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Outage Capacities and Spectral Efficiencies of Multiuser Receivers in a CDMA Cellular Environment[1]

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Abstract

A simple multi-cell model is considered for a randomly spread direct-sequence code division multiple access (DS-CDMA) system with *slow* flat fading. The model adheres to Wyner's (1994) infinite linear cell-array setting, where only adjacent-cell interference is present, and characterized by a *single* parameter $0 \leq \alpha \leq 1$. The discussion is confined to asymptotic analysis, where both the number of users per cell and the processing gain go to infinity, while their ratio goes to some finite constant. Assuming single-cell-site processing, four multiuser detection strategies, which differ in the type and amount of information on the multiuser interference utilized by the receiver, are analyzed and compared. First, the spectral efficiency is considered, specifying the ultimate performance. Next, a practically oriented transmission and decoding strategy is analyzed. Assuming that all users employ equal rates and transmit powers, a *strongest-users-only decoding* scheme is considered where the receiver ranks the intra-cell users according to their received powers, and decodes only a subset of these users, equal in number to the largest integer for which decoding is successful. The total capacities under an outage constraint of the detection strategies, derived as functions of the fraction of users that *cannot* be decoded (equivalent to the outage probability), are analyzed and compared. The performance degradation with the respect to the spectral efficiency is discussed as well.

1 Introduction

Cellular systems employing direct-sequence code division multiple access (DS-CDMA) schemes have been the focus of a multitude of works of recent years, in view of their planned use in current and future generation commercial cellular networks. See [1] – [34] for a recent sample of the abundance of relevant literature. Particularizing to the uplink channel, the use of (pseudo) random spreading sequences seems to be the common practice in many planned systems, as it requires no user-coordination, and robustly achieves low average cross-correlations between spreading sequences of different users. Analytical information theoretic performance analyses of randomly spread DS-CDMA systems may in general become prohibitively complex, as they involve random matrix eigenvalue distribution problems that cannot be handled analytically in the general finite dimensional setup. However, it has been identified in recent years, that when both the number of users and the processing gain go to infinity, while their ratio goes to some *finite* constant, limiting *analytical* and *deterministic* system performance measures of interest can be obtained while harnessing relevant results from the theory of random matrices. Asymptotic analyses of this kind can be found in works such as [1]–[3], [9]–[11], [17], [18], [32], [35]. See [36] for an outstanding summary of the state of the art in this field. The above analytical asymptotic results are of particular interest as they give a good indication on system performance in practical finite dimensional systems (see for example [1], [9], [37], [36]). It is also noted that the channel models and random matrix theory results mentioned above hold verbatim for multiple-transmit multiple-receive antenna systems, see [36] and references therein.

The ultimate system performance measure for cellular systems is the per-cell *spectral efficiency*, defined as the maximum number of bits/sec/Hz that can be transmitted arbitrarily reliably in each of the cells. Focusing on a *single-cell* setup, the spectral efficiency of optimally coded (in the information-theoretic sense) DS-CDMA systems with *random* spreading sequences has been thoroughly analyzed in [2] and [17]. Both papers consider the asymptotic scenario, where both the number of users K and the processing gain N go to infinity, while their ratio goes to some *finite* constant $\beta < \infty$ (referred to as the *"cell load"*). Assuming equal received powers and no fading, expressions are presented in [2] for the spectral efficiency of the optimum receiver, the matched-filter receiver, the decorrelator, and the linear minimum-mean-squared-error (MMSE) receiver. The results are extended in [17] to examine the impact of frequency-flat fading on these receivers. Assuming full channel state information (CSI) is available to both transmitter and receiver, the optimum per user power control policy, given as a function of the individual fade levels, has been analyzed and discussed (see also [14] for a related result).

Although analyses of the single-cell setup provide much insight into the fundamental aspects of system performance, the analysis of system performance in multi-cell setups should not be overlooked, as eventually practical systems usually include a multiplicity of cells. In particular, it is interesting to examine the effect of out-of-cell interference on system performance. Except for the particular case in which the receivers of *all* cell-sites can cooperate and joint processing of all received signals can be preformed (e.g., see [38] [39]), the uplink of the multi-cell setup falls within the framework of the multiple-access plus interference channel, the capacity of which is still an open problem in the general case (see the discussion in [40]). In this respect, an attractive multi-cell model is the one suggested by Wyner in [38]. This simple model allows for analytical tractability on the one hand, while giving insight to practical systems on the other. Accordingly, the system's cells compose an infinite *linear* array, and the received signal at each cell-site is the sum of the signals received from intra-cell users, plus a factor α $(0 \leq \alpha \leq 1)$ times the sum of the signals generated by users in the two adjacent cells. Non-adjacent cell users are assumed to produce *no interference*. The received signal is embedded in ambient Gaussian noise. The multi-cell effect on performance is thus specified by a *single* parameter (α). The analysis in [38] considered non-fading channels and assumed "wideband" transmissions in which all bandwidth is available for coding, and no DS-spreading is employed. The results were extended to independent (over users) identically distributed flat-fading channels in [41], were a comparative analysis of time division multiple access (TDMA) and the wideband transmission scheme is also presented. The effects of partial multiple cell-site processing (as opposed to the case in which joint processing of the signals received by all cell-sites is employed [38] [39]), and inter-cell time sharing, are also considered in [41]. The same model is also used by the authors in [21], which is devoted to non-fading channels and extends some of the results of [2] and [41] to the randomly-spread DS-CDMA multi-cell setup. In [39], a *full* multiple-cell receiver as in [38] is considered, and it is demonstrated that fading may turn beneficial even with the optimally coded setting, where no spreading is imposed.

In this paper, we continue the investigation of the effect out-of-cell interference on system performance in the optimally coded randomly-spread DS-CDMA regime, while employing again the Wyner [38] linear cell-array model. Independent identically distributed flat fading channels are assumed, as in [17], and again the asymptotic regime is considered, in which denoting now the number of *intra-cell* users by K (assumed constant and equal in all cells), $K, N \to \infty$, while $\frac{K}{N} \to \beta < \infty$. The first part of the paper aims at the ultimate system performance measure, the spectral efficiency, while assuming full CSI is available to the receiver but not to the transmitter (boiling down to equal transmit powers for each of the intra-cell users). Assuming *single cell-site processing*, four types of multiuser detection strategies are considered (as in [21]):

I. The *"conventional" matched-filter receiver* that treats *all* interference (either intra-cell or inter-cell) as additive white Gaussian noise (AWGN);

II. A *single-cell optimum (SCO)* receiver that "optimally" processes the transmissions of *intra-cell* users (assuming all their signatures are known at the receiver), while treating *inter-cell* interference as AWGN;

III. The *linear MMSE receiver* that knows the signatures (spreading sequences) of all interfering users (both intra-cell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter;

IV. The *"single cell-site processing optimum (SCPO) receiver"*, defined as the *optimum receiver* in terms of spectral efficiency, given that the receiver is unaware of the codebooks of adjacent-cell users, and only their signatures are known at the receiver.

It is emphasized that neither the linear MMSE receiver, nor the SCPO receiver, try to *decode* the transmissions of adjacent-cell users (which might be prohibitive if α is small). In fact, the cell-site receiver may actually be ignorant regarding codebooks or code-mask sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. It is also noted that for Gaussian inputs (conforming with the capacity achieving statistics), the SCPO receiver is equivalent in terms of spectral efficiency to a receiver that employs *MMSE based successive interference cancellation (MMSE-SC)* to decode transmissions of intra-cell users, while *inter-cell* interference is mitigated by means of a linear MMSE filter (see [21], [42]). The use of blind adaptive MMSE multiuser detection may be attractive in this respect, as it makes no distinction between intra-cell and adjacent-cell interference.

The spectral efficiency of the above four detection strategies is derived in this paper, and comparatively examined. The results are then compared to analogous results without fading (appropriately reproduced from [21]). Finally, the penalty in system performance due to random spreading is also examined, by comparison (following [41]) to the spectral efficiency of corresponding receivers, in the setting in which *all* bandwidth is available for coding (as opposed to bandwidth expansion by DS-spreading). Extensions of the some of the results obtained here to *joint multiple-cell-site processing* with *chip-level* interleaving, were recently derived in [34].

The second part of this paper considers a *suboptimum practically oriented* transmission and reception approach to be described in the following. The spectral efficiency analyses, as presented in [17], and in the first part of the paper, tacitly assume that all active users are (reliably) decoded *regardless of their received powers*. In *slow-fading* channels, where the channel fading gains are assumed to be constant within the codeword duration, the above assumption actually implies that the users adjust their rates (and possibly also their powers) as a function of the channel fading level they experience, via delayless feedback from the receiver. Unfortunately, in practice, such feedback and ideal tuning of the users' transmissions cannot always be accomplished. One possible approach, excluding any form of feedback from the receiving cell-site, but an ACK/NACK type, is to let *all* users transmit at *equal rates* and *equal powers* regardless of the individual fade levels (note that in [17] *rate*-adjustment feedback is still tacitly assumed in the equal transmit power setup). In such case, due to the presence of fading, the receiver at the cell-site can no longer guarantee reliable decoding of all active users. The notion of *"outage probability"* arises then as an appropriate system performance specifying parameter, defined as the probability that a randomly selected user will not be received with strong enough power to be successfully decoded. It should be noted at this point that this comes in contrast to the setup of (relatively) *fast*-fading channels, where the per-user fading process is stationary and ergodic, in which case *all* users can be reliably decoded at the receiver while employing (appropriately chosen) fixed and equal rates, and *without the requirement of either rate or power feedback from the receiver* (see [40] and references therein). Methods for obtaining the outage probability of linear CDMA receivers as a function of the rate, $\frac{E_b}{N_0}$, and the fading distribution, are shown in [17] and [43]. See also [44] [45] for a related analysis.

Restricting the discussion to slow-fading channels, and in conjunction with the above equal-rate equal-transmit-power scheme, it is assumed that the cell-site receiver ranks all active *intra-cell* users by their received powers, and then only decodes the transmissions of a *subset* of these users, comprising the (intra-cell) users with strongest received powers. The scheme is hence referred to as *"strongest-users decoding"*. The number of decoded users is the largest integer for which decoding is successful, and it depends on the realizations of both the fade levels and the additive background noise (fairness among users thus depends on the channel fading process, and a scheduling protocol can be incorporated so that all users are eventually decoded reliably, see for example [46] [47]). In order to analyze system performance under the strongest-users decoding scheme, the total achievable sum-rate over all *decodable* users is considered as the figure of merit of interest. This figure of merit is referred to as the *"outage constrained capacity"*. The strongest-users decoding strategy was first considered in [48] for a single-cell setup, without the constraint of random spreading (i.e., when all bandwidth is available for coding), and later on in [49] for randomly-spread DS-CDMA (again in a single-cell setup).

The underlying assumption of the analysis is that the system designer sets the transmission rate of all users in order to achieve a target fraction of *undecodable* users (FUU) per cell, which is completely equivalent to the notion of outage probability. Since different users experience independent fades, as the number of users grows, the percentage of undecodable users converges to a deterministic constant. Specifically, denoting by J the number of (reliably) *decoded* users per cell, it is assumed that at the limiting scenario $\frac{J}{K} \to 1 - \mathcal{Q}$, where $\mathcal{Q} \in [0, 1)$ denotes the limiting FUU per cell. Optimization of the overall throughput in this regime, by choosing the optimum $\mathcal{Q}$, is also considered. The rationale behind such optimization is the tradeoff between the (fixed and equal) rate to be employed by each individual user, and the FUU. As shown in Section 3, the above tradeoff does not lead to trivial results for the optimum $\mathcal{Q}$.

