,\nu_d) \triangleq \left\{\nu_r : \varepsilon\left[\log\left(1+P\frac{\nu_d(1+\nu_r P)+\nu_r Q}{1+\nu_r P+Q}\right) + \log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{Q\nu_r\nu_d}}{(1+\nu_d P+Q)\sqrt{1+\nu_r P}}\right)^2}\right)\right] + (1-\varepsilon)\log\left(1+\nu_d P+Q\right) > R\right\}. \tag{31}$$

For $a \le \nu_d \le \frac{1}{P}\left(e^R-1\right)$, the conditional probability of successful decoding given $\nu_d$ is given similarly by

$$\Pr\left\{I_{\text{AQDF}} > R|\nu_d\right\} = \int_0^\infty 1_{\{\mathcal{D}(R,P,Q,\nu_d)\}}(\nu_r)e^{-\nu_r}\,\mathrm{d}\nu_r \tag{32}$$

where

$$\mathcal{D}(R,P,Q,\nu_d) \triangleq \left\{\nu_r : \varepsilon\log\left(1+\nu_d P+\frac{\nu_r PQ}{1+bP+\nu_r P\frac{1+bP}{1+aP}+Q}\right) + (1-\varepsilon)\log\left(1+\nu_d P+Q\right) > R\right\}.$$

For $\nu_d > \frac{e^R-1}{P}$, $\Pr\left\{I_{\text{AQDF}} > R|\nu_d\right\} = 1$. Finally, the expected throughput of the AQDF protocol is given by

$$\begin{aligned} R_{\text{av}}^{\text{AQDF}}(R,P,Q,a) &= R\Pr\left\{I_{\text{AQDF}} > R\right\} \\ &= R\int_0^\infty \Pr\left\{I_{\text{AQDF}} > R|\nu_d\right\} e^{-\nu_d}\,\mathrm{d}\nu_d \\ &= Re^{-\frac{1}{P}(e^R-1)} + R\int_0^a \int_{\frac{e^R-1}{P}}^\infty 1_{\{\mathcal{C}(R,P,Q,\nu_d)\}}(\nu_r)e^{-\nu_r-\nu_d}\,\mathrm{d}\nu_r\,\mathrm{d}\nu_d + \\ &\quad + R\int_a^{\frac{1}{P}(e^R-1)} \int_0^\infty 1_{\{\mathcal{D}(R,P,Q,\nu_d)\}}(\nu_r)e^{-\nu_r-\nu_d}\,\mathrm{d}\nu_r\,\mathrm{d}\nu_d \end{aligned}$$

where we have chosen to follow the intuition gained by the numerical optimization of the QF protocol with respect to the parameter $b$, and assumed that $b = \frac{1}{P}\left(e^R - 1\right)$. This throughput can further be optimized over $a$.

## V. Successive Quantization and Broadcast Transmission Using Two Information Levels

As was mentioned earlier, the QF protocol does not handle well the situation where the fading gain of the destination is less than $a$. In such a case, the destination has to rely on its own to decode the message, a task which is impossible, as $a$ is less than $\frac{1}{P}\left(e^R - 1\right)$. The AQDF protocol tries to improve this drawback by letting the relay amplify and forward in such a case. We now take a different approach for handling this situation. We wish to extend the QF scheme by introducing the concept of successive quantization and combining it with a broadcast based transmission approach [59] for the relay to destination link.

To be more specific, the relay, which does not know the fading realization $\nu_d$, performs two successive quantization operations of $Y_r$. In the first case, it assumes that the fading realization of the destination is in the interval $[a,b]$ for some $0 \le a \le b$. This implies that the side information available at the destination is at least $Y_d^{(a)}$, while the capacity of the link between the relay and the destination is at least $C_{\text{rd}}(b)$. The relay codes (a degraded version $U$ of) its received signal $Y_r$ assuming the above side information using a rate $\rho_1$. Next, the relay performs successive quantization of $Y_r$ assuming that $\nu_d$ is less than $a$. Since the relay does not know the amount of side information actually available (depending on the actual realization of $\nu_d$), it assumes that no side information at all is available at the destination. For this purpose it codes $Y_r$ using an additional rate $\rho_2$. By the results of Appendix D the coding rates $\rho_1$ and

$\rho_2$ are given by

$$\rho_1 = I(U;Y_r) - I\left(U;Y_d^{(a)}\right) = \log\left(\frac{1+\sigma^2+\frac{\nu_r P}{1+aP}}{\sigma^2}\right)$$
$$\rho_2 = I\left(U;Y_d^{(a)}\right) = \log\left(\frac{1+\sigma^2+\nu_r P}{1+\sigma^2+\frac{\nu_r P}{1+aP}}\right).$$

where the parameter $\sigma^2$ is the controlled distortion between the received signal $Y_r$ and its quantized version $U$, in other words the quantization "noise".

The meaning of these two rates is the following. If the realization of $\nu_d$ is in the interval $[a,b]$, then by Corollary 1, a mutual information of $I(X;Y_d,U)$ can be conveyed to the destination provided that the capacity of the link between the relay and the destination is at least $\rho_1$. If, however, $\nu_d < a$, then the destination will not be able to reproduce the quantized version of $Y_r$ without receiving an additional rate of $\rho_2$. Recall that the coding scheme which is used to establish achievability of the rate $I(X;Y_d,U)$ uses a Wyner-Ziv type of scheme, where the mutual information term $I\left(U;Y_d^{(a)}\right)$ represents the side information which is needed to resolve the ambiguity present in the relay's transmission. If this mutual information is not available to the destination, it must be supplied to it via the relay to destination link, and that is represented by the additional rate $\rho_2$.

To conclude, if $\nu_d \in [a,b]$ the relay has to deliver *less* information to the destination, because the destination has *more* side information. On the other hand, if $\nu_d < a$, the destination has little side information, and the relay has to compensate this by sending more information in the relay to destination link. Note that this behavior is in perfect match with the available capacity in the relay to destination link. When $\nu_d \in [a,b]$ the capacity of the link can be assumed at least $C_{\text{rd}}(b)$, while in the case where $\nu_d < a$ the capacity is at least $C_{\text{rd}}(a) > C_{\text{rd}}(b)$.

We now address the communication between the relay and the destination. We propose a broadcast approach [52], where the relay transmits two streams of information, allocating the power between the streams according to some parameter $\alpha \in [0,1]$. The rates of the two information streams are given by

$$R_1 = \log\left(1+\frac{\alpha Q}{1+bP+\bar{\alpha}Q}\right)$$
$$R_2 = \log\left(1+\frac{\bar{\alpha}Q}{1+aP}\right)$$

where the first rate can be decoded by the destination as long as $\nu_d < b$, and the second rate can be decoded if $\nu_d < a$ by decoding the first stream and cancelling it out from the received signal. The first stream is used for the first coding rate $\rho_1$, while the second stream is used for the additional rate $\rho_2$ which is needed if the side information is not sufficient at the destination ($\nu_d < a$).

In other words, when the channel conditions associated with the link between the relay and the destinations are good, i.e., $\nu_d < a$, the destination has little side information, so it will decode both streams of information needed to reproduce $U$. When the channel is poor, the destination can decode only one stream, however this is enough as it has better side information in this case. The question is how the relay should choose $\sigma^2$ and $\alpha$ and how it should match the quantization rates $\rho_1, \rho_2$ to the channel rates $R_1, R_2$.

We now state a more precise definition of this strategy. Fix the values of $a$ and $b$, and assume that the fading gain of the relay in some fading block is $\nu_r$. The relay selects a power allocation parameter $\alpha = 1 - \bar{\alpha} = f(P, Q, a, b, \nu_r)$ and a distortion parameter $\sigma^2 = g(P, Q, a, b, \nu_r)$ such that

$$\rho_1 = \log\left(\frac{1 + \sigma^2 + \frac{\nu_r P}{1+aP}}{\sigma^2}\right) \leq \log\left(1 + \frac{\alpha Q}{1 + bP + \bar{\alpha} Q}\right) = R_1.$$

This selection ensures that if the fading $\nu_d$ is in the interval $[a, b]$, the auxiliary random variable $U$ associated with the value of $\sigma^2$ will be reproduced exactly by the destination. Next, the relay uses its remaining power to send the rate $\rho_2$. This rate will be decoded if

$$\rho_2 = \log\left(\frac{1 + \sigma^2 + \nu_r P}{1 + \sigma^2 + \frac{\nu_r P}{1+aP}}\right) \leq \log\left(1 + \frac{\bar{\alpha} Q}{1 + \nu_d P}\right). \tag{33}$$

The mutual information conveyed to the destination is given by

$$I_{\text{SQF}} = \begin{cases} I(X; Y_d, U), & \nu_d \in [a, b] \text{ or } \left\{\nu_d \in [0, a] \text{ and eq. (33) is satisfied}\right\} \\ I(X; Y_d), & \text{otherwise} \end{cases} \tag{34}$$

The best expected throughput which can be obtained by this strategy is given by

$$R_{\text{av}}^{\text{SQF}}(R, P, Q) = \max_{a,b,f,g} R \Pr\left\{I_{\text{SQF}} > R\right\} \tag{35}$$

where the maximization is carried out over all constants $a$ and $b$ and over all functions $f$ and $g$. By choosing small values of $\sigma^2$ (with appropriate values of $\alpha$ close to unity to accommodate the required rate demand $\rho_1$) the relay increases the mutual information enjoyed by the destination in the interval $[a, b]$. But this makes the reception of the second rate less likely to occur, implying an information rate of only $I(X; Y_d)$ in the event that $\nu_d \notin [a, b]$. Larger values of $\sigma^2$ enable to reduce the necessary power allocation $\alpha$ and increase the probability that the second stream gets decoded. In this case, the rate $I(X; Y_d, U)$ has a better chance of being delivered to the destination even if $\nu_d < a$. Note that the QF protocol is actually one special case of this generalized strategy where

$$f(P, Q, a, b, \nu_r) \equiv 1$$
$$g(P, Q, a, b, \nu_r) = \frac{1}{Q}\left(1 + bP + \nu_r P \frac{1 + bP}{1 + aP}\right).$$

in which case the relay maximizes $I(X;Y_d,U)$, but this rate is delivered to the destination only if $\nu_d \in [a,b]$.

