Skyless Dehazing

CCIT Report #558 October 2005

Sarit Shwartz, Einav Namer and Yoav Y. Schechner
Department of Electrical Engineering
Technion - Israel Institute of Technology.
Haifa 32000, Israel
psarit@tx.technion.ac.il, einav@ee.technion.ac.il
yoav@ee.technion.ac.il

Abstract

Turbidity problems caused by scattering such as in haze or water, can be eliminated by processing of several different pictures of the same scene taken under different ambient scattering conditions. For example, in haze these are different polarization states. From this image set, the scene is recovered as it would have looked like if there were no turbidity. In addition, the distance map of the scene is recovered as well. This reconstruction requires estimation of parameters of the ambient scattering. For example, in haze, these parameters are the polarization degree and the saturation value of the airlight.

In previous work these parameters were estimated using image pixels corresponding to objects that are effectively at an infinite distance from the camera. Nevertheless, the need to measure such objects limited the applicability of visibility recovery methods. In this work we present several alternative methods for estimation of the required parameters, without using samples from infinite distance objects. Therefore, the recovery methods have a wider applicability.

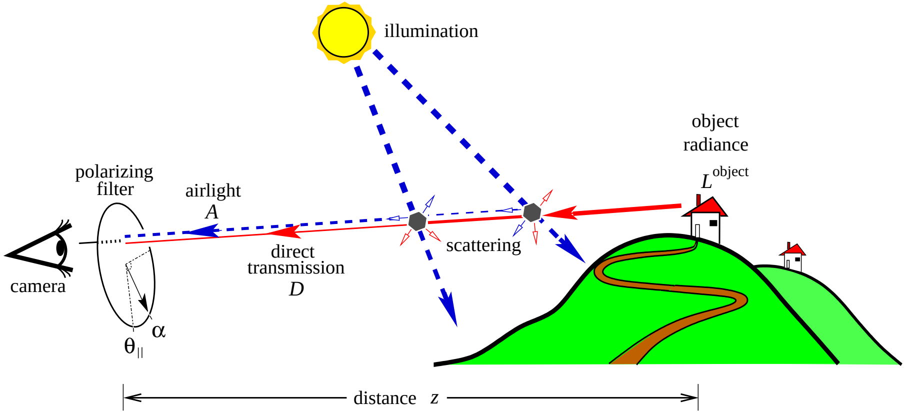

Figure 1: (Dashed rays) Light coming from the sun and scattered towards the camera by atmospheric particles is the airlight A. (Solid ray) Light emanating from the object L^{object} is attenuated by the medium along the line of sight, resulting in direct transmission D. Both A and D depend on the distance z. The scene is imaged through a polarizing filter oriented at angle α.

1 Introduction

The dehazing algorithm described in Ref. [**?**] show a way to recover L^{object}, based on p and A_∞. These parameters are measured from the raw images by looking at pixels which correspond to objects at infinity. These pixels can be assign to the sky near the horizon. However, the sky near the horizon is not always visible. Sometimes due to a very strong haze and sometimes when the sky are not photographed at all. In this paper, we show different methods by which this problem can be overcome, and those infinity parameters can be achieved without the existence of infinity in the picture.

2 Theoretical Background

In this section we describe the principles behind the basic dehazing algorithm of Ref. [?]. Consider Fig. 1. When acquiring an outdoor scene, the resulting image is a combination of two main components. The first originated from the object radiance. Let us denote by L^{object} the object radiance as if was taken in a clear atmosphere, without scattering on the line of sight. Due to attenuation in the atmosphere, the camera senses a fraction of this radiance,[1] which is the direct transmission

$$D = L^{\text{object}} t \ , \tag{1}$$

where

$$t = e^{-\beta z} \tag{2}$$

[1]There is a proportion factor between the scene radiance and image irradiance that depends on the imaging system, but does not depend on the medium and its characteristics. We thus leave this factor out.

is the transmittance of the atmosphere. The transmittance depends on the distance z between the object and the camera, and on the atmospheric attenuation coefficient β.