Focusing again on single cell-site processing, four multiuser detection strategies are considered, equivalent (under the strongest-users decoding scheme) to those considered for the preceding spectral efficiency analysis. In fact, the two linear receivers, the matched-filter, and the linear MMSE receiver, remain unchanged, and the difference in system performance as compared to the spectral efficiency simply emanates from the fact that part of the intra-cell users cannot be decoded reliably. The equivalent SCO and SCPO receivers are defined in the context of the strongest-users decoding scheme in the following way. The SCO receiver *"optimally"* decodes the transmissions of the maximum decodable subset of intra-cell users, while taking into account the *structure* of the multiuser interference generated by all remaining undecodable *intra-cell* users (assuming their signatures are known at the receiver). However, *out-of-cell interference* is treated as AWGN. The SCPO receiver is in principle similar to the SCO receiver, but takes in into account the structure of *all* undecodable multiuser interference (*both intra-cell and out-of-cell*), assuming all signatures are known at the receiver (as perviously assumed for the spectral efficiency analysis). The above four detection strategies are analyzed and compared in terms of their outage constrained capacities, analytically derived as functions of the FUU per cell. The results are also compared to the corresponding spectral efficiencies of the receivers, and the performance

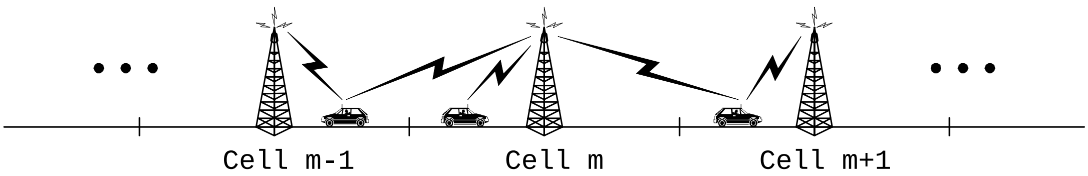

Figure 1: Linear cell-array model.

degradation due the use of the strongest-users decoding scheme is demonstrated and discussed. The outage constrained capacity of the two linear receivers is derived following [17]. The expression for the outage constrained capacity of the SCO or SCPO receivers has been derived in [49] for the *single-cell setup*, which is equivalent to setting $\alpha = 0$ (please note that both receivers are equivalent in this case), and the result is extended in this paper to account for the presence of undecodable out-of-cell interference. For purposes of the analysis, it is assumed that the system designer may in general modify the target FUU per cell as a function of the signal-to-noise ratio (SNR), or $\frac{E_b}{N_0}$, in order to optimize the outage constrained capacity. It is noted however that in some scenarios, practical considerations such as the receiver's complexity, or its available resources (translating to a maximum number of simultaneously decodable users), may restrict the *minimum* FUU. System performance can be further optimized by modifying the cell load β (as shown in [2], [17], [21] and [34]). Analogous results for a *single-cell* setup when no random spreading is employed are presented in [48], considering the matched-filter equivalent receiver, that treats all interference as additive noise, the SCO/SCPO equivalent receiver, and a receiver that performs successive interference cancellation, using the matched-filter equivalent receiver at each stage of the cancellation process. See also [50] for a related work in the framework of the single-user multiple-input multiple-output (MIMO) channel.

The structure of the rest of this paper is as follows. Section 2 presents the spectral efficiency analysis of the four multiuser detection strategies in concern. Section 3 includes the system performance analysis under the strongest-users decoding scheme. Finally, Section 4 ends the paper with a summary and some concluding remarks. The notation $(\cdot)_{\text{mf}}$, $(\cdot)_{\text{sco}}$, $(\cdot)_{\text{ms}}$, and $(\cdot)_{\text{scpo}}$ is used throughout to designate entries related to the matched-filter receiver, the SCO receiver, the linear MMSE receiver, and the SCPO receiver, respectively.

2 Spectral Efficiency Analysis

2.1 Multi-Cell System Model

Following [38] and [41], the uplink of a fully synchronous multi-cell DS-CDMA system is considered, whose cells are ordered in an *infinite* linear array, as depicted in Fig. 1. All cells are assumed completely identical from all aspects, and are therefore equivalent for the sake of analysis. Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the ith symbol, is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{H}_i \boldsymbol{x}_i + \alpha \boldsymbol{S}_i^- \boldsymbol{H}_i^- \boldsymbol{x}_i^- + \alpha \boldsymbol{S}_i^+ \boldsymbol{H}_i^+ \boldsymbol{x}_i^+ + \boldsymbol{n}_i \ . \tag{2-1}$$

The vector $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{K,i}]^T$ in (2-1) comprises the K code symbols transmitted by intra-cell users at the ith discrete time. The vectors $\boldsymbol{x}_i^\pm = \left[x_{1,i}^\pm, \ldots, x_{K,i}^\pm\right]^T$ denote the vectors of code symbols originated from users operating in adjacent cells. These symbols are assumed to be i.i.d., proper complex Gaussian random variables (which conforms with the capacity achieving statistics), with $E\{x_{k,j}\} = 0$ and $E\{|x_{k,j}|^2\} = \bar{P} \ \forall\, k, j$, where $\bar{P}$ is the equal transmit power of all users. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [41]).

The matrices $\boldsymbol{S}_i$ and $\boldsymbol{S}_i^\pm$ are $N \times K$ matrices, whose columns are the N-chip long spreading sequences (signatures) of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero mean random variables, with variance $1/N$. The vector $\boldsymbol{n}_i$ represents a zero mean white proper complex additive Gaussian noise vector, with $E\{\boldsymbol{n}_i \boldsymbol{n}_i^\dagger\} = \boldsymbol{I}$, $\forall i$. Without loss of generality all received powers are thus normalized with respect to the noise spectral level, and represent in fact the *signal to noise ratios* (SNRs) at the input to the multiuser receivers.

Finally, $\boldsymbol{H}_i \triangleq \text{diag}\,\left(h_{1,i}, \ldots, h_{K,i}\right)$ and $\boldsymbol{H}_i^\pm \triangleq \text{diag}\,\left(h_{1,i}^\pm, \ \ldots, h_{K,i}^\pm\right)$, where $\{h_{k,i}\}_{k=1}^K$ and $\left\{h_{k,i}^\pm\right\}_{k=1}^K$ designate the assumed i.i.d. zero-mean channel fading gains associated with the signals of the different users at the ith discrete time. In this setup, as the system size becomes large ($N, K \to \infty$, $\frac{K}{N} \to \beta < \infty$), the empirical distribution of the channel fading (power) levels, $\nu_{k,i}^{(\pm)} \triangleq \left|h_{k,i}^{(\pm)}\right|^2$, converges almost surely (a.s.) to a distribution $\mathcal{F}_\nu$. The fading levels are assumed throughout to be normalized so that $E_{\mathcal{F}_\nu}\{\nu\} = 1$. It is noted that the analysis to follow applies for a general fading distribution, however Rayleigh fading is assumed whenever explicit results are obtained. Some restrictions on the fading distribution shall be introduced for the analysis of the strongest-users decoding scheme, as stated in Section 3.

2.2 Spectral Efficiency of the Multiuser Receivers

The spectral efficiency of a linear receiver is conveniently expressed in terms of its *multiuser efficiency* (see [51]), defined as the ratio between the receiver's output *signal-to-interference-plus-noise ratio (SINR)* and the SNR (note that the multiuser efficiency may depend, as is the case with the linear MMSE receiver, on the presence of fading, see [17] and equation (2-22)). Denoting the multiuser efficiency by η, and following central limit results showing that the interference at the output of each of the two linear receivers, i.e., the matched-filter receiver and the linear MMSE receiver, is well approximated by a Gaussian noise (see [2], [21] and references therein for

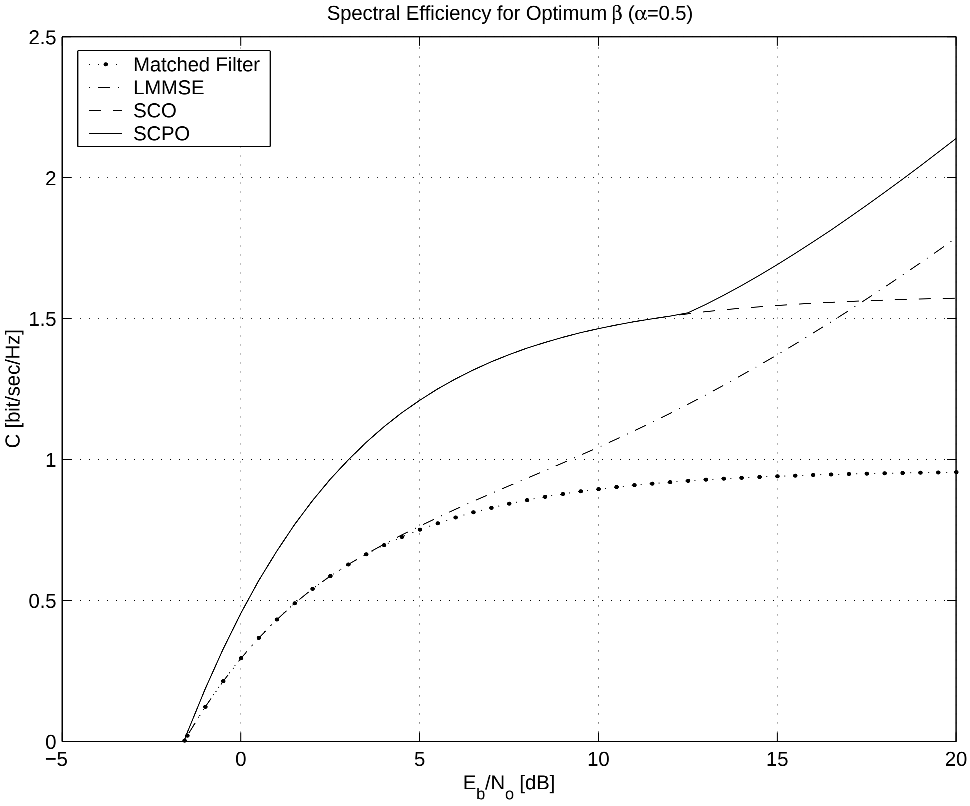

Figure 2: Spectral efficiency comparison for $\alpha = \frac{1}{2}$, and *optimized* cell load β.

justification of this Gaussian approximation), the spectral efficiency of these receivers is given by[1]

$$\tilde{C} = \beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\nu\eta\bar{P}\right)\right\} \underset{\text{Rayleigh}}{=} \beta\, e^{\frac{1}{\bar{P}\eta}} E_1\left(\frac{1}{\bar{P}\eta}\right)\log e\ , \tag{2-2}$$

where $E_{\mathcal{F}_\nu}$ denotes the expectation with respect to $\mathcal{F}_\nu$, and $E_1(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}\,dt$ $(t > 0)$, is the exponential integral function. The spectral efficiency of the two non-linear receivers, i.e., the SCO receiver and the SCPO receiver, is most conveniently determined using inter-relations between the spectral efficiency of the optimum multiuser receiver and that of the *linear* MMSE receiver, as described below.