### A. The Simplified-Successive-Quantize-and-Forward (SSQF) Protocol

We now examine a simplified SQF protocol. Here the relay selects the smallest possible distortion parameter $\sigma^2$, such that the auxiliary random variable $U$ is always reconstructed at the receiver as long as $\nu_d \in [0,b]$, implying that a rate of $I(X;Y_d,U)$ will always be delivered to the destination as long as $\nu_d \in [0,b]$. If $\nu_d \in [0,a]$ the destination receiver needs both information streams to decode $U$ (because it has little side information), and when $\nu_d \in [a,b]$, the destination decodes only the first information stream which is enough to reconstruct $U$ with the help of the side information available. As before we assume that $a$ and $b$ are fixed parameters (to be optimized later on).

In this case, the mutual information conveyed to the receiver whenever $\nu_d \in [0,b]$ is always given by $\log\left(1+\nu_d P+\frac{\nu_r P}{1+\sigma^2}\right)$, and the relay wishes to select the smallest possible $\sigma^2$ such that the following two constraints are met,

$$\rho_1 = \log\left(\frac{1+\sigma^2+\frac{\nu_r P}{1+aP}}{\sigma^2}\right) \leq \log\left(1+\frac{\alpha Q}{1+bP+\bar{\alpha}Q}\right) = R_1$$

$$\rho_2 = \log\left(\frac{1+\sigma^2+\nu_r P}{1+\sigma^2+\frac{\nu_r P}{1+aP}}\right) \leq \log\left(1+\frac{\bar{\alpha}Q}{1+aP}\right) = R_2$$

These two conditions are equivalently written as

$$\sigma^2 \geq \frac{1}{\alpha Q}\left(1+\frac{\nu_r P}{1+aP}\right)(1+bP+\bar{\alpha}Q) \triangleq g_1\left(\nu_r,P,Q,a,b,\alpha\right) \tag{36}$$

$$\sigma^2 \geq \frac{\nu_r P\left(aP-\frac{\bar{\alpha}Q}{1+aP}\right)}{\bar{\alpha}Q} - 1 \triangleq g_2\left(\nu_r,P,Q,a,\alpha\right) \tag{37}$$

Note that $g_1\left(\nu_r,P,Q,a,b,1\right)$ and $g_2\left(\nu_r,P,Q,a,0\right)$ are both finite. Moreover, $g_1$ and $g_2$ are both monotonic functions of $\alpha$ in the interval $[0,1]$, but of opposite direction. For $\alpha \to 0$, $g_1$ is unbounded, and for $\alpha \to 1$ $g_2$ is unbounded. It is therefore desirable to select $\alpha$ such that

$$\max\left\{g_1\left(\nu_r,P,Q,a,b,\alpha\right), g_2\left(\nu_r,P,Q,a,\alpha\right)\right\} \tag{38}$$

is minimized. This selection is obtained by choosing $\alpha$ which satisfies:

$$g_1\left(\nu_r,P,Q,a,b,\alpha\right) = g_2\left(\nu_r,P,Q,a,\alpha\right).$$

Solving for $\alpha$ one gets

$$\alpha^* = \frac{(1+aP+\nu_r P)(1+bP+Q)}{(1+aP+\nu_r P)(1+bP+Q)+aP^2\nu_r(1+aP)}. \tag{39}$$

Substituting (39) in $g_2$ we get that the optimal $\sigma^2$ satisfies

$$\frac{\nu_r P}{1+\sigma^2} = \frac{\nu_r P Q}{1 + bP + Q + \nu_r P\left[\frac{1+bP}{1+aP} + aP\right]} \tag{40}$$

Therefore, the mutual information conveyed to the destination is given by

$$I_{\text{SSQF}} = \log\left(1 + \nu_d P + \frac{\nu_r P Q}{1 + bP + Q + \nu_r P\left[\frac{1+bP}{1+aP} + aP\right]}\right). \tag{41}$$

By comparing $I_{\text{SSQF}}$ with $I_{\text{QF}}$ (equation (27)) it can be seen that we are reducing the mutual information conveyed by this scheme, but hopefully making it more likely to actually be delivered when channel conditions are less favorable. We now compute the probability of successful decoding. As usual, we first consider the conditional probability given $\nu_d$. For the case $\nu_d \in [0, b]$ and $\nu_d < \frac{e^R-1}{P}$ we have

$$\begin{aligned}\Pr\left\{I_{\text{SSQF}} > R|\nu_d\right\} &= \Pr\left\{\frac{\nu_r P Q}{1 + bP + Q + \nu_r P\left[\frac{1+bP}{1+aP} + aP\right]} > e^R - 1 - \nu_d P\middle|\nu_d\right\} \\ &= \Pr\left\{\nu_r > \frac{\left(e^R - 1 - \nu_d P\right)(1 + bP + Q)}{PQ - \left(e^R - 1 - \nu_d P\right)P\left[\frac{1+bP}{1+aP} + aP\right]}\middle|\nu_d\right\}\end{aligned}$$

Note that this probability vanishes for $\nu_d < \frac{1}{P}\left(e^R - 1 - \frac{Q}{\frac{1+bP}{1+aP}+aP}\right)$ and assumes the value of unity whenever $\nu_d > \frac{e^R-1}{P}$. Averaging the conditional probability over $\nu_d$ we get

$$\Pr\left\{I_{\text{SSQF}} > R\right\} = e^{-\frac{e^R-1}{P}} + \int_0^b 1_{[\eta_1,\eta_2]}(\nu)\exp\left\{-\frac{\left(e^R - 1 - \nu P\right)(1 + bP + Q)}{PQ - \left(e^R - 1 - \nu P\right)P\left[\frac{1+bP}{1+aP} + aP\right]}\right\}e^{-\nu}\,\mathrm{d}\nu \tag{42}$$

where

$$\begin{aligned}\eta_1 &= \frac{1}{P}\left(e^R - 1 - \frac{Q}{\frac{1+bP}{1+aP} + aP}\right) \\ \eta_2 &= \frac{1}{P}\left(e^R - 1\right)\end{aligned}$$

and the expected throughput is thus

$$\begin{aligned}R_{\text{av}}^{\text{SSQF}}(R, P, Q, a, b) &= Re^{-\frac{e^R-1}{P}} + \\ &+ R\int_0^b 1_{[\eta_1,\eta_2]}(\nu)\exp\left\{-\frac{\left(e^R - 1 - \nu P\right)(1 + bP + Q)}{PQ - \left(e^R - 1 - \nu P\right)P\left[\frac{1+bP}{1+aP} + aP\right]}\right\}e^{-\nu}\,\mathrm{d}\nu\end{aligned} \tag{43}$$

Finally, this throughput can be further optimized (numerically) over $a$ and $b$.

## VI. NUMERICAL RESULTS, COMPARISONS, AND DISCUSSION

In this section we report the results of numerically computing the expected throughput of the various protocols described in the previous sections. It is instructive to compare these protocols with some or all of the four cases described below:

1) No-cooperation (NC): In this case, the source is not helped by the relay who remains silent throughout the transmission. The expected throughput in this case is given [50] by

$$R_{\text{av}}^{\text{NC}}(R,P) = Re^{-\frac{e^R-1}{P}}.$$

2) Transmitting to the best user (BU): This is when the source transmits only to the best user, and the other user remains silent. The corresponding expected throughput is given by [50]

$$R_{\text{av}}^{\text{BU}}(R,P) = 2Re^{-\frac{e^R-1}{P}} - Re^{-2\frac{e^R-1}{P}}.$$

3) Transmit Diversity (TD): Here the relay is assumed to fully cooperate with the source. This situation is equivalent to the assumption $\nu_r \to \infty$. The corresponding throughput is

$$\begin{aligned} R_{\text{av}}^{\text{TD}}(R,P,Q) &= R\,\text{Pr}\Big\{\log(1+\nu_d P + Q) > R\Big\} \\ &= \begin{cases} R, & R < \log(1+Q) \\ Re^{-\frac{e^R-1-Q}{P}}, & R > \log(1+Q) \end{cases}. \end{aligned}$$