The second component is known as *path radiance*, or *airlight*. It originates from the scene illumination (e.g., by the sun), portion of which is scattered into the line of sight by atmospheric particles. It is given by

$$A = A_\infty (1 - t) \ , \tag{3}$$

where A_∞ is the of airlight, which depends on the atmospheric and illumination conditions. Contrary to the direct transmission, this component increases with the distance and dominates the acquired image irradiance

$$I_{\text{total}} = D + A \tag{4}$$

at long range. This is a major cause for reduction of image contrast in haze.

In haze, the airlight is often partially polarized. Hence, we can modulate it by mounting a polarizing filter at angle α in the imaging system. When rotating the polarizer, there is an orientation at which the image is least intense. Let us denote this image as I_{min}. Ref. [?] assumes that polarization is associated only with the airlight. If so, then I_{min} corresponds to the lowest amount of airlight. Hence, this is the image with the best contrast that can be achieved by optical filtering. We denote this optimal polarizer orientation as $\theta_\parallel$. We may then rotate the polarizer by 90^o relative $\theta_\parallel$. This time the image irradiance is strongest, since we sense the principle polarization component of the airlight. Denote this image as I_{max}.

Once these images are acquired, Ref. [?] describes I_{total} as

$$\hat{I}_{\text{total}} = \frac{I_{\text{max}} + I_{\text{min}}}{2} \ . \tag{5}$$

where I_{max} is also written as $I_\parallel$, and I_{min} is also written as $I_\perp$. Based on Eqs.(4,5)

$$\hat{L}^{\text{object}} = \frac{\hat{I}_{\text{total}} - \hat{A}}{\hat{t}} \ , \tag{6}$$

is the dehazing of the scene, where

$$\hat{t} = 1 - \frac{\hat{A}}{A_\infty} \tag{7}$$

is the estimated transmittance and

$$\hat{A} = \frac{I_{\text{max}} - I_{\text{min}}}{p} \tag{8}$$

is the estimated airlight. The parameter p appearing in Eq. (8) is the degree of polarization of airlight. For narrow fields of view (FOV), this parameter does not vary much. Note that we also make use of the airlight saturation value A_∞. Both of these parameters are generally unknown, and thus provide the incentive for this paper. Before detailing how these parameters are estimated, let us assume for the moment that they are known.

Eq. (6) expresses the dehazed scene radiance. Note that in addition, Eqs. (2,7) recover a distance map of the scene

$$\beta\hat{z} = -\log\left[1 - \frac{\hat{A}}{A_\infty}\right] \ . \tag{9}$$

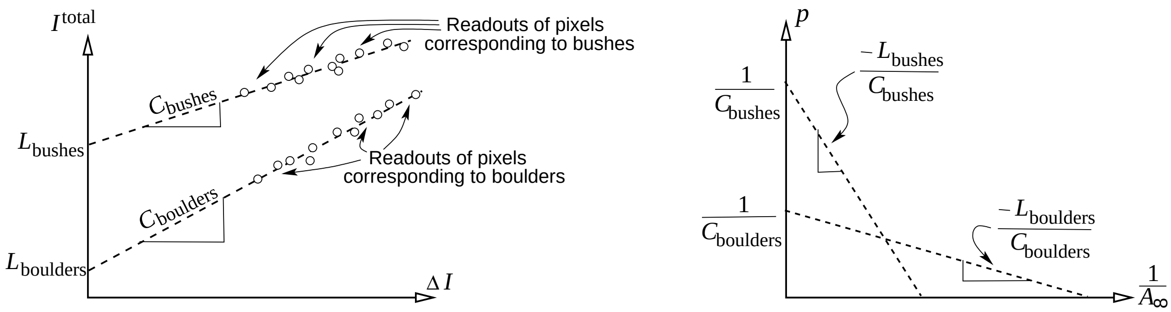

Figure 2: A hypothesized method for estimating p and A_∞. [Left] Measurements of pixels corresponding to objects of a certain class (e.g., "bushes", "boulders") are predicted to lie on a straight line in the $(\Delta I, \hat{I}^{\text{total}})$. [Right] Parameters derived from the measurements create linear constraints on p and A_∞. Intersection of such constraints uniquely determines p and A_∞.