The spectral efficiency of the four multiuser receivers is analyzed in the following subsections. It is noted that when different systems are to be compared (with possibly different spreading gains and data rates), it is useful to express the spectral efficiency in terms of $\frac{E_b}{N_0}$, following the relation $\bar{P} = \frac{1}{\beta}\,\tilde{C}\,\frac{E_b}{N_0}$ (see [17]). However, for simplicity of notation, equations are expressed here in terms of the transmit power (which is in fact the SNR, following the normalization with respect to the noise spectral level). The spectral efficiency is also analyzed in terms of its extreme-SNR characteristics

[1]The logarithms in all expressions are of arbitrary basis, however all numerical results in this paper correspond to base-2 logarithms.

[17] [52] [53], recognized in recent years as a key tool for comparing the performance of different systems in different settings. The low-SNR regime is characterized through the minimum E_b/N_0 that enables reliable communications,

$$\frac{E_b}{N_0}_{\min} \triangleq \frac{\log_e 2}{\dot{\tilde{C}}(0)} \ , \tag{2-3}$$

and the low-SNR spectral efficiency slope

$$S_0 \triangleq \frac{2\left[\dot{\tilde{C}}(0)\right]^2}{-\ddot{\tilde{C}}(0)} \ , \tag{2-4}$$

yielding the following low-SNR approximation

$$\mathsf{C}\left(\frac{E_b}{N_0}\right) \approx \frac{S_0}{3|_{\text{dB}}}\left(\left.\frac{E_b}{N_0}\right|_{\text{dB}} - \left.\frac{E_b}{N_0}_{\min}\right|_{\text{dB}}\right) \ . \tag{2-5}$$

In the above definitions $3|_{\text{dB}} = 10\log_{10} 2$, and $\dot{\tilde{C}}(0)$ and $\ddot{\tilde{C}}(0)$ are the first and second derivatives (whenever exist) of the spectral efficiency with respect to $\bar{P}$, respectively, evaluated in nats/sec/Hz at $\bar{P} = 0$. The high-SNR regime is characterized through the high-SNR slope (or "pre-log")

$$S_\infty \triangleq \lim_{\bar{P}\to\infty} \bar{P}\dot{\tilde{C}}(\bar{P}) \ , \tag{2-6}$$

with $\dot{\tilde{C}}(\bar{P})$ evaluated in nats/sec/Hz, and the high-SNR power offset

$$\mathcal{L}_\infty \triangleq \lim_{\bar{P}\to\infty}\left(\log_2 \bar{P} - \frac{\tilde{C}(\bar{P})}{S_\infty}\right) \ , \tag{2-7}$$

with $\tilde{C}(\bar{P})$ evaluated in bits/sec/Hz, yielding the following affine capacity approximation

$$\tilde{C}(\bar{P}) \underset{\bar{P}\gg 1}{\approx} \frac{S_\infty}{3|_{\text{dB}}}\left(\bar{P}|_{\text{dB}} - 3|_{\text{dB}}\mathcal{L}_\infty\right) \ . \tag{2-8}$$

For all multiuser receivers considered here it can easily be shown that the minimum transmit $\frac{E_b}{N_0}$ that enables reliable communications satisfies $\frac{E_b}{N_0}_{\min} = \ln 2 \approx -1.59$dB. In the rest of the analysis the focus is on the low- and high-SNR slopes of the different receivers. For simplicity of notation the slopes are derived while restricting the analysis to fading distribution satisfying $\Pr_{\mathcal{F}_\nu}(\nu > 0) = 1$ (that is, it is assumed that the fading distribution has no mass-point at zero). The results can however be straightforwardly extended to general fading distributions as in [17].

As said, Rayleigh fading channels were assumed throughout in order to produce numerical results, and the interference factor α was set to $\frac{1}{2}$, to mimic the case in which the average inter-cell interference power equals one half of the average power of intra-cell transmissions ($2\alpha^2 = \frac{1}{2}$). This figure is in agreement with early reports on IS-95 systems. The spectral efficiencies of all four multiuser receivers, optimized with respect to the cell load β, are plotted in Fig. 2.

2.2.1 The Matched-Filter Receiver

Proposition 2.1 *The asymptotic spectral efficiency of the matched-filter receiver equals*

$$\begin{aligned}\tilde{C}_{mf} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\frac{\bar{P}\nu}{1+\beta\left(1+2\alpha^2\right)\bar{P}}\right)\right\} \\ &\underset{Rayleigh}{=} \beta\, e^{\left[\frac{1}{\bar{P}}+\beta(1+2\alpha^2)\right]} E_1\left[\frac{1}{\bar{P}}+\beta(1+2\alpha^2)\right]\log e\ .\end{aligned} \tag{2-9}$$

The low-SNR slope of the matched filter receiver is given by

$$\mathcal{S}_{0_{mf}} = \frac{2\beta}{2\beta(1+2\alpha^2)+\kappa_\nu}\ , \tag{2-10}$$

where κ_ν is the kurtosis[2] *of the fading distribution.*

Proof: The derivation of the spectral efficiency of the matched-filter receiver in the multi-cell setting is based on the following Lemma on the convergence of the multiuser efficiency of the matched-filter receiver in a *single-cell* system.

Lemma 2.2 (Tse-Hanly [3]) *Let the empirical distribution of the received powers of all users converge a.s. as $K,N\rightarrow\infty$, $\frac{K}{N}\rightarrow\beta<\infty$, to some non-random limit $\mathcal{H}(P)$. Then, the multiuser efficiency of the matched-filter receiver converges in probability to a non-random limit η_{mf}, equal for all users, given by*

$$\eta_{mf} = \frac{1}{1+\beta E_{\mathcal{H}}\left\{P\right\}}\ , \tag{2-11}$$

where $E_{\mathcal{H}}\left\{\cdot\right\}$ represents expectation with respect to $\mathcal{H}(P)$.

Turning to the particular multi-cell model considered here, the matched-filter receiver effectively operates in an *equivalent single-cell system* of $3K$ users, one third of which (the intra-cell users) are received at powers $\left\{\nu_k\bar{P}\right\}$, $k=1,\ldots,K$, while the remaining two thirds (the adjacent-cell users) are received at powers $\left\{\alpha^2\nu_k\bar{P}\right\}$, $k=K+1,\ldots,3K$. Applying Lemma 2.2 to this equivalent single-cell setting, it is easy to see that the multiuser efficiency of the matched-filter receiver in the multi-cell model converges in probability to

$$\eta_{\mathrm{mf}} = \frac{1}{1+\beta\left(1+2\alpha^2\right)\bar{P}}. \tag{2-12}$$

Equation (2-9) then follows immediately from (2-2). Finally, the low-SNR slope is straightforwardly obtained by applying (2-4). The explicit dependency of the low-SNR slope in the kurtosis is observed by performing the derivation while expressing the spectral efficiency in terms of the *received* SNR $\bar{P}_{\mathrm{rec}} \triangleq E_{\mathcal{F}_\nu}\left\{\nu\right\}\bar{P}$ (see a discussion in this respect in App. A). ■

By examining (2-9), it is clear that the spectral efficiency of the matched-filter receiver monotonically increases with the cell load β. The optimum choice is therefore to increase β without

[2] *The kurtosis [17] of the fading distribution $\mathcal{F}_\nu$ is defined as $\kappa_\nu = E_{\mathcal{F}_\nu}\left\{\nu^2\right\}/\left(E_{\mathcal{F}_\nu}\left\{\nu\right\}\right)^2$.*

bound and the spectral efficiency at the limit of $\beta \to \infty$ admits the following expression

$$\lim_{\beta\to\infty} \tilde{C}_{\text{mf}} = \frac{1}{1+2\alpha^2}\left[\log e - \left(\frac{E_b}{N_0}\right)^{-1}\right] \ . \tag{2-13}$$

The matched-filter receiver is obviously interference limited and its spectral efficiency reaches a limit of $\frac{\log e}{1+2\alpha^2}$ as $\frac{E_b}{N_0} \to \infty$. The presence of out-of-cell interference induces a penalty factor of $\frac{1}{1+2\alpha^2}$, as compared to the corresponding spectral efficiency in the single-cell setup. The effect of out-of-cell interference is also evident in the low-SNR slope (2-10) (through the $(1+2\alpha^2)$ factor in the denominator, cf. [17]). It can also be observed that taking $\beta \to \infty$ *completely eliminates* the effect of fading, and the spectral efficiency of the receiver coincides with that attained in *non-fading* channels [21]. This result holds regardless of the fading distribution. Furthermore, taking $\beta \to \infty$ also eliminates the penalty due to the use of random spreading. The latter effect is observed by comparison to the spectral efficiency of a receiver equivalent to the matched-filter receiver (in the sense of treating all other-users interference as an AWGN), when no spreading is employed and *all* bandwidth is available for coding [41]. It is noted that the effect of fading in the latter setting is also eliminated in the infinite number of users regime. The spectral efficiency of the matched filter receiver at the limit as $\beta \to \infty$ is plotted in Fig. 2.

2.2.2 The SCO Receiver

Proposition 2.3 *The asymptotic spectral efficiency of the SCO receiver equals*

$$\begin{aligned}\tilde{C}_{sco} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left[1+\bar{P}_{eq}\nu\eta^{s\text{-}c}_{ms}\right]\right\} + \log\frac{1}{\eta^{s\text{-}c}_{ms}} + (\eta^{s\text{-}c}_{ms}-1)\log e \\ &\underset{Rayleigh}{=} \beta\, e^{\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}} E_1\left(\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}\right)\log e + \log\frac{1}{\eta^{s\text{-}c}_{ms}} + (\eta^{s\text{-}c}_{ms}-1)\log e \ ,\end{aligned} \tag{2-14}$$

with $\eta^{s\text{-}c}_{ms}$ being the unique positive solution of

$$\begin{aligned}1 &= \eta^{s\text{-}c}_{ms} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}_{eq}\nu\eta^{s\text{-}c}_{ms}}{1+\bar{P}_{eq}\nu\eta^{s\text{-}c}_{ms}}\right\} \\ &\underset{Rayleigh}{=} \eta^{s\text{-}c}_{ms} + \beta\left[1 - \frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}} e^{\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}} E_1\left(\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}\right)\right] \ ,\end{aligned} \tag{2-15}$$

and

$$\bar{P}_{eq} \triangleq \frac{\bar{P}}{1+2\beta\alpha^2\bar{P}} \ . \tag{2-16}$$

The low-SNR slope of the SCO receiver is given by

$$S_{0_{sco}} = \frac{2\beta}{\beta(1+4\alpha^2)+\kappa_\nu} \ . \tag{2-17}$$

Proof: In order to derive the spectral efficiency of the SCO receiver, the following observation [21] is applied. By definition, the SCO receiver treats adjacent-cell interference as an additive white Gaussian noise, which implies that joint nearest neighbor decoding is employed in the processing of intra-cell transmissions. However the additive interference originating from adjacent-cell users is in fact *not* an additive white Gaussian noise, which puts the setup in the framework of the mismatched decoding problem, as analyzed in [54] and [55]. According to [54] and [55], adding a mild restriction that the additive adjacent cell interference is ergodic of second moment, and under the assumption, adhered to in this analysis, that all codebooks are Gaussian, the spectral efficiency of the receiver depends on the actual noise plus interference distribution only via its power, and thus coincides with the spectral efficiency in a *white Gaussian noise channel* with signal and noise powers equal to those of the original channel. Following this result, it may be concluded that in terms of spectral efficiency the SCO receiver is equivalent to an *optimum* receiver in a *single-cell* system, where the additive white Gaussian background noise process has spectral level given by $1 + 2\beta\alpha^2\bar{P}$. Accordingly, $\bar{P}_{\text{eq}}$ of (2-16) represents the effective "single-cell" SNR for the SCO receiver in the multi-cell setting. The spectral efficiency of the optimum receiver in the single-cell setting has been derived in [17], and the result is reviewed in the following Lemma.