4) Receive Diversity (RD): Here the two co-located users are assumed to be able to fully cooperate between themselves. This situation is equivalent to the assumption $Q \to \infty$. The expected throughput is given by

$$\begin{aligned} R_{\text{av}}^{\text{RD}}(R,P) &= R\,\text{Pr}\Big\{\log\left(1+(\nu_r+\nu_d)\,P\right) > R\Big\} \\ &= R\,\text{Pr}\left\{\nu_d + \nu_r > \frac{1}{P}\left(e^R-1\right)\right\} \\ &= R\left(1+\frac{e^R-1}{P}\right)e^{-\frac{e^R-1}{P}} \qquad (44) \end{aligned}$$

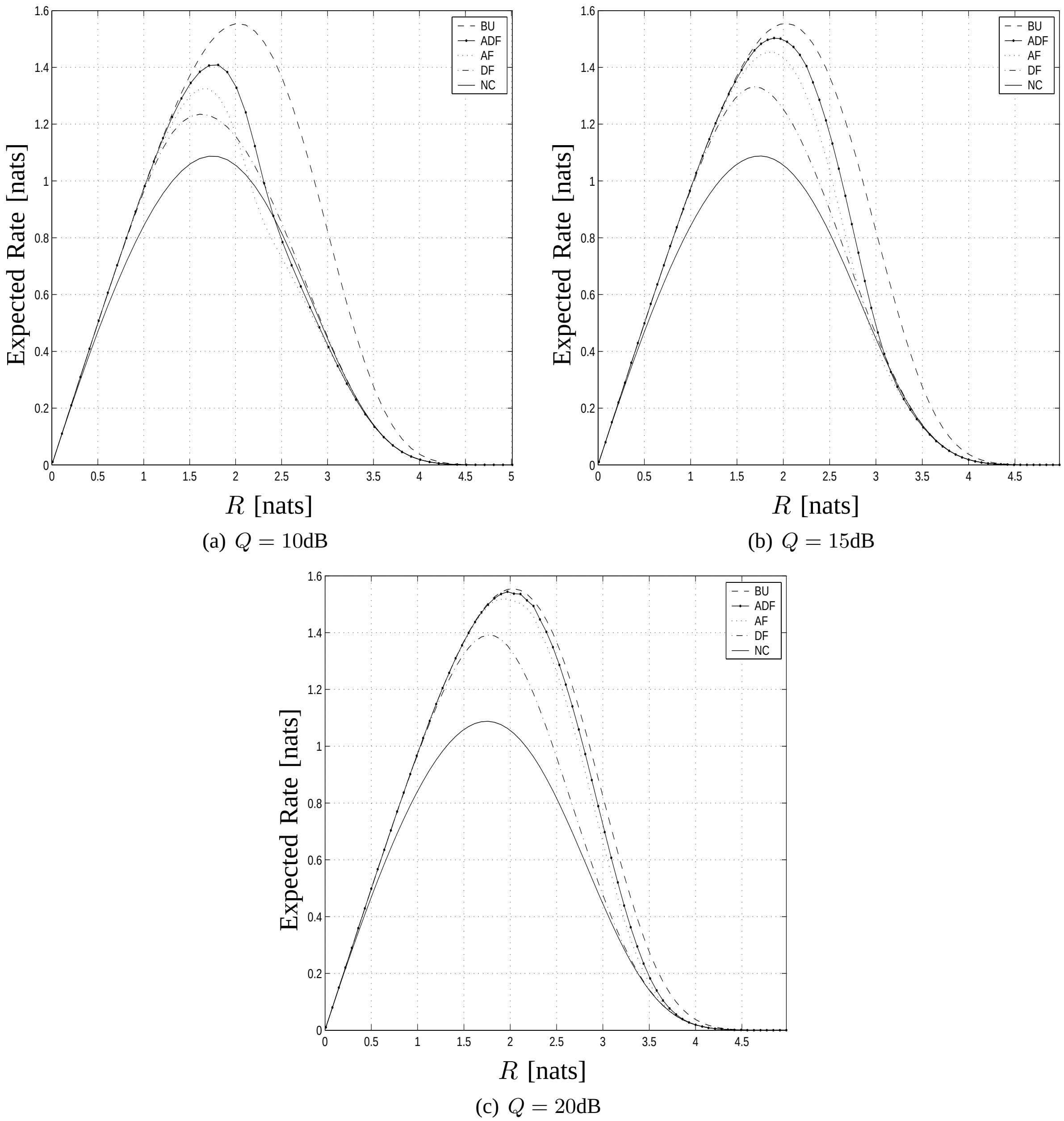
where (44) follows because $\nu_d + \nu_r$ is a chi-square random variable with 4 degrees of freedom.

Note that as opposed to all the protocols discussed in this paper, cases 1, 2, and 4 do not depend on the co-location gain $Q$.

### A. The Basic Protocols

We begin with the performance of the three basic protocols, namely, DF, AF, and ADF. It is important to realize that the throughput of all these schemes is upper bounded by the performance of the best user.

In the DF protocol, if $|a_d| > |a_r|$, then user $r$ stays silent at least until user $d$ decodes the message, so the performance is just that of the best user. If, on the other hand, $|a_r| > |a_d|$, then even if $Q$ is enormous, user $d$ will not be able to decode the message before user $r$ does. At best, it will decode the message a short time afterwards, so again we are bounded by the best user's performance. Next, by proposition 1, the performance of the AF protocol for any power policy, in particular for the policy $q^*(\cdot)$, is bounded




(a) $Q = 10$dB (b) $Q = 15$dB (c) $Q = 20$dB

Fig. 2. Expected throughput, $P = 10$ dB.

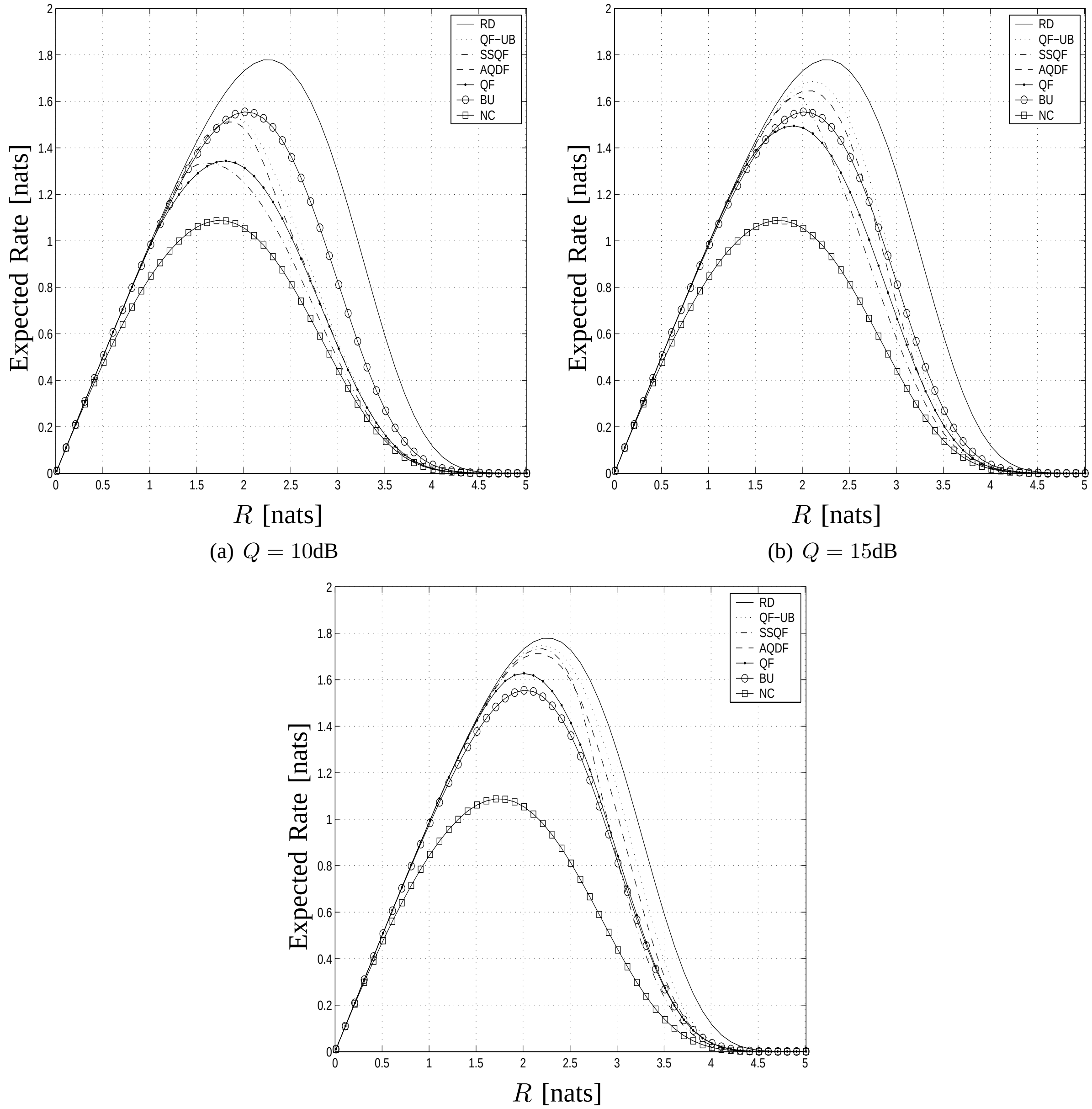
by that of the best user. Finally, consider the ADF protocol. If the relay does not succeed to decode the message before the destination does, then the scheme reduces to the AF scheme, which was shown to perform worse than the best user's performance. If the relay does succeed to decode the message before the destination does, then it was the strongest user, and in particular, the destination does not decode the message any sooner.