2.1 Prior Methods for Parameter Estimation

The dehazing method described above requires knowledge of the airlight saturation value A_∞ and of its degree of polarization p. Past methods have derived these parameters by measuring clear sky pixels [4]. However, such pixels may not always be available in the field of view. Moreover, the sky by the horizon may be cloudy, ruining that kind of estimation.

There is thus a need to estimate A_∞ and p, without resorting to sky measurements. One prior work has suggested exploiting knowledge of different objects in the scene [4]. Suppose we can mark scene points (x_k, y_k) which, in the absence of scattering, would have a similar, but unknown radiance. For example, boulders in the scene have an unknown radiance L_{boulders}. It is easy to show from the previous equations that

$$\hat{I}_{\text{total}}(k) = L_{\text{boulders}} + C_{\text{boulders}} \Delta I(k) \quad , \tag{10}$$

where $\Delta I(x,y) \equiv [\hat{I}^{\perp}(x,y) - \hat{I}^{\parallel}(x,y)]$, and

$$C_{\text{boulders}} \equiv \left(\frac{1}{p} - \frac{L_{\text{boulders}}}{p A_\infty} \right) \tag{11}$$

is constant. Boulder points at different distances from the viewer will have intensity readouts due to the effects of scattering. Therefore, they will have different values of I_{total} and $\Delta I(x,y)$. According to Eq. (10), $\hat{I}_{\text{total}}(k)$ as a function of $\Delta I(k)$ forms a straight line, as depicted on the left of Fig. 2. Extrapolating the line, its intercept yields the radiance value L_{boulders}. The slope of the fitted line is C_{boulders}. We end up with a linear equation (11) that relates the unknown $(1/p)$ to the unknown A_∞.

Ref. [4] notes that if a similar analysis is done for an additional class of objects, say bushes, then an additional linear constraint is derived. This determines the required parameters, as depicted on the right of Fig. 2. However, this method has shortcomings: it requires the presence of several types of distinct scene features in the scene. It would not be possible to operate this method is an environment having only a single type of ground or ground-cover (forest). Moreover, it is inconvenient and less reliable to identify objects of different classes.

3 Estimating p using ICA method

3.1 Problem Formulation

One of our goals is to decouple the airlight and direct transmission. We assume that direct transmission is not polarized, hence its energy is evenly distributed between the polarization components. The variations due to the polarizer rotation are assumed to be mainly due to airlight. The light ray from the light source to a scatterer and the line of sight from the camera to the scatterer define a plane of incidence. We can divide the airlight to two polarization components that are parallel and perpendicular to this plane. Define $A\|$, $A\perp$ as the parallel and perpendicular airlight components. Then,

$$A = A^{\|} + A^{\perp} \ , \tag{12}$$

and the degree of polarization is

$$p = (A^{\perp} - A^{\|})/A \ . \tag{13}$$

When the polarizing filter is oriented such that the image irradiance is minimal, we measure

$$I^{\|} = A(1-p)/2 + D/2 \ . \tag{14}$$

This is the best state of the polarizer because here the image irradiance is the closest to the irradiance corresponding to the direct transmission (except for a factor of 1/2). There is a difference between $I^{\|}$ and $D/2$, because the airlight is not completely polarized $A^{\|} \neq 0$.

We can acquire two polarized images. The first image is $I^{\|}$, taken with polarizer angle of $\theta^{\|}$. The second image is acquired when the filter is oriented perpendicular to $\theta^{\|}$.

$$I^{\perp} = A(1+p)/2 + D/2 \ . \tag{15}$$

Note that $I^{\perp}$ is the worst state of the polarizer, because the airlight is enhanced relative to the direct transmission.

To dehaze the image, we first have to remove the airlight A. The key step here is the estimation of p, the degree of polarization of airlight. p relates the unknown airlight A to the difference between the image irradiances $I^{\|}$ and $I^{\perp}$.