Lemma 2.4 (Shamai-Verdú, [17] Theorem IV.1) *Let the empirical distribution of the received powers of all users converge a.s. as* $K, N \to \infty$, $\frac{K}{N} \to \beta < \infty$, *to some non-random limit* $\mathcal{H}(P)$. *Then, the spectral efficiency of the optimum multiuser receiver is given by*

$$\tilde{C}_{sco} = \tilde{C}_{ms}^{s\text{-}c} + \log\frac{1}{\eta_{ms}^{s\text{-}c}} + (\eta_{ms}^{s\text{-}c} - 1)\log e \ , \tag{2-18}$$

where $\eta_{ms}^{s\text{-}c}$ *is the limiting multiuser efficiency of the corresponding linear MMSE receiver, as given by Lemma 2.6 (Eq.* (2-25)*), and*

$$\tilde{C}_{ms}^{s\text{-}c} = \beta E_{\mathcal{H}}\left\{\log(1 + \eta_{ms}^{s\text{-}c}P)\right\} \tag{2-19}$$

is the spectral efficiency of this receiver.

The above relation between the spectral efficiency of the optimum receiver and that of the linear MMSE receiver in a single-cell setting, was found to hold also for binary coded symbols (as opposed to the Gaussian symbols assumed here) via the replica method of statistical physics [30] (see also [56] for the fundamental relation between mutual information and MMSE in Gaussian channels). Using the observation described at the beginning of the proof, the proposition follows straightforwardly by substituting $\bar{P}_{\text{eq}}$ of (2-16), instead of $\bar{P}$, into (2-25), (2-18) and (2-19). The low-SNR slope of (2-17) is derived by applying (2-4) to (2-14). ■

In short, what Proposition 2.3 basically states is that the spectral efficiency of the SCO receiver in the multi-cell setting equals the spectral efficiency of the optimum multiuser receiver in a single-cell setting, but with the SNR replaced by the "effective" SNR of (2-16). As shown in [17], the spectral efficiency of the optimum receiver in the single-cell setting is maximized by taking $\beta \to \infty$. Therefore this result must also hold for the SCO receiver in the current multi-cell model as well (for the low-SNR regime this property is clearly observed by examining the low-SNR slope (2-17)).

Taking $\beta \to \infty$, the spectral efficiency of the SCO receiver admits the following limiting expression

$$\lim_{\beta\to\infty} \tilde{C}_{\text{sco}} = \log\left(1 + \frac{\beta\bar{P}}{1+2\alpha^2\beta\bar{P}}\right) = \log\left(1 + \frac{\tilde{C}_{\text{sco}}\frac{E_b}{N_0}}{1+2\alpha^2\tilde{C}_{\text{sco}}\frac{E_b}{N_0}}\right) \ . \tag{2-20}$$

Considering the low-SNR regime at the limit of $\beta \to \infty$, it is observed that the low-SNR slope goes to the limit of $\frac{2}{1+4\alpha^2}$, exhibiting a penalty factor of $\frac{1}{1+2\alpha^2}$ due to the presence of inter-cell-interference. For the particular case of $\alpha = \frac{1}{2}$ the slope is reduced by a factor of $\frac{1}{2}$, as compared to the corresponding result in the single-cell setting. Also, in obvious contrast to the single-cell setting, the introduction of inter-cell interference, and the fact that the receiver treats this interference as AWGN, turn the receiver interference limited, and the spectral efficiency reaches a limit of $\log\left(1+\frac{1}{2\alpha^2}\right)$ as $\frac{E_b}{N_0} \to \infty$ (for any fixed strictly positive α). As observed for the matched-filter receiver, increasing the cell load β without bound completely eliminates both the effect of fading (also shown for the single-cell setting in [17]), and the penalty due to the use of random spreading. This can be seen by comparing (2-20) to the corresponding results in the no-fading regime [21], and when no spreading is employed [41], respectively. The limiting spectral efficiency of the SCO receiver as $\beta \to \infty$ is plotted in Fig. 2.

2.2.3 The Linear MMSE receiver

Proposition 2.5 *The asymptotic spectral efficiency of the linear MMSE receiver equals*

$$\begin{aligned}\tilde{C}_{ms} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\nu\eta_{ms}\bar{P}\right)\right\} \\ &\underset{Rayleigh}{=} \beta\, e^{\frac{1}{\bar{P}\eta_{ms}}} E_1\left(\frac{1}{\bar{P}\eta_{ms}}\right)\log e \ ,\end{aligned} \tag{2-21}$$

where η_{ms} is the unique positive solution to the following implicit equation

$$\begin{aligned}1 &= \eta_{ms} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}\nu\eta_{ms}}{1+\bar{P}\nu\eta_{ms}} + \frac{2\alpha^2\bar{P}\nu\eta_{ms}}{1+\alpha^2\bar{P}\nu\eta_{ms}}\right\} \\ &\underset{Rayleigh}{=} \eta_{ms} + \beta\left[3 - \frac{1}{\bar{P}\eta_{ms}}\, e^{\frac{1}{\bar{P}\eta_{ms}}} E_1\left(\frac{1}{\bar{P}\eta_{ms}}\right) - \frac{2}{\alpha^2\bar{P}\eta_{ms}}\, e^{\frac{1}{\alpha^2\bar{P}\eta_{ms}}} E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{ms}}\right)\right] \ .\end{aligned} \tag{2-22}$$

The low-SNR and high-SNR slopes of the linear MMSE receiver are given, respectively, by

$$\mathcal{S}_{0_{ms}} = \frac{2\beta}{2\beta(1+2\alpha^2)+\kappa_\nu} \ , \tag{2-23}$$

and (assuming $\Pr_{\mathcal{F}_\nu}(\nu > 0) = 1$)

$$\mathcal{S}_{\infty_{ms}} = \begin{cases} \beta & \beta < \frac{1}{3}, \\ \frac{1}{6} & \beta = \frac{1}{3}, \\ 0 & \beta > \frac{1}{3} \ . \end{cases} \tag{2-24}$$

Proof: In order to prove the proposition we use again a result from [3], as it is formulated in [17], that applies to single-cell systems and flat fading channels.

Lemma 2.6 (Tse-Hanly [3]) *Let the empirical distribution of the received powers of all users converge a.s. as* $K, N \to \infty$, $\frac{K}{N} \to \beta < \infty$, *to some non-random limit* $\mathcal{H}(P)$. *Then, the multiuser efficiency of the linear MMSE receiver converges a.s. to a non-random limit* η_{ms}, *equal for all users, given by the unique positive solution to the implicit equation*

$$\eta_{ms} + \beta E_{\mathcal{H}} \left\{ \left[\frac{\eta_{ms} P}{1 + \eta_{ms} P} \right] \right\} = 1 \ , \tag{2-25}$$

where $E_{\mathcal{H}} \{\cdot\}$ *represents expectation with respect to* $\mathcal{H}(P)$.

Using the single-cell $3K$-user system interpretation as in the proof of Proposition 2.1, it is straightforward to see that the multiuser efficiency of the linear MMSE receiver in the multi-cell setting is given by the unique positive solution to (2-22). The spectral efficiency of the receiver is then evaluated by substituting the result into (2-2). The extreme-SNR slopes are derived by applying (2-4) and (2-6) to (2-21), while extending Properties 3 to 6 of [17] to the multi-cell setting considered here. ■

In contrast to the matched-filter and the SCO receivers, the linear MMSE receiver is not interference limited, provided that the cell load β is appropriately chosen (see (2-24)). Analysis of the result of Proposition 2.5 shows that for low $\frac{E_b}{N_0}$, see (2-23), it is optimum to increase the cell load β without bound. In such case, the spectral efficiencies of the linear MMSE receiver coincides with that of the matched-filter receiver. In fact, this equivalence holds for $\beta \to \infty$ regardless of $\frac{E_b}{N_0}$, and in the wideband (low-SNR) limit for any β (note that the low-SNR slopes for both receivers are exactly the same). However, beyond some critical $\frac{E_b}{N_0}$ the optimum choice for the cell load β starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$ in the high $\frac{E_b}{N_0}$ region (as can be deduced from (2-24)), and the spectral efficiency of the receiver grows without bound with $\frac{E_b}{N_0}$. In fact, the the high-SNR slope of the receiver (2-24) indicates that the linear MMSE receiver becomes *interference limited* if the cell load β is set to values higher than $\frac{1}{3}$. This threshold value emanates from the fact that the receiver processes the signals of $3K$ users (K intra-cell users, and $2K$ users in the two adjacent cells), and it is in agreement with the threshold value of unity observed in the single-cell analysis of [17] (see also [21]). It is interesting to note in this respect that unlike the low-SNR slope (2-23), assuming the fading distribution has no mass-point at zero, the high-SNR slope of the linear MMSE receiver (2-24) is independent of neither the fading distribution, nor the inter-cell interference factor α (provided it is *strictly* positive), and the degradation with respect to the single-cell setting is only due to the fact the the receiver "sees" the signals of $3K$ users (instead of K).

The critical $\frac{E_b}{N_0}$ beyond which the optimum cell load takes on finite values can be analytically expressed, and the result is summarized in the following proposition.

Proposition 2.7 *Suppose that* $E_{\mathcal{F}_\nu} \left\{ \nu^2 \right\} < \infty$. *Then, for the linear MMSE receiver, the optimum value in terms of the spectral efficiency of the cell load* β *is finite if and only if*

$$\frac{E_b}{N_0} > \frac{E_b}{N_0}_{cr} \triangleq \frac{2(1 + 2\alpha^4)}{1 - 2\alpha^2 + 4\alpha^4} \ln 2. \tag{2-26}$$

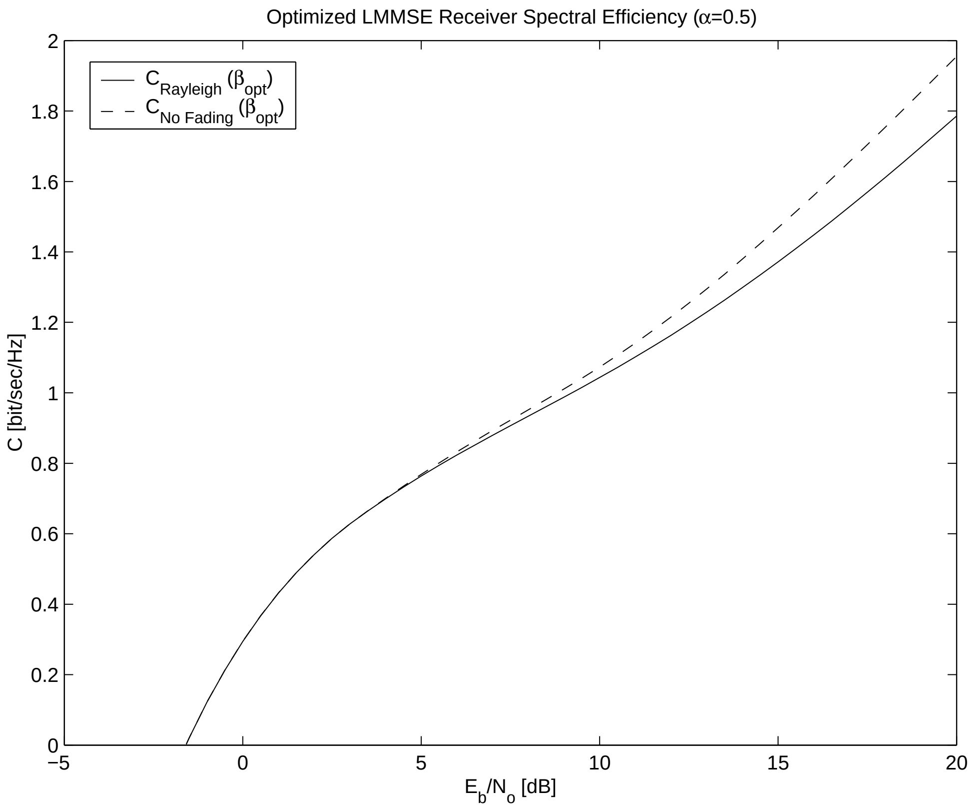

Figure 3: The effect of fading on the spectral efficiency of the linear MMSE receiver for $\alpha = \frac{1}{2}$: Optimum β.