The performance of these protocols is shown in Fig. 2 for $P = 10$ dB. Since we are considering a setting were the users are co-located, it is natural to assume that the co-location gain is typically larger than the power level at which the users hear the remote source. We therefore examine three values of co-location gain, namely, $Q = 10, 15,$ and $20$ dB. These values correspond to situations where the co-location is limited, moderate, or high, respectively. A common feature of the expected throughput analysis of these protocols is that they all exhibit a peek for some choice of rate $R$, which we call the *maximizing attempted rate* or simply the *maximizing rate*. One way of comparing between the protocols is by examining their maximal achievable expected rates at the maximizing rate. In this respect, it seems that the ADF outperform both the AF and DF protocols, where the DF has the weakest performance. However, when the co-location gain is only limited, both the AF and ADF suffer some degradation for rates above the maximizing rate. This degradation can be attributed to the suboptimal decentralized power control policy which we employed in those schemes. Recall that $q^*(\cdot)$ was shown to be optimal only when the co-location gain is large. The results seem to confirm this result. It seems that this choice is good when the co-location gain approaches $\infty$, but becomes far from optimal when the co-location gain is small. Of course, as we increase the co-location gain $Q$, all protocols approach the performance of the best user, reflecting the fact that they are all bounded by this performance.

### *B. Quantization Based Protocols*

In all of the preceding protocols, the performance was bounded by the performance of the best user. We turn now to the more efficient protocols, which are all based on Wyner-Ziv quantization. Fig. 3 depicts the performance of the QF, AQDF, and SSQF protocols. For high co-location gains, the SSQF protocol seems to achieve the highest maximal throughput, almost closing the gap with respect to QF-UB upper bound (which assumes the relay knows $\nu_d$). However, for rates higher than the maximizing rate, and for lower co-location gains, the performance of the SSQF falls below that of the other protocols. The AQDF protocol seems to be more robust in those situations, keeping its close proximity to the QF-UB upper bound in a wide range of rates and co-location gains.

It is interesting to examine the relationship between the SSQF and AQDF protocols more closely.




(a) $Q = 10$dB

(b) $Q = 15$dB

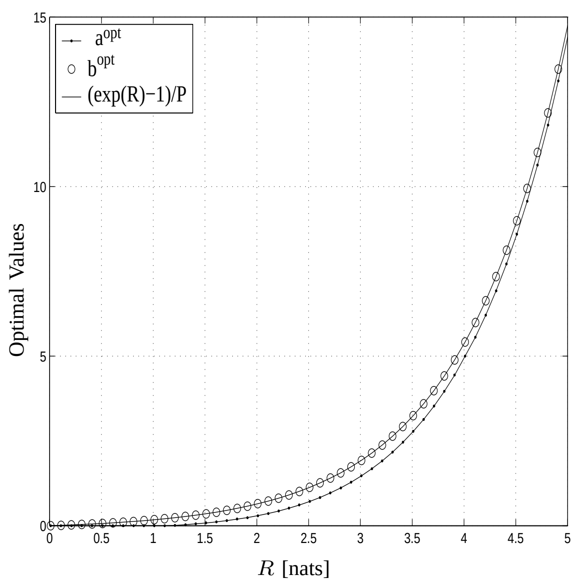
(c) $Q = 20$dB

Fig. 3. Expected throughput for quantization based protocols, $P = 10$ dB.

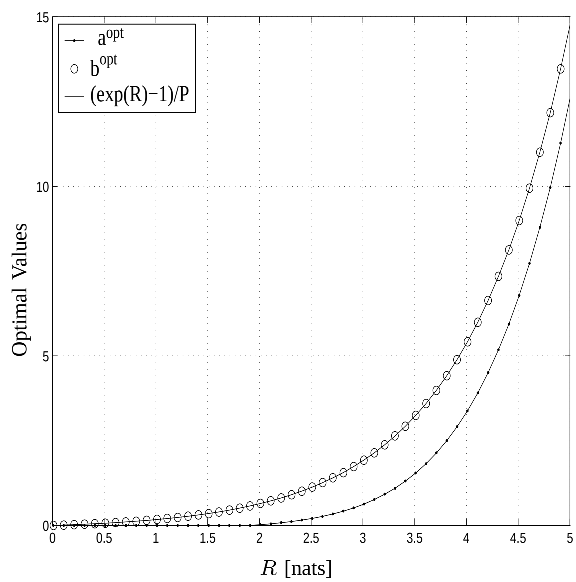
Recall that the quantization procedure is different in both cases. In AQDF the quantization is performed only if $\nu_d \in [a, b]$, and in SSQF it is performed for all $\nu_d \in [0, b]$. That is why the quantization noise in SSQF is larger than in AQDF (Compare eq. (26) for AQDF with eq. (36) for SSQF). When the co-location gain factor $Q$ is high, then despite the fact that the quantization in SSQF is slightly more noisy (as to allow reconstruction for low values of $\nu_d$), and despite that it does not contain the DF part, it is still better than AQDF. This happens for two reasons: a) By results of [12] it is known that QF is better than DF when the relay is very close to the destination (i.e. when $Q$ is large), so the lack of the DF component in SSQF is not too harmful. b) The QF part for $\nu_d < a$ is better than just AF in this case. On the other hand, when $Q$ is small, the quantization in SSQF becomes very inefficient, such that it is worse than AF, and the lack of the DF part becomes more dominant.

These results further show that the usage of such protocols seems to be beneficial when the co-location gain is at least moderately higher than the power level of the received source's transmission. The QF protocol appears to yield large benefit only when the co-location is relatively large.

Another interesting question which arises with respect to these protocols is how one should select the endpoints of the interval $[a, b]$ in which the fading gain of the destination must fall in order for the quantization and forwarding scheme to work. Some insight may be gained by looking at the optimal values of $a$ and $b$ which were obtained numerically for the QF protocol (see Fig. 4).




(a) $Q = 10$ dB




(b) $Q = 20$ dB

Fig. 4. Optimal values of $a$ and $b$ for the QF protocol, $P = 10$ dB.

Fig. 4 depicts the behavior of the optimal values of $a$ and $b$ for $P = 10$ dB and for two values of co-location gain. As was mentioned at the end of section IV-B, the value of $b$ can be restricted to $b \leq \frac{1}{P}\left(e^R - 1\right)$ with no loss in performance. The numerical results confirm this fact, and further suggest that $b$ should be exactly equal to $\frac{1}{P}\left(e^R - 1\right)$ in a wide range of co-location gains and attempted coding rates. In other words, we would like the destination receiver to receive helpful information from the relay starting at the moment when its fading gain no longer allows it to decode the message alone (i.e. when $\nu_d < \frac{1}{P}\left(e^R - 1\right)$).

As for $a$, one would like to select the smallest $a$ possible, so as to cover as much "bad" fading realization of the destination, but not too small, as then the amount of side information which is assumed available to the destination will be too small, causing an overloading of the link from the relay to the destination with the quantized information from the relay. The optimal value of $a$ determines the best trade off from the standpoint of the expected throughput.

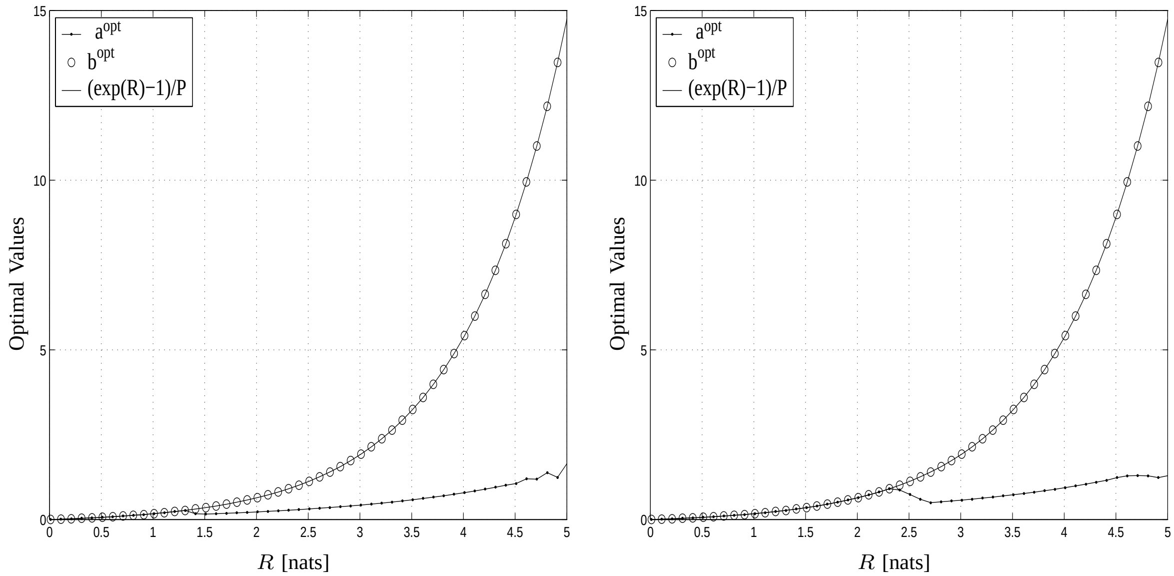
Another observation can be made with respect to the size of the quantize-and-forward fading window $[a, b]$. When the co-location gain is large, it allows to assume less side information available at the destination, since the channel between the relay and the destination can accommodate transmission of more information. When capacity of the relay to destination link is small, however, more side information is needed by the destination in order to decode the quantized information, rendering the value of $a$ higher.