Assume that p is uniform over the acquired scene. Then, the two acquired images constitute the following equation system:

$$\begin{cases} I^{\perp} = (1+p)A/2 + D/2 \\ I^{\|} = (1-p)A/2 + D/2 \end{cases} . \tag{16}$$

Assume for a moment that p is known. Then, the equation system is easy to invert and the direct transmission and the airlight can be calculated from the acquired images as

$$\begin{cases} A = (I^{\perp} - I^{\|})/p \\ D = (p-1)I^{\perp}/p + I^{\|}(p+1)/p \end{cases} . \tag{17}$$

However, typically p is unknown. Nevertheless, we propose a method for estimating p based on the acquired images.

3.2 Independent Component Analysis

There are applications in which the acquired signals are pointwise mixtures of statistically independent sources. We know neither the original sources nor the mixing process. The goal of ICA is: given only the acquired mixtures, find the separation process that yields the original sources. This is done by finding the separation process that minimize the statistical independence between the estimated sources (see [2] and references therein).

Denote $\{s_1, \ldots, s_K\}$ as the set of K original independent sources, and $\{u_1, \ldots, u_K\}$ as the set of acquired sources. Denote, M as the linear mixing process that transform $\{s_1, \ldots, s_K\}$ to $\{u_1, \ldots, u_K\}$. Each one of the original sources and the acquired sources can be rearranged in row vectors. In this case M is represented by a simple matrix called the separation matrix.

$$\begin{bmatrix} u_1 \\ \vdots \\ u_K \end{bmatrix} = \mathbf{M} \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} . \tag{18}$$

Denote $\{\hat{s}_1, \ldots, \hat{s}_K\}$ as the set of estimated sources. Denote $\mathbf{W}$ as the linear separation process that transform $\{u_1, \ldots, u_K\}$ to $\{\hat{s}_1, \ldots, \hat{s}_K\}$. By rearranging the estimated sources and the acquired sources in row vectors we can express $\mathbf{W}$ as a simple matrix termed the separation matrix.

$$\begin{bmatrix} \hat{s}_1 \\ \vdots \\ \hat{s}_K \end{bmatrix} = \mathbf{W} \begin{bmatrix} u_1 \\ \vdots \\ u_K \end{bmatrix} . \tag{19}$$

Mutual information (MI) is a natural measure for statistical dependency. Therefore MI is commonly use in ICA algorithms. MI of two signals can be expressed as

$$\mathcal{I}_{\hat{s}_1,\hat{s}_2} = \mathcal{H}_{\hat{s}_1} + \mathcal{H}_{\hat{s}_2} - \mathcal{H}_{\hat{s}_1,\hat{s}_2} . \tag{20}$$

Here $\mathcal{H}_{\hat{s}_k}$, $k = 1, 2$ is the marginal entropy of source k, and $\mathcal{H}_{\hat{s}_1,\hat{s}_2}$ is the joint entropy of the sources. Therefore, in order to estimate the MI of the sources we need to estimate the joint and marginal entropies of the sources. Nevertheless, in the special case of pointwise mixtures the MI expression can be simplified to

$$\mathcal{I}(\hat{s}_1, \hat{s}_2) = \mathcal{H}_{\hat{s}_1} + \mathcal{H}_{\hat{s}_2} - \log|\det(\mathbf{W})| - \mathcal{H}_{u_1,u_2} . \tag{21}$$

Here $\mathcal{H}_{u_1,u_2}$ is independent of $\mathbf{W}$ and is thus constant for a given measurements set $\{u_1, u_2\}$. For this reason, we ignore it in the optimization process. This way the only terms we need to estimate from the data are the marginal entropies.

3.3 Dehazing by ICA

The dehazing formulation Eqs. (16) and (17) can be ragarded as a special case of ICA. By comparing Eqs. (16) and (17) to Eqs. (18) and (19) respectively, we can identify A, D with s_1, s_2 and $I^\perp, I^\parallel$ with u_1, u_2 respectively. In addition, the mixing matrix and the separation matrix in this case have a special structure:

$$\mathbf{M} = \begin{bmatrix} (1+p)/2 & 1/2 \\ (1-p)/2 & 1/2 \end{bmatrix} \quad , \quad \mathbf{W} = \begin{bmatrix} 1/p & -1/p \\ (p-1)/p & (p+1)/p \end{bmatrix} . \tag{22}$$

It is important to note that the assumption that the direct transmission and the airlight signals are statistically independent is not obvious. For instance, the airlight signal increases with the distance z while the direct transmission decreases with z. Therefore, the airlight image will be brighter where the direct transmission image is darker. Nevertheless, the direct transmission image contains color information which is independent of the objects distance z. Moreover, the amount of airlight between the objects and the camera does not depend on the objects color rather it depends only on z. Therefore, the independence assumption is valid for natural scenes, which is typically colorful.