Proof: See Appendix A. ■

As can be observed, as long as $E_{\mathcal{F}_\nu}\left\{\nu^2\right\} < \infty$, the value of $\frac{E_b}{N_0}_{\text{cr}}$ is independent of the fading distribution, and the result holds in particular for the non-fading regime [21]. For $\alpha = \frac{1}{2}$, one gets $\frac{E_b}{N_0}_{\text{cr}} = 3\ln 2 \approx 3.18$dB.

The spectral efficiency of the linear MMSE receiver with the *optimum* choice of cell load β is plotted both in Figs. 2 and 3. The sort of "knee effect" observed in the spectral efficiency curve of the receiver designates the region in which the optimum choice for the cell load starts to decrease from infinity (and the linear MMSE receiver starts to outperform the matched-filter receiver, as observed in Fig. 2). Comparing the spectral efficiency results to those of the SCO receiver, it is also observed that beyond some critical $\frac{E_b}{N_0}$, the relatively simpler *linear* MMSE receiver, is preferable over the interference limited SCO receiver, that employs *non-linear* processing of the intra-cell users' transmissions. The above result emanates from the fact the linear MMSE receiver is more informed regarding adjacent-cell interference, while the SCO receiver treats inter-cell interference as AWGN.

In order to examine the impact of fading on the spectral efficiency of the linear MMSE receiver, the *optimized* spectral efficiency obtained in the non-fading regime was also included in Fig. 3.

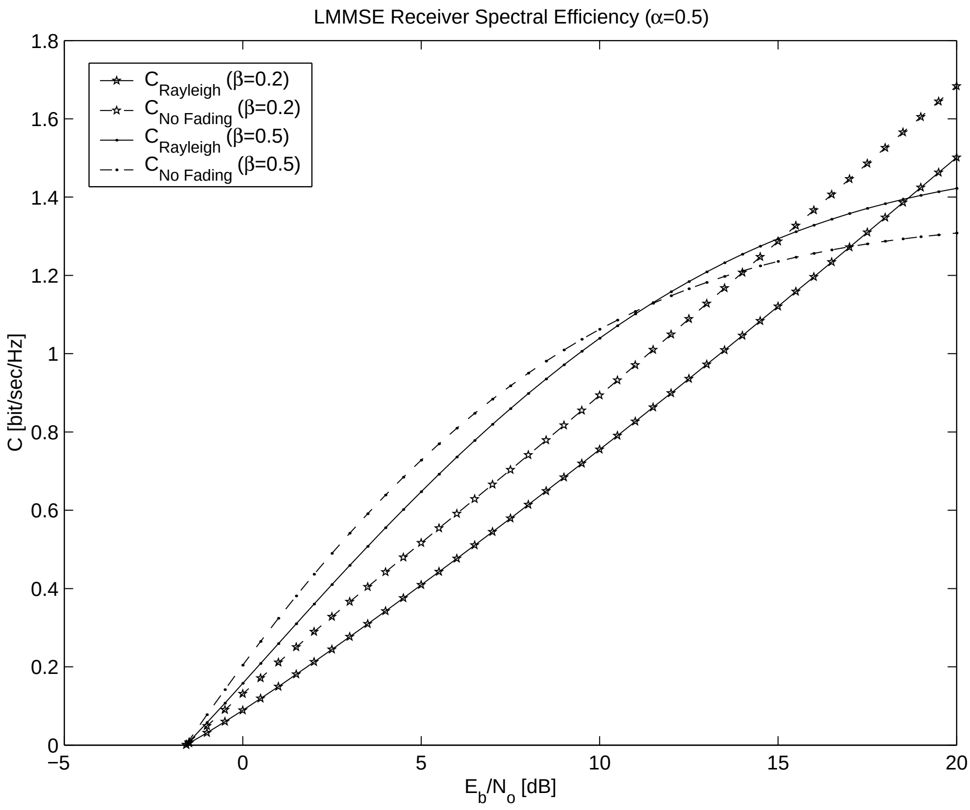

Figure 4: The effect of fading on the spectral efficiency of the linear MMSE receiver for $\alpha = \frac{1}{2}$: Fixed β.

As can be observed, the introduction of fading induces a clear degradation in system performance beyond $\frac{E_b}{N_0}_{\rm cr}$, below which taking $\beta \to \infty$ is optimum, the effect of fading (and spreading) is completely eliminated, and the results coincide with those obtained for the matched-filter receiver. It is noted however that for a *fixed* cell load $\beta > \frac{1}{3}$, fading becomes, in fact, *beneficial* in terms of spectral efficiency beyond some critical (β depended) $\frac{E_b}{N_0}$, and the spectral efficiency *with* fading surpasses that of non-fading channels. The result is explained by the "interference population control" effect of fading, effectively reducing the cell load as seen by the receiver (see an elaboration on this phenomena in [17]). This behavior is demonstrated for Rayleigh fading in Fig. 4, where the spectral efficiency of the linear MMSE receiver is plotted taking $\beta = 0.2 < \frac{1}{3}$, for which the spectral efficiency without fading always surpasses the spectral efficiency in Rayleigh fading channels, and taking $\beta = 0.5 > \frac{1}{3}$, for which Rayleigh fading becomes beneficial beyond some critical $\frac{E_b}{N_0}$.

2.2.4 The SCPO Receiver

Proposition 2.8 *The asymptotic spectral efficiency of the SCPO receiver equals*

$$\begin{aligned}
\tilde{C}_{scpo} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\bar{P}\nu\eta_{\mathfrak{C}}\right)\right\} + 2\beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}\right)\right\} + \log\frac{1}{\eta_{\mathfrak{C}}} + (\eta_{\mathfrak{C}}-1)\log e \\
&\quad - \left[2\beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}\right)\right\} + \log\frac{1}{\eta_{\mathfrak{S}}} + (\eta_{\mathfrak{S}}-1)\log e\right] \\
&\underset{Rayleigh}{=} \beta\log e\left[e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}}E_1\left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}}\right) + 2e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}}E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}\right) - 2e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}}E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}\right)\right] \\
&\quad + \log\frac{\eta_{\mathfrak{S}}}{\eta_{\mathfrak{C}}} + (\eta_{\mathfrak{C}}-\eta_{\mathfrak{S}})\log e\ ,
\end{aligned} \tag{2-27}$$

where $\eta_{\mathfrak{C}}$ and $\eta_{\mathfrak{S}}$ are the respective unique positive solutions of the following two implicit equations:

$$\begin{aligned}
1 &= \eta_{\mathfrak{C}} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}\nu\eta_{\mathfrak{C}}}{1+\bar{P}\nu\eta_{\mathfrak{C}}}\right\} + 2\beta E_{\mathcal{F}_\nu}\left\{\frac{\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}}{1+\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}}\right\} \\
&\underset{Rayleigh}{=} \eta_{\mathfrak{C}} + \beta\left[3 - \frac{1}{\bar{P}\eta_{\mathfrak{C}}}e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}}E_1\left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}}\right) - \frac{2}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}}E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}\right)\right]\ ,
\end{aligned} \tag{2-28}$$

and

$$\begin{aligned}
1 &= \eta_{\mathfrak{S}} + 2\beta E_{\mathcal{F}_\nu}\left\{\frac{\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}}{1+\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}}\right\} \\
&\underset{Rayleigh}{=} \eta_{\mathfrak{S}} + 2\beta\left[1 - \frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}}E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}\right)\right]\ .
\end{aligned} \tag{2-29}$$

The low-SNR and high-SNR slopes of the SCPO receiver are given, respectively, by

$$S_{0_{scpo}} = \frac{2\beta}{\beta(1+4\alpha^2)+\kappa_\nu}\ , \tag{2-30}$$

and (assuming $\Pr_{\mathcal{F}_\nu}(\nu>0)=1$)

$$S_{\infty_{scpo}} = \begin{cases} \beta & \beta \le \frac{1}{3}, \\ 1-2\beta & \frac{1}{3} < \beta \le \frac{1}{2}, \\ 0 & \beta > \frac{1}{2}\ . \end{cases} \tag{2-31}$$

Proof: The SCPO receiver is defined as the receiver that optimally decodes the transmissions of intra-cell users, while fully taking into account the structure of the interference generated by out-of-cell users (assuming the receiver is unaware of the codebooks employed by these users). Hence, the asymptotic spectral efficiency of the SCPO receiver is equal to the limiting normalized mutual information between the channel input due to intra-cell users only, and the channel output, i.e.,

$$\tilde{C}_{\text{scpo}} = \lim_{N,K\to\infty,\frac{K}{N}\to\beta<\infty}\frac{1}{N}I\left(\boldsymbol{x}\,;\,\boldsymbol{y}\right)\ , \tag{2-32}$$

with $\boldsymbol{x}$ and $\boldsymbol{y}$ defined as in (2-1). Using Kolmogorov's identity, the above mutual information can be expressed as

$$I\left(\boldsymbol{x}\,;\,\boldsymbol{y}\right) = I\left(\boldsymbol{x}, \boldsymbol{x}^{-}, \boldsymbol{x}^{+}\,;\,\boldsymbol{y}\right) - I\left(\boldsymbol{x}^{-}, \boldsymbol{x}^{+}\,;\,\boldsymbol{y}\,|\,\boldsymbol{x}\right)\ , \tag{2-33}$$

and defining

$$\begin{aligned}\tilde{C}_{\mathfrak{C}} &= \lim_{N,K\to\infty,\frac{K}{N}\to\beta<\infty} \frac{1}{N} I\left(\boldsymbol{x}, \boldsymbol{x}^{-}, \boldsymbol{x}^{+}\,;\,\boldsymbol{y}\right) \\ \tilde{C}_{\mathfrak{S}} &= \lim_{N,K\to\infty,\frac{K}{N}\to\beta<\infty} \frac{1}{N} I\left(\boldsymbol{x}^{-}, \boldsymbol{x}^{+}\,;\,\boldsymbol{y}\,|\,\boldsymbol{x}\right)\end{aligned} \tag{2-34}$$

this yields

$$\tilde{C}_{\text{scpo}} = \tilde{C}_{\mathfrak{C}} - \tilde{C}_{\mathfrak{S}}\ . \tag{2-35}$$