We can further examine the optimal values of $a$ and $b$ for the SSQF protocol. These values are depicted in Fig. 5. Note that as in the QF protocol we can restrict the optimization to $b \leq \frac{1}{P}\left(e^R - 1\right)$ at no performance cost. Here also it appears that the best choice is in fact $b = \frac{1}{P}\left(e^R - 1\right)$. A common behavior in all cases of co-location gain is that when the rate is small, it is best to assume no side information at all and simply transmit the whole quantized information by the relay. As the rate increases, it is better to assume that some side information might be available to the destination, and rely on the opposite match between the side information available at the destination and the capacity of the relay to destination link.

## VII. Conclusions

We have explored several cooperative strategies tailored for situations in wireless networks where co-located users can act as potential relays for one another, and where channel state information is available only to the receivers. Two groups of protocols were examined. The first group of protocols includes variations of decode-and-forward and amplify-and-forward. A common feature of these protocols is that their performance is upper bounded by the performance of the best user, and that they approach this performance as the co-location gain increases. Among these protocols, the hybrid ADF protocol showed the best behavior.

The second group of protocols was based on the notion of Wyner-Ziv quantization at the relay. That is, the relay quantizes its received observation of the source's transmitted symbol relying on the side information available at the destination, and sends this quantized information to the destination over




(a) $Q = 10$ dB (b) $Q = 20$ dB

Fig. 5. Optimal values of $a$ and $b$ for the SSQF protocol, $P = 10$ dB.

an error free link in which the source's transmission is considered by the destination as noise. Since channel state information is known only locally (by the receivers), the relay has trouble knowing even the statistics of the signal received by the destination. In order to be able to perform the efficient quantization, a sub-optimal scheme was proposed where the relay assumes some minimal level of side information available when performing the quantization. An inverse relation which exists between the amount of side information available to the destination, and the capacity of the error free link between the relay and the destination was utilized, where a successive quantization at the relay was matched with a double stream broadcast transmission strategy. Finally a hybrid scheme which encompasses three notions of relay operation in one scheme was proposed and analyzed.

Numerical results demonstrated the superiority of the quantization approaches with respect to the basic schemes, which do not involve quantization. Among these protocols, the scheme based on successive quantization and a broadcast approach yields the best maximal expected rate for high co-location gains. However, when one operates at lower co-location gains or higher coding rates than the maximizing rate, the performance of the hybrid AQDF protocol seems to give better performance. It is noted that the schemes based on quantization all approach the performance of a full receive diversity scheme employing two fully cooperating receive antennas.

While the optimal relaying strategy remains unknown in general, it has been observed [12] that decode-and-forward schemes are more suited in cases where the relay enjoys favorable channel conditions with respect to the destination. For instance, in some wireless networks this strategy even achieves capacity (see discussion in section I). On the other hand, when the relay is close to the destination, quantization schemes offer better performance. Our setting deals with the latter scenario, and in this respect conforms to the reported findings.

These observations together with the successful performance of the AQDF scheme point to the direction of hybrid schemes. It appears that a good relaying scheme should be flexible in the sense that the relay could adapt its operation depending upon channel conditions (to which it has access to). From our results it appears that the ability of the relay to decode the message or the inability thereof plays a crucial role in determining in which way the relay should best act. This role is more pronounced when the co-location gain is high.

It would be of interest to examine ways in which these quantization schemes can be extended to settings with multiple co-located relays. While a general extension seems somewhat challenging, our results can be readily used to gain some insight into the possible performance of a multi–relay system. For instance, consider the case where there are $K$ co-located users. One can examine a suboptimal scheme where the strongest relay (in the sense that it enjoys the best channel conditions) quantizes and forwards its observation to the destination. Such a scheme, which serves as a lower bound to the general multi–relay system, can be analyzed by applying our results for the two-relay case, after modifying the statistics of the best relay's fading gain. An upper bound can be obtained by allowing all the relays to cooperate. This situation is equivalent to a single super–relay which has $K-1$ receive antennas and $K-1$ transmit antennas, and the results here can be used to assess this case as well after some straight forward modifications.

## APPENDIX A
## AN ISI CHANNEL

The channel (3) can equivalently be written as

$$Y_d(n) = a_d X(n) + \beta a_r X(n-1) + \tilde{Z}_d(n), \quad n = 1, 2, \ldots, N \tag{45}$$

where $\tilde{Z}_d(n) = \beta Z_r(n-1) + Z_d(n)$ and $\mathbb{E}\left[\tilde{Z}_d(n)\tilde{Z}_d^*(m)\right] = (\beta^2+1)\delta_{nm}$. In this case, the average mutual information between the source and the destination assuming i.i.d. signalling over long blocks is given by [60]–[62]

$$\begin{aligned}
I_{\text{AF}} &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left(1+\frac{P}{\beta^2+1}\left(|a_d+\beta a_r e^{-j\omega}|^2\right)\right)\,\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left(1+\frac{P}{\beta^2+1}\left(\nu_d+\beta^2\nu_r+2\beta\sqrt{\nu_r\nu_d}\cos\omega\right)\right)\,\mathrm{d}\omega \\
&= \frac{1}{\pi}\int_{0}^{\pi}\log\left(1+\frac{P\left(\nu_d+\beta^2\nu_r\right)}{\beta^2+1}+\frac{2P\beta\sqrt{\nu_r\nu_d}}{\beta^2+1}\cos\omega\right)\,\mathrm{d}\omega \\
&= \log\left(\frac{1+\frac{P(\nu_d+\beta^2\nu_r)}{\beta^2+1}+\sqrt{\left(1+\frac{P(\nu_d+\beta^2\nu_r)}{\beta^2+1}\right)^2-\left(\frac{2P\beta\sqrt{\nu_r\nu_d}}{\beta^2+1}\right)^2}}{2}\right) \\
&= \log\left[\left(1+\frac{P\left(\nu_d+\beta^2\nu_r\right)}{\beta^2+1}\right)\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\beta\sqrt{\nu_r\nu_d}}{\beta^2+1+P\left(\nu_d+\beta^2\nu_r\right)}\right)^2}\right)\right] \\
&= \log\left(1+P\frac{\nu_d(1+\nu_r P)+\nu_r q(\nu_r)}{1+\nu_r P+q(\nu_r)}\right)+\log\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\left(\frac{2P\sqrt{q(\nu_r)\nu_r\nu_d}}{(1+\nu_d P+q(\nu_r))\sqrt{1+\nu_r P}}\right)^2}\right).
\end{aligned}$$

## APPENDIX B

## PERFORMANCE OF AF IS BOUNDED BY THE PERFORMANCE OF THE BEST USER

By definition of the expected throughput it suffices to show that for every policy $q(\cdot)$ satisfying (4)

$$I_{\text{AF-UB}}(q) \le \log\bigl(1+\max\{\nu_d,\nu_r\}\,P\bigr).$$

Consider the function $f(x)$ defined for $x \in [0, Q]$ by

$$f(x) \triangleq \frac{\nu_d(1+\nu_r P)+\nu_r x}{1+\nu_r P+x}$$

whose derivative equals

$$f'(x) = \frac{(\nu_r-\nu_d)\,(1+\nu_r P)}{(1+\nu_r P+x)^2}.$$

If $\nu_r > \nu_d$ the function $f(x)$ achieves its maximum value at $x = Q$. Conversely, if $\nu_r \le \nu_d$ the function achieves its maximum value for $x = 0$. Now consider the case where the relay is the stronger user, i.e.

$\nu_r > \nu_d$. Then

$$\begin{aligned}
I_{\text{AF-UB}} - \log\bigl(1 + \max\{\nu_d, \nu_r\} P\bigr) &= I_{\text{AF-UB}} - \log(1 + \nu_r P) \\
&= \log\left(1 + P\frac{\nu_d(1+\nu_r P) + \nu_r q(\nu_r)}{1 + \nu_r P + q(\nu_r)}\right) - \log(1 + \nu_r P) \\
&\le \log\left(1 + P\frac{\nu_d(1+\nu_r P) + \nu_r Q}{1 + \nu_r P + Q}\right) - \log(1 + \nu_r P) \\
&= \log\frac{(1 + \nu_d P + Q)(1 + \nu_r P)}{1 + \nu_r P + Q} - \log(1 + \nu_r P) \\
&= \log\frac{1 + \nu_d P + Q}{1 + \nu_r P + Q} \\
&< 0
\end{aligned} \tag{46}$$

Next, if the destination is strongest, i.e. $\nu_r \le \nu_d$, we have

$$\begin{aligned}
I_{\text{AF-UB}} - \log\bigl(1 + \max\{\nu_d, \nu_r\} P\bigr) &= I_{\text{AF-UB}} - \log(1 + \nu_d P) \\
&= \log\left(1 + P\frac{\nu_d(1+\nu_r P) + \nu_r q(\nu_r)}{1 + \nu_r P + q(\nu_r)}\right) - \log(1 + \nu_d P) \\
&\le \log(1 + \nu_d P) - \log(1 + \nu_d P) \\
&= 0
\end{aligned} \tag{47}$$

proving the proposition.