MI optimization has three inherent ambiguities. The first ambiguity is scale ambiguity. The order in which the reconstructed sources appear does not change their MI. Therefore, the estimated sources given by ICA optimization can appear in arbitrary order. This ambiguity does not concern us in this work.

The remaining ambiguities are the scale and sign ambiguities. Suppose we have two statistically independent sources. Multiplying each one of these sources with some arbitrary constant does not influence their MI which is zero. Therefore, the estimated sources given by ICA optimization can have any arbitrary scale. The sign ambiguity is the special case in which the scale is -1. This ambiguity implies that we can estimate the direct transmission and the airlight images up to a scale and sign. Moreover, the scale ambiguity implies that we have infinite number of solutions to the minimization problem. This can destabilize the numerical optimization. However, in the special case of the dehazing problem we can exploit the special structure of the mixing and the separation matrices (Eq. 22) in order to bypass all the ICA ambiguities as detailed in Sec. 3.3.3.

3.3.1 Dehazing Optimization

Let us look at the structure of the separation matrix $\mathbf{W}$ (Eq. 22). This structure implies that up to a scale p the airlight A is a simple substraction of the two acquired images. Hence, we denote $\hat{A}$ as our estimation for the airlight (up to a scale p)

$$\hat{A} = I^{\perp} - I^{\parallel} \ . \tag{23}$$

Denote

$$w_1 = (p-1) \quad , w_2 = (p+1) \ . \tag{24}$$

and $\hat{D}$ as the estimation of the direct transmission up to a scale p. Then Eq. (22) implies that

$$\hat{D} = w_1 I^{\perp} + w_2 I^{\parallel} \ . \tag{25}$$

Recall that our goal is to separate the airlight and the direct transmission signals by minimizing the statistical dependency between $\hat{A}$ and $\hat{D}$. We do that by minimizing the MI between the two estimated sources. By substituting Eqs. (23) and (25) into Eq. (21) we get

$$\mathcal{I}(\hat{D}, \hat{A}) = \mathcal{H}_{\hat{D}} + \mathcal{H}_{\hat{A}} - \log \left| \det \begin{bmatrix} 1 & -1 \\ w_1 & w_2 \end{bmatrix} \right| = \mathcal{H}_{\hat{D}} + \mathcal{H}_{\hat{A}} - \log |(w_2 + w_1)| \ . \tag{26}$$

Note that $\hat{A}$ does not depend on w_1, w_2. Therefore, $\mathcal{H}_{\hat{A}}$ is constant and can be ignored in the optimization process. Therefore, the optimization problem we need to solve is

$$\min_{w_1,w_2} \left\{ \mathcal{I}(\hat{D}, \hat{A}) \right\} = \left\{ \min_{w_1,w_2} \mathcal{H}_{\hat{D}} - \log |(w_2 + w_1)| \right\} . \tag{27}$$

By definition, the degree of polarization is limited to $0 < p < 1$ this means that $0 < w_2 + w_1 < 2$. In this region $-\log |(w_2 + w_1)|$ is a convex function. Hence, if we use an entropy estimator which is convex in w_1, w_2 we can guaranty convergence to a unique solution. We are interested in an entropy estimator which is not only convex in w_1, w_2 but also accurate and efficient to calculate. In the next section we detail such an estimator.