But $\tilde{C}_{\mathfrak{C}}$ is actually the asymptotic spectral efficiency of the *optimum* receiver in the $3K$-users single-cell interpretation, as described in the proof of Proposition 2.1. Similarly, it follows that $\tilde{C}_{\mathfrak{S}}$ equals the asymptotic spectral efficiency of the optimum receiver in a single-cell setting with $2K$ users, *all* received with powers $\left\{\alpha^2\nu\bar{P}\right\}$. Both $\tilde{C}_{\mathfrak{C}}$ and $\tilde{C}_{\mathfrak{S}}$ can be obtained using Lemma 2.4, and it is straightforward to show that (2-28) and (2-29) define the two multiuser efficiencies of the corresponding linear MMSE receivers (as defined in Lemma 2.4), for the respective $3K$-users and $2K$-users single-cell settings, as described above. The proof of (2-27) is completed by substituting the two spectral efficiency results into (2-35). The extreme-SNR slopes of (2-30) and (2-31) can be straightforwardly derived in a similar manner to the previous receivers. However, a much simpler derivation of (2-31) can be obtained by noticing, following [57] (see also [34]), that the high-SNR slopes of $\tilde{C}_{\mathfrak{C}}$ and $\tilde{C}_{\mathfrak{S}}$ of (2-34) are given, respectively, by $\min(3\beta, 1)$ and $\min(2\beta, 1)$. The difference between the two slopes immediately yields (2-31). ■

It is worth noting at this point that the spectral efficiency of the SCPO receiver can also be attained by a receiver that employs an *MMSE based successive interference cancellation (MMSE-SC)* to decode transmissions of intra-cell users, while *inter-cell* interference is mitigated by means of a linear MMSE filter [21], [42]. That is, starting from the first intra-cell user, the MMSE-SC receiver uses linear MMSE processing to mitigate the interference generated by $3K-1$ users ($K-1$ intra-cell, and $2K$ at adjacent cells), as experienced by user 1. After decoding, the signal generated by user 1 is reconstructed, subtracted from the total received signal, and the result is passed on to the receiver for the second intra-cell user, now experiencing interference generated by $3K-2$ users ($K-2$ intra-cell, and $2K$ at adjacent cells). The procedure is repeated until the last intra-cell user (user K), who due to the cancellation process experiences only interference generated by the $2K$ adjacent-cell users. Since by the underlying assumption the cell-site receiver is unaware of the codebooks of adjacent-cell users, no decoding and cancellation are performed with respect to the signals generated by these users. The spectral efficiency equivalence of the MMSE-SC receiver and the SCPO receiver is evident by noticing the information preserving property of the MMSE estimator in the Gaussian regime, while emphasizing again the underlying assumption that the receiver has *no knowledge of the codebooks used in the adjacent cells*, and that those codebooks are randomly selected per message (as is indeed assumed in the system model considered, see Subsection 2.1). By exactly the same arguments used in [58] for the MMSE-SC receiver in a single-cell scenario, the sum of rates attained in the present model by the successive cancellation process, can be shown to correspond to the chain decomposition rule for mutual information, and the equivalence of the two receivers then follows from (2-35). It is noted, however, that in the case in which the codebooks of adjacent-cell users *are* known or chosen once for good, the MMSE-SC is *no longer* the optimum receiver, as the problem falls within the difficult framework of joint multiple-access/interference channels.

The SCPO receiver is also not interference limited, as should have been expected, provided that the cell load β is appropriately chosen (see (2-31)). In the low-SNR regime, the low-SNR slope of the SCPO receiver coincides with the slope of the SCO receiver, and it is optimum (as is in fact the case for all other receivers) to increase the cell load β without bound. In such case, the spectral efficiency of the SCPO receiver coincides with that of the SCO receiver (again this equivalence holds for $\beta \to \infty$ regardless of $\frac{E_b}{N_0}$). However, beyond some critical $\frac{E_b}{N_0}$ the optimum cell load starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$, and the spectral efficiency the SCPO receiver grows without bound with $\frac{E_b}{N_0}$. As observed from (2-31), the high-SNR slope attains its maximum value of $\frac{1}{3}$ for $\beta = \frac{1}{3}$, and the receiver becomes interference limited for $\beta \geq \frac{1}{2}$. Also, as noted with respect to the linear MMSE receiver, the high-SNR slope of the SCPO receiver is independent of neither the fading distribution (assuming no mass-point as zero) nor of the out-of-cell interference factor α.

The spectral efficiency of the SCPO receiver obtained for the *optimum* choice of cell load β is plotted both in Figs. 2 and 5. As observed for the linear MMSE receiver, the region in which the optimum choice for β starts to decrease from infinity (and the SCPO receiver starts to outperform the SCO receiver, as observed in Fig. 2) induces a "knee effect" in the spectral efficiency curve.

Referring to Fig. 5, and comparing the spectral efficiency to that obtained in the non-fading regime, while optimizing performance with respect to β, it is observed again that fading degrades system performance beyond the critical $\frac{E_b}{N_0}$ above which finite cell loads are optimum. Below this critical $\frac{E_b}{N_0}$, i.e., for $\beta \to \infty$, the effect of fading (and in fact also of random spreading [21]) is completely eliminated and the spectral efficiency curves of both regimes coincide. As observed for the linear MMSE receiver, fading may become beneficial for a *fixed* cell load $\beta > \frac{1}{3}$, beyond some critical $\frac{E_b}{N_0}$, due to its population control effect. One can obtain similar results in this case to those presented in Fig. 4 for the linear MMSE receiver.

To complete the discussion (in view of [21]), the spectral efficiency of what is referred to in [41] as the *adjacent-cell decoder (ACD)* was also considered. This receiver, with *no spreading employed*, also knows the *codebooks* of users in adjacent cells, and either decodes their transmissions, or treats them as additive Gaussian noise, whichever is preferable in terms of spectral efficiency. The spectral efficiency of this receiver is given by (note that without spreading $K = \beta$)

$$\tilde{C}^{\text{n.s.}}_{\text{ACD}} = \max\left[\log\left(1 + \frac{K\bar{P}}{1 + 2\alpha^2 K\bar{P}}\right), \min\left(\frac{1}{3}\log\left(1 + (1 + 2\alpha^2)K\bar{P}\right), \frac{1}{2}\log\left(1 + 2\alpha^2 K\bar{P}\right)\right)\right], \tag{2-36}$$

and the corresponding numerical results are provided in Fig. 5. As can be seen, for low $\frac{E_b}{N_0}$ it is preferable not to decode adjacent-cell transmissions, and the spectral efficiency of this receiver

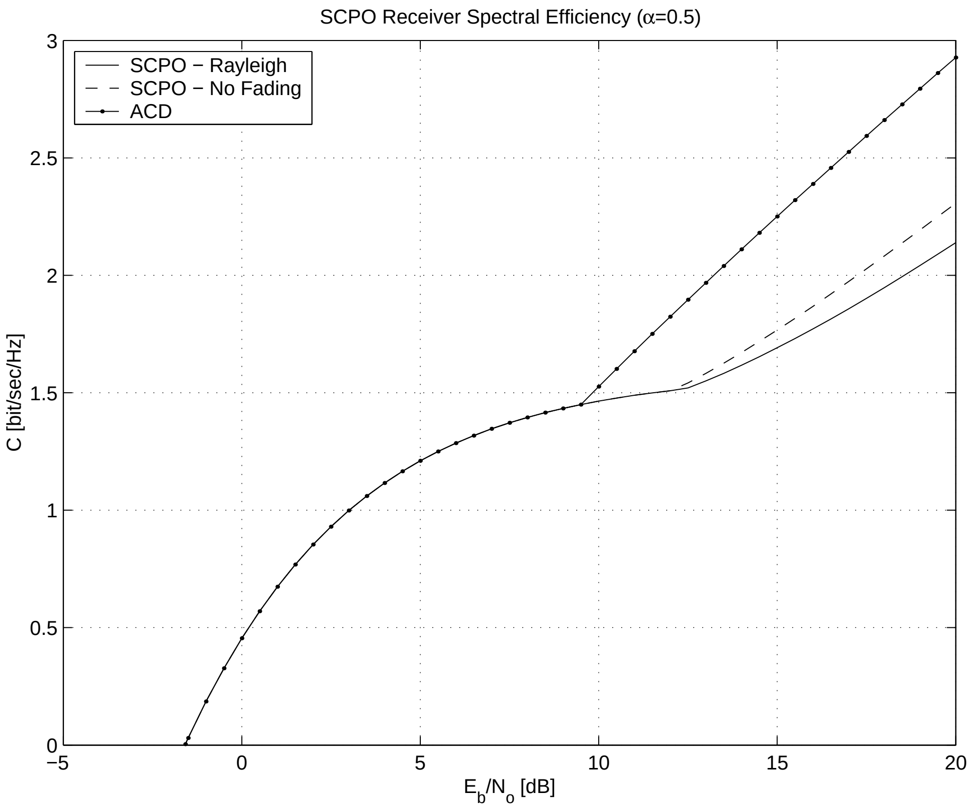

Figure 5: Optimized spectral efficiency of the SCPO receiver and the ACD for $\alpha = \frac{1}{2}$.

coincides with that of the SCPO receiver (and also the SCO receiver), for the optimum choice of β which is $\beta \to \infty$. However beyond some critical $\frac{E_b}{N_0}$, where decoding is preferable, the curves depart and the spectral efficiency of the ACD grows quite rapidly with $\frac{E_b}{N_0}$ as compared to that of the SCPO receiver. It should be noted in this respect that the high-SNR slope of the ACD equals $\frac{1}{3}$, which upper bounds the corresponding high-SNR slope of the SCPO receiver for any choice of β (see (2-31)).

3 Analysis of the Strongest-Users Decoding Scheme

Following the investigation of the ultimate performance of the four multiuser detection strategies, the performance under the strongest-users decoding scheme described in Section 1 is analyzed next. In general, the same system model defined for the analysis of the ultimate performance applies for the current analysis as well. However two restrictions are introduced. First, it is now assumed that the fading channel changes *slowly*, and therefore the matrices of channel fading-gains $\boldsymbol{H}^{(\pm)}$ are assumed to be constant for the whole codeword duration. Hence, the resulting signal vector received at some arbitrary cell-site, at the ith discrete time, is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{H} \boldsymbol{x}_i + \alpha \boldsymbol{S}_i^- \boldsymbol{H}^- \boldsymbol{x}_i^- + \alpha \boldsymbol{S}_i^+ \boldsymbol{H}^+ \boldsymbol{x}_i^+ + \boldsymbol{n}_i \ , \tag{3-1}$$

which is the same as (2-1), except for the fact that the time index attributed to the matrices $\boldsymbol{H}^{(\pm)}$ has been removed. A second restriction is posed on the fading distribution, by assuming that its density $\frac{\partial \mathcal{F}_\nu(\nu)}{\partial \nu}$ exists, and is continuous on its supports [48]. Apart from the above restrictions the notation and all assumptions as detailed in Subsection 2.1 apply verbatim. Again, Rayleigh fading is assumed whenever explicit results are obtained, and an interference factor of $\alpha = \frac{1}{2}$ is used to produce numerical results.