□

## APPENDIX C

## ASYMPTOTIC OPTIMALITY OF $q^*(\cdot)$

We prove the proposition in several steps. We first show that the policy $q^*(\cdot)$ maximizes the expected throughput associated with the mutual information upper bound, namely

$$R_{\text{av}}^{\text{AF-UB}}(R, P, Q, q(\cdot)) = R\,\Pr\{I_{\text{AF-UB}}(q) > R\}\,. \tag{48}$$

To this end, denote by $p_d^{\text{AF-UB}}(q)$ the probability of successful decoding by the destination associated with (48). This probability is given by

$$\begin{aligned}
p_d^{\text{AF-UB}}(q) &= \Pr\left\{I_{\text{AF-UB}}(q) > R\right\} \\
&= \Pr\left\{\log\left(1 + \frac{P\left(\nu_r\nu_d P + \nu_d + \nu_r q(\nu_r)\right)}{1 + \nu_r P + q(\nu_r)}\right) > R\right\} \\
&= \Pr\left\{\log\frac{(1 + \nu_d P + q(\nu_r))(1 + \nu_r P)}{1 + \nu_r P + q(\nu_r)} > R\right\} \\
&= \Pr\left\{\nu_d > \frac{1}{P}\left[e^R\left(1 + \frac{q(\nu_r)}{1 + \nu_r P}\right) - 1 - q(\nu_r)\right]\right\} \\
&= \int_0^\infty \Pr\left\{\nu_d > \frac{1}{P}\left[e^R\left(1 + \frac{q(u)}{1 + uP}\right) - 1 - q(u)\right] \,\middle|\, \nu_r = u\right\} p_{\nu_r}(u)\,\mathrm{d}u
\end{aligned}$$

Since $\Pr\left\{\nu_d > \alpha \,|\, \nu_r = \beta\right\} = e^{-\alpha}$, $\alpha \geq 0$, $\beta \geq 0$, and $p_{\nu_r}(\alpha) = e^{-\alpha}$, it follows that

$$p_d^{\text{AF-UB}}(q) = \int_{\mathcal{A}(R,P,Q)} \exp\left\{-\frac{1}{P}\left[e^R\left(1 + \frac{q(u)}{1 + uP}\right) - 1 - q(u)\right] - u\right\}\mathrm{d}u + \int_{\overline{\mathcal{A}}(R,P,Q)} e^{-u}\,\mathrm{d}u, \quad (49)$$

where $\mathcal{A}(R,P,Q) \triangleq \{u \in [0,\infty) : f_q(u) \geq 0\}$ and where $f_q(u)$ is defined by

$$f_q(u) \triangleq \frac{1}{P}\left[e^R\left(1 + \frac{q(u)}{1 + uP}\right) - 1 - q(u)\right], \quad u \geq 0.$$

We now wish to characterize the set $\mathcal{A}(R,P,Q)$. Note that from the constraints (4) it follows that

$$\begin{aligned}
f_q(0) &= \frac{1}{P}\left[e^R\left(1 + \frac{q(0)}{1 + uP}\right) - 1 - q(0)\right] \\
&= \frac{e^R - 1}{P}.
\end{aligned}$$

Next, observe that since

$$\begin{aligned}
f_q(u) &= \frac{1}{P}\left[e^R\left(1 + \frac{q(u)}{1 + uP}\right) - 1 - q(u)\right] \\
&= \frac{1}{P}\left[\frac{q(u)(e^R - 1 - uP)}{1 + uP} + e^R - 1\right] \quad (50)
\end{aligned}$$

it follows that for any $u < \frac{e^R-1}{P}$, there holds $f_q(u) \geq 0$. Finally, note that from the derivative of $f_q(u)$, namely,

$$\begin{aligned}
Pf_q'(u) &= e^R\frac{q'(u)(1 + uP) - q(u)P}{(1 + uP)^2} - q'(u) \\
&= \frac{q'(u)(1 + uP)(e^R - 1 - uP) - e^R P q(u)}{(1 + uP)^2} \quad (51)
\end{aligned}$$

it follows that the function $f_q(u)$ is monotonically decreasing for all $u > \frac{e^R-1}{P}$. It therefore follows that $f_q(u)$ either has one zero crossing, or it is nonnegative for all $u$. Since $\lim_{u\to\infty} f_q(u) = \frac{e^R-1-Q}{P}$, then if $R < \log(1+Q)$ there is one zero crossing, and if $R \geq \log(1+Q)$ there is no zero crossing. We can therefore rewrite (49) as follows

$$p_d^{\text{AF-UB}}(q) = \int_0^{u^*} \exp\left\{-\frac{1}{P}\left[e^R\left(1+\frac{q(u)}{1+uP}\right)-1-q(u)\right]-u\right\} \mathrm{d}u + e^{-u^*}, \tag{52}$$

where $u^*$ is the unique solution to the equation $f_q(u) = 0$ for $R < \log(1+Q)$, and $u^* = \infty$ for $R \geq \log(1+Q)$. We now claim that this probability is maximized for the power control policy

$$q^*(u) = \begin{cases} 0, & u < \frac{e^R-1}{P} \\ Q, & u \geq \frac{e^R-1}{P} \end{cases}. \tag{53}$$

To verify this it suffices to show that $f_{q^*}(u) \leq f_q(u), \quad \forall u \geq 0$, for any power control $q(u)$ satisfying (4). But this holds because for all $u \geq 0$ there holds

$$\begin{aligned} f_{q^*}(u) - f_q(u) &= \frac{1}{P}\left[e^R\left(1+\frac{q^*(u)}{1+uP}\right)-1-q^*(u)\right] - \frac{1}{P}\left[e^R\left(1+\frac{q(u)}{1+uP}\right)-1-q(u)\right] \\ &= \frac{1}{P}\left(\frac{e^R}{1+uP}-1\right)(q^*(u)-q(u)) \\ &\leq 0. \end{aligned}$$

It therefore follows that the expected throughput (48) is maximized for this selection. The second step is to show that the performance associated with the mutual information upper bound and $q^*(\cdot)$ approaches the performance of the best user as $Q \to \infty$, namely

$$\lim_{Q\to\infty} R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) = R_{\text{av}}^{\text{BU}}(R,P).$$

To see this compute the expected throughput corresponding to $q^*(\cdot)$ for all $Q$, by substituting $q^*(\cdot)$ into (52), namely

$$\begin{aligned} R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) &= R p_d^{\text{AF-UB}}(q^*) \\ &= R\int_0^{\frac{e^R-1}{P}} \exp\left\{-\frac{e^R-1}{P}-u\right\} \mathrm{d}u + \\ &\quad + R\int_{\frac{e^R-1}{P}}^{u^*} \exp\left\{-\frac{1}{P}\left[e^R\left(1+\frac{Q}{1+uP}\right)-1-Q\right]-u\right\} \mathrm{d}u + Re^{-u^*}, \end{aligned}$$

where

$$u^* = \begin{cases} \frac{(1+Q)(e^R-1)}{(1+Q-e^R)P}, & R < \log(1+Q) \\ \infty, & R \geq \log(1+Q) \end{cases}.$$

For fixed $R$, and large enough $Q$ we can write

$$\begin{aligned} R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) &= Re^{-\frac{e^R-1}{P}}\left(1-e^{-\frac{e^R-1}{P}}\right) \\ &\quad + R\int_{\frac{e^R-1}{P}}^{\frac{(1+Q)\left(e^R-1\right)}{\left(1+Q-e^R\right)P}} \exp\left\{-\frac{1}{P}\left[e^R\left(1+\frac{Q}{1+uP}\right)-1-Q\right]-u\right\}\mathrm{d}u \\ &\quad + Re^{-\frac{(1+Q)\left(e^R-1\right)}{\left(1+Q-e^R\right)P}}. \end{aligned} \tag{54}$$

Since $u^* \xrightarrow[Q\to\infty]{} \frac{e^R-1}{P}$, it follows that the second term in (54) approaches zero, yielding

$$\begin{aligned} \lim_{Q\to\infty} R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) &= Re^{-\frac{e^R-1}{P}}\left(1-e^{-\frac{e^R-1}{P}}\right)+Re^{-\frac{e^R-1}{P}} \\ &= 2Re^{-\frac{e^R-1}{P}} - Re^{-2\frac{e^R-1}{P}} \\ &= R_{\text{av}}^{\text{BU}}(R,P) \end{aligned} \tag{55}$$