3.3.2 Entropy Estimation by Sparsity

We are interested in an accurate and efficient entropy estimator which is also a convex function in w_1, w_2. Such an estimator exists for sparse signals. However, natural images are not sparse. Therefore, apparently we can not exploit sparsity. Nevertheless, a natural image can be easily transformed into a sparse signal by simple linear transformation such as Sobel operator, or wavelets transformation. Such transformations do not change the mixing matrix since they are linear and therefore commutative with the transformation as shown in [3] and references therein. Hence, we can apply a sparsifying linear transformation on the original sources. Then, we apply the ICA separation algorithm described in this work to estimate p. Finally, we can reconstruct the dehazed image from the original images using the estimated p.

A PDF model that is widely used in the literature to model sparse images is the generalized Laplacian (see for example [5])

$$p(\hat{D}) = c(\rho) \exp(-|\hat{D}|^{\rho}) \ , \ \text{where} \quad 0 < \rho < 2 \ . \tag{28}$$

Here $c(\rho)$ is the normalization factor of $p(\hat{D})$. The sparsity of the represented signal is determined by the parameter ρ. The smaller ρ is, the narrower the PDF, representing a sparser signal. Note that this PDF model assumes that the source has a unit variance. This implicit normalization makes the optimization robust to some numerical issues. However, this implicit normalization does not solve the scale ambiguity problem, the true scale of the signals is still unknown. We detail how to reconstruct the true scale in Sec. 3.3.3.

We now exploit this prior of image statistics to formalize estimation of entropies in our optimization. Entropy is defined as (see for example [1])

$$\hat{\mathcal{H}}_{\hat{D}} = E \left\{ \log[p(\hat{D})] \right\} . \tag{29}$$

Substituting Eq. (28) into Eq. (29) and replacing the expectation with empirical averaging, we obtain the channel entropy estimator:

$$\hat{\mathcal{H}}_{\hat{D}} = \frac{1}{N} \sum_{n=1}^{N} |\hat{D}(n)|^{\rho} + C(\rho) \ . \tag{30}$$

Here $C(\rho) = \log[c(\rho)]$. Since $C(\rho)$ does not depend on $\hat{D}$, it can be ignored in the optimization process. The generalized Laplacian model yields a very simple formula (30): the entropy is a simple expression of the variables. Moreover, the computational complexity of this entropy estimator is $\mathcal{O}(N)$.

The channel entropy (Eq. 30) is a convex function of $\hat{D}(n)$ only if $\rho \geq 1$. However, the PDF of the transformed images are typically very sparse, i.e. $\rho < 1$. Therefore, we have a tradeoff between accuracy and convexity: for efficient optimization, we require a convex function, while for accuracy we require $\rho < 1$. The most sparse PDF that still yields a convex function in (30) is described by $\rho = 1$. Therefore, we choose to use $\rho = 1$.

Substituting Eq. (30) in Eq. (35) yields the following MI minimization

$$\min_{w_1,w_2} \mathcal{I}_{\hat{D},\hat{A}} = \min_{w_1,w_2} \left\{ \frac{1}{N} \sum_{n=1}^{N} |\hat{D}(n)| - \log |w_1 + w_2| \right\} . \tag{31}$$

This is the core of our optimization. In addition, we use a local gradient based optimization which is very efficient for convex functions allowing for fast convergence to a global minimum. For this purpose we need to calculate the gradient and the hessian of Eq. (31). However, the absolute value function used in Eq. (31) is not differentiable. Nevertheless, we solve this problem by substituting the absolute value function with a function that is a smooth approximation to the absolute value function. There are several options for such a function, we use Ref. ([3])

$$f(\hat{D}) = \nu[|\hat{D}/\nu| - \log(1 + |\hat{D}/\nu|)] . \tag{32}$$

Here ν is the smoothing parameter, $\nu \to \inf \Rightarrow f(\hat{D}) \to |\hat{D}|$ but the function is not smooth. while $\nu \to 0$ yield a very smooth function but not accurate enough. We use $\nu = 0.1$. By substituting $|\hat{D}(n)|$ with $f[\hat{D}(n)]$ in Eq. (31) and differentiating it we get the gradient expression:

$$\nabla_{w_1,w_2} \mathcal{I}_{\hat{D},\hat{A}} = \begin{bmatrix} \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)] I^{\perp}(n) - 1/(w_1 + w_2) \\ \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)] I^{\parallel}(n) - 1/(w_1 + w_2) \end{bmatrix} . \tag{33}$$

Where $\dot{f}$ is the derivative of f.