Starting with the linear receivers, their outage constrained capacity, for a given FUU, is again expressed in terms of their *multiuser efficiency*, as is the case with the spectral efficiency. The derivation of the outage constrained capacity of the SCO and SCPO receiver is more involved (see Propositions 3.6 and 3.7 below), and the details are presented in Subsection 3.3. The reader is also referred to Appendix B for a short summary of some basic order statistics results that are used in the derivations. When a linear processing strategy is employed, the kth strongest intra-cell user is decodable, if the (equal) transmission rate per user (bits/sec/Hz) satisfies:

$$\tilde{R} \leq \frac{1}{N} \log\left(1 + \nu_{i_k} \eta \bar{P}\right) \ , \tag{3-2}$$

where i_k denotes the index of the kth strongest intra-cell user at the receiver (i.e., $\nu_{i_1} \geq \nu_{i_2} \geq \cdots \geq \nu_{i_K}$), and η is the multiuser efficiency of the receiver (as before). Hence, the outage constrained capacity for a given FUU of $\mathcal{Q}$ equals[3] (recall that J denotes the number of reliably decoded users per cell)

$$\begin{aligned} \tilde{R}_T(\mathcal{Q}) &= \lim_{\substack{K,N,J\to\infty,\\ \frac{K}{N}\to\beta,\frac{J}{K}\to 1-\mathcal{Q}}} \frac{1}{N} J \log\left(1 + \nu_{i_J} \eta \bar{P}\right) \\ &= \beta(1-\mathcal{Q}) \log\left(1 + \mathcal{F}_\nu^{-1}(\mathcal{Q}) \eta \bar{P}\right) \underset{\text{Rayleigh}}{=} \beta(1-\mathcal{Q}) \log\left(1 - \ln(1-\mathcal{Q}) \eta \bar{P}\right) \ . \end{aligned} \tag{3-3}$$

As in the spectral efficiency analysis of Section 2, different multiuser receiver strategies are compared while investigating the outage constrained capacity as a function of the system average received $\frac{E_b}{N_0}$. However, for simplicity of natation the results are presented again in terms of the SNR, through the relation

$$\bar{P} = \frac{1}{\beta} \tilde{R}_T \frac{E_b}{N_0} \ . \tag{3-4}$$

With the above relation in mind, focusing on the low-SNR regime, the following important property of the strongest-users decoding scheme is derived.

Proposition 3.1 *The minimum system average received $\frac{E_b}{N_0}$ that enables reliable communications*

[3]For Rayleigh fading $\mathcal{F}_\nu(y) = 1 - e^{-y}$, $\mathcal{F}_\nu^{-1}(x) = -\ln(1-x)$, and $d\mathcal{F}_\nu(y) = e^{-y}\, dy$.

under the strongest-users decoding scheme, for a given FUU of $\mathcal{Q}$, equals

$$\frac{E_b}{N_0}_{min} = \frac{\ln 2}{(1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})} \underset{Rayleigh}{=} -\frac{\ln 2}{(1-\mathcal{Q})\ln(1-\mathcal{Q})} . \tag{3-5}$$

Proof: See Appendix E. ■

Proposition 3.1 indicates that the strongest-users decoding scheme induces a penalty in the minimum received $\frac{E_b}{N_0}$ that enables reliable communications. Comparing (3-5) to $\frac{E_b}{N_0}_{\min} = \ln 2 \approx -1.59$dB, the corresponding value when *all* users are decoded [17], the penalty is observed to be by a factor of $\left[(1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})\right]^{-1}$. This penalty is explained by the power "wasted" when users fail to be decoded. Focusing on Rayleigh fading, and further minimizing (3-5) with respect to Q, it follows that $\frac{E_b}{N_0}_{\min} = e\ln 2 \approx 2.75$dB, exhibiting a *severe* penalty of $10\log_{10} e \approx 4.34$dB. This minimum value is achieved for $\mathcal{Q}^\star = 1 - 1/e$, being hence the *optimum FUU* in the low-SNR regime, for Rayleigh fading channels.

It is also interesting to note that the same degradation has been identified in the broadcast strategy of communicating over (slowly) fading channels with no rate feedback to the transmitter [59] (see also [50]). According to the latter strategy, the transmitter views the fading channel as a degraded Gaussian broadcast channel with a continuum of receivers, each experiencing a different signal to noise ratio according to the possible values of the fade level (assumed continuous as well). The transmitter is then envisaged as transmitting in parallel infinitely many information streams, with rates corresponding to the possible fade levels. Each of the above streams can be reliably decoded by receivers corresponding to equal, or lower, fade levels. The analogy to the setting considered in this report is straightforwardly observed.

Comparative outage constrained capacity results in Rayleigh fading channels, for the four multiuser receivers, are presented in Fig. 6. The results are discussed in the following subsections. The outage constrained capacities of all receivers were evaluated for the (numerically obtained unless otherwise stated) optimum values of both the cell load β and $\mathcal{Q}$ (and for $\alpha = \frac{1}{2}$). Both optimum parameters are in general functions of $\frac{E_b}{N_0}$. For the sake of comparison, the *spectral efficiencies* of the four receivers are also included in Fig. 6 (designated by the dotted curves). Fig. 7 shows the corresponding optimum FUU as a function of $\frac{E_b}{N_0}$.

3.1 The Matched-Filter Receiver

Proposition 3.2 *The asymptotic outage constrained capacity of the matched-filter receiver for a target FUU of $\mathcal{Q}$ is given by:*

$$\begin{aligned}\tilde{R}_{T\,mf}(\mathcal{Q}) &= \beta(1-\mathcal{Q})\log\left(1+\mathcal{F}_\nu^{-1}(\mathcal{Q})\frac{\bar{P}}{1+\beta(1+2\alpha^2)\bar{P}}\right)\\ &\underset{Rayleigh}{=} \beta(1-\mathcal{Q})\log\left(1-\ln(1-\mathcal{Q})\frac{\bar{P}}{1+\beta(1+2\alpha^2)\bar{P}}\right) .\end{aligned} \tag{3-6}$$

Proof: The result is trivially obtained by substituting the expression for the multiuser efficiency of the matched-filter receiver, as given by (2-12), into (3-3). ■

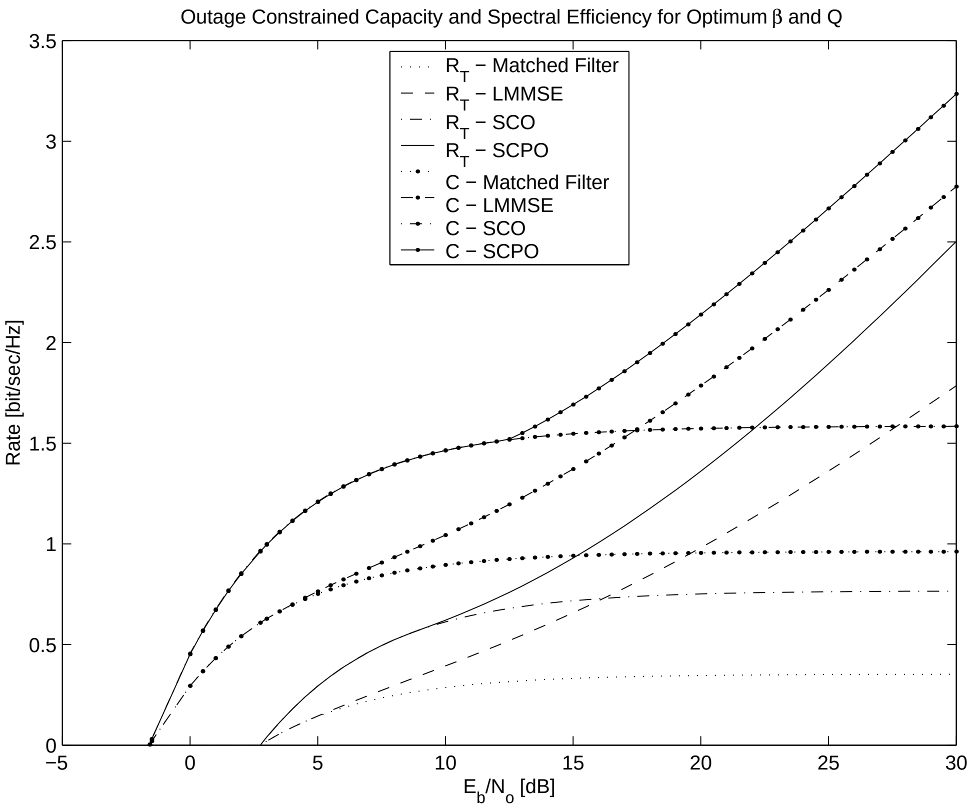

Figure 6: Spectral efficiency and outage constrained capacity for optimum choice of β and FUU, and for $\alpha = \frac{1}{2}$ (Rayleigh fading).

Examining (3-6), it is observed that the outage constrained capacity of the matched-filter receiver monotonically increases with the cell load β. The limiting outage constrained capacity is given by

$$\begin{aligned}\lim_{\beta\to\infty} \tilde{R}_{T_{\mathrm{mf}}}(\mathcal{Q}) &= \frac{1}{1+2\alpha^2}\left[(1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})\log e - \left(\frac{E_b}{N_0}\right)^{-1}\right] \\ &\underset{\text{Rayleigh}}{=} \frac{1}{1+2\alpha^2}\left[-(1-\mathcal{Q})\log(1-\mathcal{Q}) - \left(\frac{E_b}{N_0}\right)^{-1}\right] \ ,\end{aligned} \tag{3-7}$$

which coincides with the corresponding result in the non-spreading regime [48] (using the equivalence of β and K in the latter case), as observed for the spectral efficiency in Subsection 2.2.1. The optimum FUU for *all* $\frac{E_b}{N_0}$ values, in Rayleigh fading channels, is hence $\mathcal{Q}^\star = 1 - 1/e$ for which

$$\lim_{\beta\to\infty} \tilde{R}_{T_{\mathrm{mf}}}(\mathcal{Q})|_{\mathcal{Q}^\star} = \frac{1}{1+2\alpha^2}\left[\frac{\log e}{e} - \left(\frac{E_b}{N_0}\right)^{-1}\right] \ . \tag{3-8}$$

The interference limited behavior of the receiver is observed again, and the outage constrained capacity reaches the limit of $\frac{\log e}{e(1+2\alpha^2)}$ bits/sec/Hz as $\frac{E_b}{N_0} \to \infty$ (0.3538 bits/sec/Hz for $\alpha = \frac{1}{2}$). Recall that the corresponding *spectral efficiency* of the receiver reaches the limit of $\frac{\log e}{1+2\alpha^2}$ (0.9618