Finally, we show that the expected throughput associated with the upper bound and the expected throughput associated with the true performance coincide for the policy $q^*(\cdot)$ as $Q\to\infty$, i.e.

$$\lim_{Q\to\infty}\left[R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) - R_{\text{av}}^{\text{AF}}(R,P,Q,q^*(\cdot))\right] = 0. \tag{56}$$

Recall that

$$\begin{aligned} R_{\text{av}}^{\text{AF-UB}}(R,P,Q,q^*(\cdot)) &= R\,\Pr\left\{T_1(Q)>R\right\} \\ R_{\text{av}}^{\text{AF}}(R,P,Q,q^*(\cdot)) &= R\,\Pr\left\{T_1(Q)+T_2(Q)>R\right\} \end{aligned}$$

where $T_1(Q)$ and $T_2(Q)$ are the first and second terms, respectively, of the AF mutual information (5) for the policy $q^*(\cdot)$. Note that for all $0<\epsilon<\log 2$

$$\begin{aligned} \Pr\left\{|T_2(Q)|>\epsilon\right\} &= \int_0^\infty \Pr\left\{\frac{\sqrt{q^*(\nu_r)\nu_r\nu_d}}{(1+\nu_dP+q^*(\nu_r))\sqrt{1+\nu_rP}} > \frac{1}{2P}\sqrt{1-(2e^{-\epsilon}-1)^2}\,\middle|\,\nu_r\right\} e^{-\nu_r}\,\mathrm{d}\nu_r \\ &= \int_{\frac{e^R-1}{P}}^\infty \Pr\left\{\frac{\sqrt{Q\nu_r\nu_d}}{(1+\nu_dP+Q)\sqrt{1+\nu_rP}} > \frac{1}{2P}\sqrt{1-(2e^{-\epsilon}-1)^2}\,\middle|\,\nu_r\right\} e^{-\nu_r}\,\mathrm{d}\nu_r \\ &\le \int_{\frac{e^R-1}{P}}^\infty \Pr\left\{\frac{\sqrt{Q(1+\nu_rP)\nu_d}}{(1+\nu_dP+Q)\sqrt{1+\nu_rP}} > \frac{1}{2\sqrt{P}}\sqrt{1-(2e^{-\epsilon}-1)^2}\,\middle|\,\nu_r\right\} e^{-\nu_r}\,\mathrm{d}\nu_r \\ &= e^{-\frac{e^R-1}{P}}\Pr\left\{\frac{\sqrt{Q\nu_d}}{1+\nu_dP+Q} > \frac{1}{2\sqrt{P}}\sqrt{1-(2e^{-\epsilon}-1)^2}\right\} \\ &\le e^{-\frac{e^R-1}{P}}\Pr\left\{\nu_d > \frac{Q}{4P}\left(1-\left(2e^{-\epsilon}-1\right)^2\right)\right\} \\ &= e^{-\frac{e^R-1}{P}}\exp\left\{-\frac{Q}{4P}\left(1-\left(2e^{-\epsilon}-1\right)^2\right)\right\} \xrightarrow[Q\to\infty]{} 0 \end{aligned}$$

which implies that $T_2(Q) \xrightarrow[Q\to\infty]{P} 0$. Next, observe that (55) implies that

$$T_1(Q) \xrightarrow[Q\to\infty]{D} \log\left(1+\max\left\{\nu_r,\nu_d\right\}P\right). \tag{57}$$

By [57, Theorem C.3] it follows that $T_1(Q)+T_2(Q) \xrightarrow[Q\to\infty]{D} \log\left(1+\max\left\{\nu_r,\nu_d\right\}P\right)$, which implies (56). Combining (55) and (56) yields the result.

## APPENDIX D

## DERIVATION OF EQUATIONS (14) AND (15)

We start with the object function

$$\begin{aligned} I(X;Y_d,U) &= I(X;Y_d) + I(X;U|Y_d) \\ &= I(X;Y_d) + H(U|Y_d) - H(U|X,Y_d) \\ &\overset{(a)}{=} I(X;Y_d) + H(U|Y_d) - H(U|X) \end{aligned} \tag{58}$$

where $(a)$ is due to the Markovity of $Y_d \Longleftrightarrow X \Longleftrightarrow Y_r \Longleftrightarrow U$. The first term in (58) is easily given by

$$I(X;Y_d) = \log\left(1+|a_d|^2P\right).$$

As for the third term in (58), since $U = a_rX + Z_r + W$ it follows that

$$H(U|X) = H(Z_r+W|X) = \log\left(\pi e\left(1+\sigma^2\right)\right).$$

We now address the second term in (58). Consider the random vector $[U\ Y_d]^T$ whose covariance matrix is given by

$$\Sigma = \mathbb{E}\begin{bmatrix} U \\ Y_d \end{bmatrix}[U^*,\ Y_d^*] = \begin{pmatrix} \nu_rP+1+\sigma^2 & a_ra_d^*P \\ a_da_r^*P & |a_d|^2P+1 \end{pmatrix}$$

and whose log-determinant is given by

$$\begin{aligned} \log\det\Sigma &= \log\Big(\left(|a_r|^2P+1+\sigma^2\right)\left(|a_d|^2P+1\right) - |a_r|^2|a_d|^2P^2\Big) \\ &= \log\Big(|a_r|^2P + \left(1+\sigma^2\right)\left(|a_d|^2P+1\right)\Big) \end{aligned}$$

The second term in (58) can now be written as

$$\begin{aligned} H(U|Y_d) &= H(U,Y_d) - H(Y_d) \\ &= \log\det\left(\pi e\Sigma\right) - \log\left[\pi e\left(|a_d|^2P+1\right)\right] \\ &= \log \pi e\left(1+\sigma^2+\frac{|a_r|^2P}{|a_d|^2P+1}\right) \end{aligned} \tag{59}$$

Combining all three terms in (58) we have

$$\begin{aligned} I(X;Y_d,U) &= \log\left(1+|a_d|^2P\right)+\log\left(\pi e\left(1+\sigma^2+\frac{|a_r|^2P}{|a_d|^2P+1}\right)\right)-\log\left(\pi e\left(1+\sigma^2\right)\right) \\ &= \log\left(1+|a_d|^2P\right)+\log\left(1+\frac{|a_r|^2P}{\left(|a_d|^2P+1\right)\left(1+\sigma^2\right)}\right). \end{aligned}$$

Next, we wish to express the Wyner-Ziv constraint (15). First, note that

$$\begin{aligned} I(U;Y_r) &= \log\left(1+\frac{|a_r|^2P+1}{\sigma^2}\right) \\ &= \log\left(\frac{1+\sigma^2+|a_r|^2P}{\sigma^2}\right). \end{aligned} \tag{60}$$

Next, since $I(U;Y_d)=H(U)-H(U|Y_d)$ and since $H(U)=\log \pi e\left(|a_r|^2P+1+\sigma^2\right)$, using (59) we get

$$\begin{aligned} I(U;Y_d) &= \log \pi e\left(1+\sigma^2+|a_r|^2P\right)-\log \pi e\left(1+\sigma^2+\frac{|a_r|^2P}{1+\nu_dP}\right) \\ &= \log\left(\frac{1+\sigma^2+|a_r|^2P}{1+\sigma^2+\frac{|a_r|^2P}{1+\nu_dP}}\right) \end{aligned} \tag{61}$$

By combining (60) and (61) we get the Wyner-Ziv constraint

$$I(U;Y_r)-I(U;Y_d)=\log\left(\frac{1+\sigma^2+\frac{|a_r|^2P}{1+\nu_dP}}{\sigma^2}\right).$$

## REFERENCES


[1] E. C. Van Der Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 121–154, 1971.

[2] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sept. 1979.

[3] A. Høst-Madsen and J. Zhang, "Capacity bounds and power allocation for the wireless relay channel," Sept. 2003, submitted to IEEE Trans. Inform. Theory.

[4] B. Wang, J. Zhang, and A. Host-Madsen, "On capacity of MIMO relay channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 29–43, Jan. 2005.

[5] A. El Gamal, M. Mohseni, and S. Zahedi, "On reliable communication over additive white Gaussian noise relay channels," Sept. 2004, submitted to IEEE Trans. Inform. Theory.

[6] A. El Gamal and S. Zahedi, "Capacity of a class of relay channels with orthogonal components," to appear in IEEE Trans. Inform. Theory.

[7] S. Zahedi, M. Mohseni, and A. El Gamal, "On the capacity of AWGN relay channels with linear relaying functions," in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 399.

[8] A. El Gamal and S. Zahedi, "Minimum energy communication over a relay channel," in *IEEE Int. Symp. Inform. Theory*, Yokohama, Japan, June 29 – July 4 2003, p. 344.

[9] A. Reznik, S. R. Kulkarni, and S. Verdu, "Degraded Gaussian multirelay channel: Capacity and optimal power allocation," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3037–3046, Dec. 2004.

[10] B. Schein and R. G. Gallager, "The Gaussian parallel relay network," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 25–30 2000, p. 22.

[11] P. Vanroose and E. C. Van Der Meulen, "Uniquely decodable codes for deterministic relay channels," *IEEE Trans. Inform. Theory*, vol. 38, no. 4, pp. 1203–1212, July 1992.