By differentiating a second time we get the Hessian expression:

$$\nabla_{w_1,w_2} \mathcal{I}_{\hat{D},\hat{A}} = \begin{bmatrix} \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)][I^{\perp}(n)]^2 & \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)][I^{\perp}(n) I^{\parallel}(n)] \\ \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)][I^{\parallel}(n) I^{\perp}(n)] & \frac{1}{N} \sum\limits_{n=1}^{N} \dot{f}[\hat{D}(n)][I^{\parallel}(n)]^2 \end{bmatrix} + 1/(w_1+w_2)^2 . \tag{34}$$

3.3.3 Image Reconstruction

ICA optimization yields the sources up to a scale. Therefore, we do not reconstruct the dehazed image from w_1 and w_2. Rather we first extract p from w_1 and w_2. Then, we reconstruct the dehazed image from Eq. (17).

Let us look at the ratio $\alpha = w_1/w_2$. From Eq. (24) we get

$$\alpha = \frac{w_1}{w_2} = (p-1)/(p+1) \ . \tag{35}$$

Simple manipulation of Eq. (35) yields

$$p = \frac{w_1 + w_2}{w_2 - w_1} = \frac{1+\alpha}{1-\alpha} \ . \tag{36}$$

Now we can reconstruct the direct transmission image using Eq. (17). Note that in this reconstruction scheme we bypass all inherent ICA ambiguities: permutation, sign and scale.

In order to perform complete dehazing reconstruct $\hat{L}^{\text{object}}$ we need to divide each pixel in the direct transmission image by $e^{(-\beta z)}$ (see Eq. 1). However, neither β nor z are known.

4 Exploiting Distance Information

The method described here is based on distance estimation in several points in the field of view (FOV). Let us select two points in the FOV, corresponding to similar objects (thus having the same underlying L^{object}). The two points, however, should be at different distances from the camera. For example they could correspond to two trees placed in different distances, or two buildings. Let the image coordinates of these two points be (x_1, y_1) and (x_2, y_2), respectively, and their distances are z_1 and z_2.

A distances z in the picture can be calculated in several ways. One is by using a map or a DTM, assuming you know where you are. Another is by using instruments to estimate distance, such as laser beam. If the camera and lens type are known, a simple magnification calculation can be done, to estimate the distance of an object, that you know roughly its real size.

Using Eq. (4),

$$I_1^{\parallel} = L_1^{\text{object}} \cdot e^{-\beta z_1} + A_\infty^{\parallel} \cdot [1 - e^{-\beta z_1}] \tag{37}$$

$$I_1^{\perp} = L_1^{\text{object}} \cdot e^{-\beta z_1} + A_\infty^{\perp} \cdot [1 - e^{-\beta z_1}] \tag{38}$$

where $I_1^{\parallel} = I^{\parallel}(x_1, y_1)$ and $I_1^{\perp} = I^{\perp}(x_1, y_1)$.

From Eqs. (37,38) we extract A_∞ as

$$A_\infty = \frac{A_\infty^{\parallel} + A_\infty^{\perp}}{2} = N_1 + L_1^{\text{object}} \cdot N_2 \tag{39}$$

where $N_1 = (I_1^{\parallel} + I_1^{\parallel})/(2[1 - e^{-\beta z_1}])$ and $N_2 = e^{-\beta z_1}/(e^{-\beta z_1} - 1)$.

Similarly,

$$A_\infty = \frac{A_\infty^{\parallel} + A_\infty^{\perp}}{2} = N_3 + L_1^{\text{object}} \cdot N_4 \tag{40}$$

where $N_3 = (I_2^{\parallel} + I_2^{\parallel})/(2[1 - e^{-\beta z_2}])$, $N_4 = e^{-\beta z_2}/(e^{-\beta z_2} - 1)$, and $I_2^{\parallel} = I^{\parallel}(x_2, y_2)$, $I_2^{\perp} = I^{\perp}(x_2, y_2)$.