[12] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," Feb. 2004, submitted to IEEE Trans. Inform. Theory.

[13] G. Kramer, "Models and theory for relay channels with receive constraints," in *42nd Annual Allerton Conf. on Commun., Control, And Computing*, Monticello, IL, USA, Sept. 29– Oct. 1 2004.

[14] B. Smith and S. Vishwanath, "Cooperative communications in sensor networks: Relay channels with correlated sources," in *42nd Annual Allerton Conf. on Commun., Control, And Computing*, Monticello, IL, USA, Sept. 29– Oct. 1 2004.

[15] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *IEEE Int. Symp. Inform. Theory*, Cambridge, MA, USA, Aug. 16–21 1998, p. 156.

[16] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I: system description, part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1948, Nov. 2003.

[17] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[18] N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless netwroks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[19] A. Stefanov and E. Erkip, "Cooperative information transmission in wireless networks," in *2nd Asian-European Workshop on Information Theory*, Breisach, Germany, June 26–29 2002.

[20] T. E. Hunter and A. Nosratinia, "Cooperation diversity through coding," in *IEEE Int. Symp. Inform. Theory*, Lausanne, Switzerland, June 30 – July 5 2002, p. 220.

[21] P. Herhold, E. Zimmermann, and G. Fettweis, "On the performance of cooperative amplify-and-forward relay networks," in *International ITG Conference on Source and Channel Coding (SCC)*, Erlangen, Germany, Jan. 14–16 2004.

[22] A. Høst-Madsen and J. Zhang, "Capacity bounds for cooperative diversity," 2003, submitted to IEEE Trans. Inform. Theory.

[23] A. Høst-Madsen, "A new achievable rate for cooperative diversity based on generalized writing on dirty paper," in *IEEE Int. Symp. Inform. Theory*, Yokohama, Japan, June 29 – July 4 2003, p. 317.

[24] A. Høst-Madsen, "On the achievable rate for receiver cooperation in ad-hoc networks," in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 272.

[25] R. U. Nabar, F. W. Kneubühler, and H. Bölcskei, "Performance limits of amplify-and-forward based fading relay channels," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Montreal, Quebec, Canada, May 17 – 21 2004.

[26] R. U. Nabar, H. Bölcskei, and F. W. Kneubühler, "Fading relay channels: Performance limits and space-time signal design," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[27] S. D. Servetto, "Lattice quantization with side information: Codes, asymptotics, and applications in sensor networks," 2002, submitted to IEEE Trans. Inform. Theory.

[28] Z. Dawy and H. Kamoun, “The general Gaussian relay channel: Analysis and insights,” in *International ITG Conference on Source and Channel Coding (SCC)*, Erlangen, Germany, Jan. 14–16 2004.

[29] H. Bölcskei and R. U. Nabar, “Realizing MIMO gains without user cooperation in large single-antenna wireless networks,” in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 18.

[30] Y. Liang and V. V. Veeravalli, “Gaussian frequency division relay channels: optimal bandwidth allocation and capacity,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[31] I. Maric and R. Yates, “Forwarding strategies for Gaussian parallel-relay networks,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[32] M. Khojastepour, A. Sabharwal, and B. Aazhang, “Lower bounds on the capacity of Gaussian relay channel,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[33] S. Wei, D. Goeckel, and M. Valenti, “Asynchronous cooperative diversity,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[34] K. Azarian, H. El Gamal, and P. Schniter, “On the achievable diversity-vs-multiplexing tradeoff in cooperative channels,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[35] X. Cai, Y. Yao, and G. B. Giannakis, “Achievable rates in low-power relay-links over fading channels,” 2004, submitted to IEEE Trans. Commun.

[36] D. P. Palomar, A. Agustí, O. Muñoz, and J. Vidal, “Decode-and-forward protocol for cooperative diversity in multi-antenna wireless networks,” in *38th Annual Conf. on Inform. Sci. and Sys. (CISS’04)*, Princeton, NJ, USA, Mar. 17–19 2004.

[37] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, “Coded cooperation in wireless communications: Space-time transmission and iterative decoding,” *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 362–371, Feb. 2004.

[38] M. Dohler, A. Gkelias, and H. Aghvami, “A resource allocation strategy for distributed mimo multi-hop communication systems,” *Communications Letters*, vol. 8, no. 2, pp. 99–101, Feb. 2004.

[39] H. Bölcskei, R. U. Nabar, Ö. Oyman, and A. J. Paulraj, “Capacity scaling laws in MIMO relay networks,” Apr. 2004, submitted to IEEE Trans. Wireless Comm.

[40] T. E. Hunter, S. Sanayei, and A. Nosratinia, “The outage behavior of coded cooperation,” in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 270.

[41] M. Yuksel and E. Erkip, “Diversity gains and clustering in wireless relaying,” in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 400.

[42] M. Yuksel and E. Erkip, “Diversity in relaying protocols with amplify and forward,” in *Proc. IEEE Global Communications Conference (GLOBECOM)*, San Francisco, CA, USA, Dec. 1–5 2003.

[43] J. N. Laneman and G. W. Wornell, “Exploiting distributed spatial diversity in wireless networks,” in *Allerton Conference on Communication, Control and Computing*, Oct. 4–6 2000.

[44] T. E. Hunter and A. Nosratinia, “Diversity through coded cooperation,” 2003, submitted to IEEE Journal on Selected Areas in Communications.

[45] V. Tarokh, P. Mitran, and H. Ochiai, “Capacity enhancements using space-time collaborative communications in wireless channels,” in *International Workshop on Signal Processing for Wireless Communications (SPWC)*, London, UK, June 2–4 2004.

[46] M. Yuksel and E. Erkip, “Broadcast strategies for the fading relay channel,” in *Military Communications Conference (MILCOM)*, Monterey, CA, U.S.A., Oct. 31 – Nov. 3 2004.

[47] M. C. Valenti and N. Correal, "Exploiting macrodiversity in dense multihop networks and relay channels," in *In Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC)*, New Orleans, LA, USA, Mar. 16–20 2003.

[48] J. M. Shea, T. F. Wong, A. Avudainayagam, and X. Li, "Collaborative decoding on block fading channels," Feb. 2003, submitted to IEEE Trans. Commun.

[49] M. Katz and S. Shamai (Shitz), "Communicating to co-located ad-hoc receiving nodes in a fading environment," in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 115.

[50] M. Katz and S. Shamai (Shitz), "Transmitting to co-located users in wireless ad-hoc and sensory networks," Tech. Rep. CCIT report no. 489, EE Pub. No. 1437, Technion - Israel Institute of Technology, June 2004, submitted to the IEEE Trans. Inform. Theory on Dec. 2003, revised on Sept. 2004.

[51] A. D. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Trans. Inform. Theory*, vol. 22, no. 1, pp. 1–10, Jan. 1976.

[52] S. Shamai (Shitz), "A broadcast strategy for the Gaussian slowly fading channel," in *IEEE Int. Symp. Inform. Theory*, Ulm, Germany, June 29 – July 4 1997, p. 150.

[53] S. Shamai (Shitz), "A broadcast approach for the multiple-access slow fading channel," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 25 – 30 2000, p. 128.

[54] Y. Liu, K. N. Lau, O. Y. Takeshita, and M. P. Fitz, "Optimal rate allocation for superposition coding in quasi-static fading channels," in *IEEE Int. Symp. Inform. Theory*, Lausanne, Switzerland, June 30 – July 5 2002, p. 111.

[55] S. Shamai (Shitz) and A. Steiner, "A broadcast approach for a single-user slowly fading MIMO channel," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2617–2635, Oct. 2003.

[56] Y. Steinberg and N. Merhav, "On successive refinement for the wyner-ziv problem," *IEEE Trans. Inform. Theory*, vol. 50, no. 8, pp. 1636–1654, Aug. 2004.

[57] B. Porat, *Digital Processing of Random Signals, Theory and Methods*, Prentice-Hall, Inc., 1994.

[58] N. Jindal, U. Mitra, and A. Goldsmith, "Capacity of Ad-Hoc networks with node cooperation," in *IEEE Int. Symp. Inform. Theory*, Chicago, Illinois, U.S.A., June 27 – July 2 2004, p. 271.

[59] G. Caire and S. Sesia, "Multistage trellis quantization and its applications," in *42nd Annual Allerton Conf. on Commun., Control, And Computing*, Monticello, IL, USA, Sept. 29– Oct. 1 2004.

[60] W. Hirt and J. L. Massey, "Capacity of the discrete-time Gaussian channel with intersymbol interference," *IEEE Trans. Inform. Theory*, vol. 34, no. 3, pp. 380–388, May 1988.

[61] S. Shamai (shitz), L. H. Ozarow, and A. D. Wyner, "Information rates for a discrete-time Gaussian channel with intersymbol interference and stationary inputs," *IEEE Trans. Inform. Theory*, vol. 37, no. 6, pp. 1527–1539, Nov. 1991.

[62] S. Shamai (shitz) and R. Laroia, "The intersymbol interference channel: Lower bounds on capacity and channel precoding loss," *IEEE Trans. Inform. Theory*, vol. 42, no. 5, pp. 1388–1404, Sept. 1996.