Let us assume for a moment that z_1, z_2 and β are known. Hence Eqs. (39,40) are a set of two linear equations, with two unknowns, A_∞ and L_1^{object}. The intersection between these two linear constrains yields the desired solution of A_∞.
We now determine p. Subtracting Eq. 38 from Eq. 37, and performing additional simple operations yields,

$$\Delta A = A_\infty^\| - A_\infty^\perp = \frac{I_1^\| - I_1^\perp}{1 - e^{-\beta z_1}} \tag{41}$$

Since z_1 and β are known, p can be calculated. By using Eqs. (39,41,**??**), we denote:

$$\hat{p} = \frac{A_\infty^\| - A_\infty^\perp}{A_\infty^\| + A_\infty^\perp} = \frac{2\Delta A}{A_\infty} \tag{42}$$

We thus recover the required parameters to perform dehazing based polarization.

Estimating the atmospheric attenuation coefficient β

The parameter β can be measured with Runway Visual Range (RVR) instrument (see http:////www.rvr.it). Suppose we do not know what β is. We can derive it using a **third** object of known distance z_3. By performing the same operations in Eqs. (39,40),

$$A_\infty = \frac{A_\infty^\| + A_\infty^\perp}{2} = N_5 + L_1^{\text{object}} \cdot N_6 \tag{43}$$

where $N_5 = (I_3^\| + I_3^\|)/(2[1 - e^{-\beta z_3}])$, $N_6 = e^{-\beta z_3}/(e^{-\beta z_3} - 1)$, and

Using Eq. (39,40,43) we can calculate β as well as A_∞ and p.

Using ICA method

The ICA process yields the parameter p, without any assumptions on the image. Therefore, $A = \frac{I^\perp + I^\|}{P}$, is easyly calculated. From Eq.(9, we know that $e^{-\beta z} = 1 - \frac{A}{A_\infty}$. Again, let us assume for a moment that we can estimate β and the distance in the point (x_0, y_0). Therefore, A_∞ can be calculated:

$$A_\infty(x_0, y_0) = \frac{A(x_0, y_0)}{1 - e^{-\beta z_0}} \tag{44}$$

It is reasonable to determine that $A_\infty = A_\infty(x_0, y_0)$.

Now, after estimation p and A_∞, we can use Eq. 6 and perform dehazing.

5 Combination of Ref. [4] and ICA

Let us choose two points in the picture, k_1 and k_2, which have different distances from the camera, but represent the same object. For example, a couple of boulders in the scene. Using Eq. 10 on k_1 and k_2, we get

$$\hat{I}^{\text{total}}(k_1) = L_{\text{boulders}} + C_{\text{boulders}} \Delta I(k_1) \tag{45}$$

$$\hat{I}^{\text{total}}(k_2) = L_{\text{boulders}} + C_{\text{boulders}} \Delta I(k_2) \ . \tag{46}$$

Based on Eqs. (45,46), we can calculate L_{boulders} and C_{boulders}:

$$C_{\text{boulders}} = \frac{I_{k_1}^{\text{total}} - I_{k_2}^{\text{total}}}{\Delta I_{k_1} - \Delta I_{k_2}} \tag{47}$$

and based on the estimated C_{boulders},

$$L_{\text{boulders}} = I_{k_1}^{\text{total}} - C_{\text{boulders}} \cdot \Delta I_{k_1} \tag{48}$$

The ICA process, as shown in Eq. 36, yields the parameter p, without any assumptions on the image. Therefore, at this stage we have p, C_{boulders} and L_{boulders}, based on 2 similar samples of boulders. Hance, using Eq. **??** to calculate A_∞:

$$A_\infty = \frac{L_{\text{boulders}}}{1 - p \cdot C_{\text{boulders}}} \tag{49}$$

Acknowledgments

Yoav Schechner is a Landau Fellow - supported by the Tabu Foundation, and an Alon Fellow. The work was supported by the "Dvora" Fund of the Technion. The research was conducted in the Collectors Minerva Center in the Elect. Enc. Dept. at the Technion. Minerva is funded through the BMBF.

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