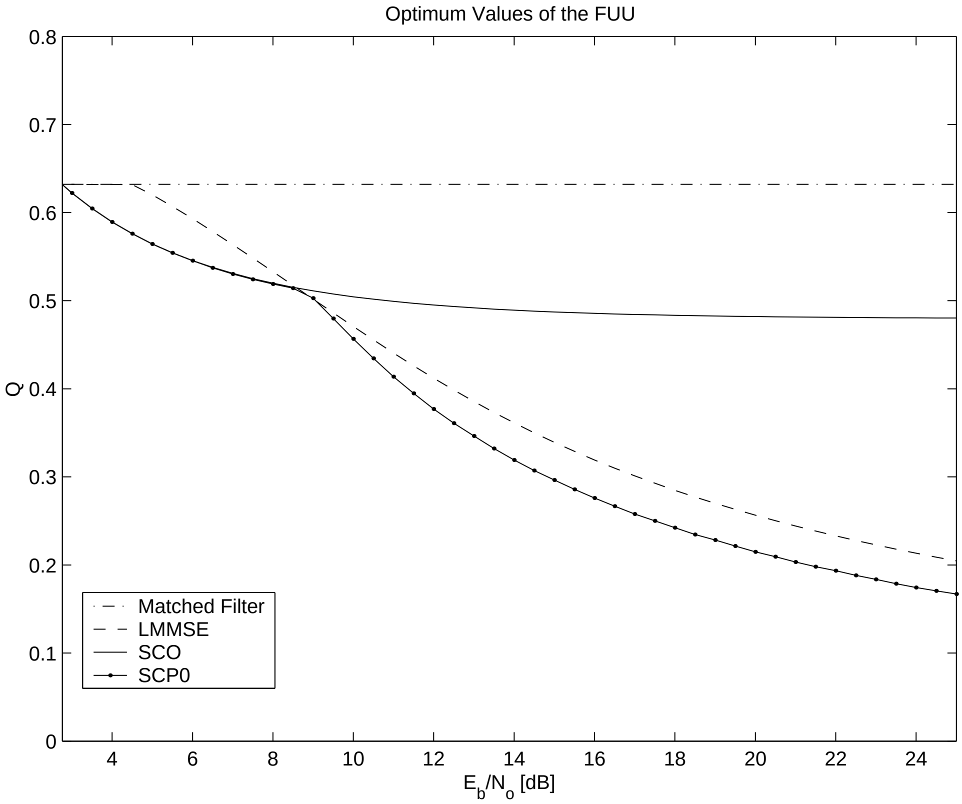

Figure 7: Optimum FUU for $\alpha = \frac{1}{2}$ (Rayleigh fading).

bits/sec/Hz for $\alpha = \frac{1}{2}$), and the strongest-users decoding is thus observed to degrade the limiting system performance as $\frac{E_b}{N_0} \to \infty$ by a factor of $e \approx 2.7183$. A plot of the outage constrained capacity of the matched filter receiver for $\beta \to \infty$, $\mathcal{Q} = \mathcal{Q}^\star$, and $\alpha = \frac{1}{2}$ is shown in Fig. 6.

3.2 The Linear MMSE receiver

Proposition 3.3 *The asymptotic outage constrained capadcity of the linear MMSE receiver equals*

$$\begin{aligned} \tilde{R}_{T\,ms}(\mathcal{Q}) &= \beta(1-\mathcal{Q})\log\left(1+\mathcal{F}_\nu^{-1}(\mathcal{Q})\eta_{ms}\bar{P}\right) \\ &\underset{Rayleigh}{=} \beta(1-\mathcal{Q})\log\left(1-\ln(1-\mathcal{Q})\eta_{ms}\bar{P}\right) \ , \end{aligned} \tag{3-9}$$

where η_{ms} is the unique positive solution to the implicit equation (2-22).

Proof: The result is immediately obtained by substituting the multiuser efficiency of the linear MMSE receiver (given by the unique solution to (2-22)) into (3-3). ■

As observed while analyzing the spectral efficiency of the receiver (and in contrast to the matched-filter receiver, and the SCO receiver discussed in the following), the linear MMSE receiver is not interference limited also in terms of outage constrained capacity, provided that the cell load

β is appropriately chosen. Analysis of the result of Proposition 3.3 shows a similar behavior to that observed for the spectral efficiency of the receiver. For low $\frac{E_b}{N_0}$ it is again optimum to increase the cell load β without bound, and the outage constrained capacity of the linear MMSE receiver coincides, with that of the matched-filter receiver (an equivalence that holds for $\beta \to \infty$ regardless of $\frac{E_b}{N_0}$). However, as observed for the spectral efficiency, beyond some critical $\frac{E_b}{N_0}$ the optimum choice for the cell load β starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$ in the high $\frac{E_b}{N_0}$ region. The optimized outage constrained capacity of the receiver grows without bound with $\frac{E_b}{N_0}$, however a clear performance degradation is observed in comparison to the spectral efficiency, as demonstrated in Fig. 6. The critical $\frac{E_b}{N_0}$ beyond which the optimum cell load in terms of outage constrained capacity takes on finite values, can be analytically expressed, and the result is summarized in the following proposition.

Proposition 3.4 *Suppose that $E_{\mathcal{F}_\nu}\left\{\nu^2\right\} < \infty$. Then, for the linear MMSE receiver, the optimum value in terms of the outage constrained capacity of the cell load β is finite if and only if*

$$\begin{aligned} \frac{E_b}{N_0} > \frac{E_b}{N_0}_{cr}^{*} &= \frac{2\kappa_\nu E_{\mathcal{F}_\nu}\{\nu\}(1+2\alpha^4)}{2\kappa_\nu E_{\mathcal{F}_\nu}\{\nu\}(1+2\alpha^4) - (1+2\alpha^2)\mathcal{F}_\nu^{-1}(\mathcal{Q})} \frac{E_b}{N_0}_{min} \\ &\underset{\text{Rayleigh}}{=} \frac{4(1+2\alpha^4)}{4(1+2\alpha^4)+(1+2\alpha^2)\ln(1-\mathcal{Q})} \frac{E_b}{N_0}_{min}, \end{aligned} \tag{3-10}$$

where $\frac{E_b}{N_0}_{min}$ is given by (3-5)

Proof: See Appendix C. ■

As can be observed, unlike the critical $\frac{E_b}{N_0}$ given in Proposition 2.7, the critical $\frac{E_b}{N_0}$ with respect to the outage constrained capacity of the linear MMSE receiver *depends* on the fading distribution. The optimum FUU monotonically decreases with $\frac{E_b}{N_0}$, beyond $\frac{E_b}{N_0}_{\text{cr}}^{*}$ defined in Proposition 3.4 (below which it equals $1 - 1/e$ for Rayleigh fading channels, and the outage constrained capacity coincides with that of the matched-filter receiver), as observed in Fig. 7. For Rayleigh fading, $\mathcal{Q} = \mathcal{Q}^\star = 1 - \frac{1}{e}$, and $\alpha = \frac{1}{2}$, it follows from (3-10) that $\frac{E_b}{N_0}_{\text{cr}}^{*} = 1.5e\ln 2 \approx 4.51\text{dB}$.

3.3 "Optimum" Receivers

Before proceeding with the presentation of the outage constrained capacity results of the SCO and the SCPO receivers, it is useful to focus for a while on the particular case in which $\alpha = 0$ (no out-of-cell interference), equivalent to the *single-cell* setting [17]. In this particular case the SCO receiver and the SCPO receiver are identical (as they differ only in their treatment of out-of-cell interference), and both receivers are referred to here in the single-cell context as the "optimum" receiver (the quotation marks are introduced to designate this receiver from the truly optimum multiuser receiver, that is not restricted by the strongest-users decoding scheme). As shall be observed, the nature of the results obtained for the single-cell setup emphasizes the effect of out-of-cell interference on system performance, to be discussed later on.

The outage constrained capacity of the "optimum" receiver under the strongest-users decoding scheme has been derived in [49], and the result is reviewed in the following proposition.

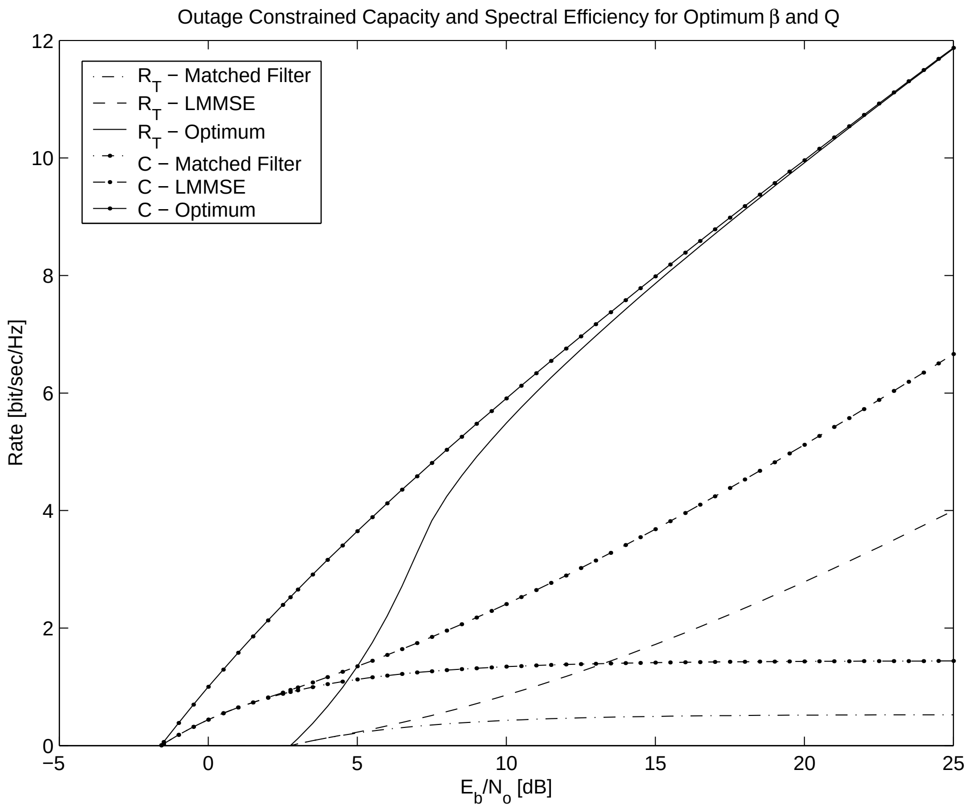

Figure 8: Spectral efficiency and outage constrained capacity of the multiuser receivers, for optimum choice of cell load β and FUU, and for $\alpha = 0$ (Rayleigh fading).

Proposition 3.5 (Shamai–Verdú [49]) *Let $\eta_{opt}(\delta\,;\bar{P})$ be the unique solution to the equation*

$$1-\beta(1-\delta)=\eta-\beta\Theta_{opt}(\delta\,;\bar{P}\eta)\ , \tag{3-11}$$

where

$$\begin{aligned}\Theta_{opt}(\omega\,;\varsigma) &\triangleq \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} \frac{1}{1+\varsigma\phi}\, d\mathcal{F}_\nu(\phi) \\ &\underset{\text{Rayleigh}}{=} \int_0^{-\ln\omega} \frac{1}{1+\varsigma\phi}\, e^{-\phi}\, d\phi = \frac{1}{\varsigma}\, e^{\frac{1}{\varsigma}} \left[\mathcal{E}_1\!\left(\frac{1}{\varsigma}\right) - \mathcal{E}_1\!\left(\frac{1}{\varsigma}-\ln\omega\right)\right] \ .\end{aligned} \tag{3-12}$$

Define

$$\begin{aligned}\Gamma_{opt}(w\,;\varsigma) &\triangleq \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} \log\left(1+\varsigma\phi\right)\, d\mathcal{F}_\nu(\phi) \underset{\text{Rayleigh}}{=} \int_0^{-\ln\omega} \log\left(1+\varsigma\phi\right) e^{-\phi}\, d\phi \\ &= -\omega\log(1-\varsigma\ln\omega) + e^{\frac{1}{\varsigma}} \left[\mathcal{E}_1\!\left(\frac{1}{\varsigma}\right) - \mathcal{E}_1\!\left(\frac{1}{\varsigma}-\ln\omega\right)\right] \log e\end{aligned} \tag{3-13}$$

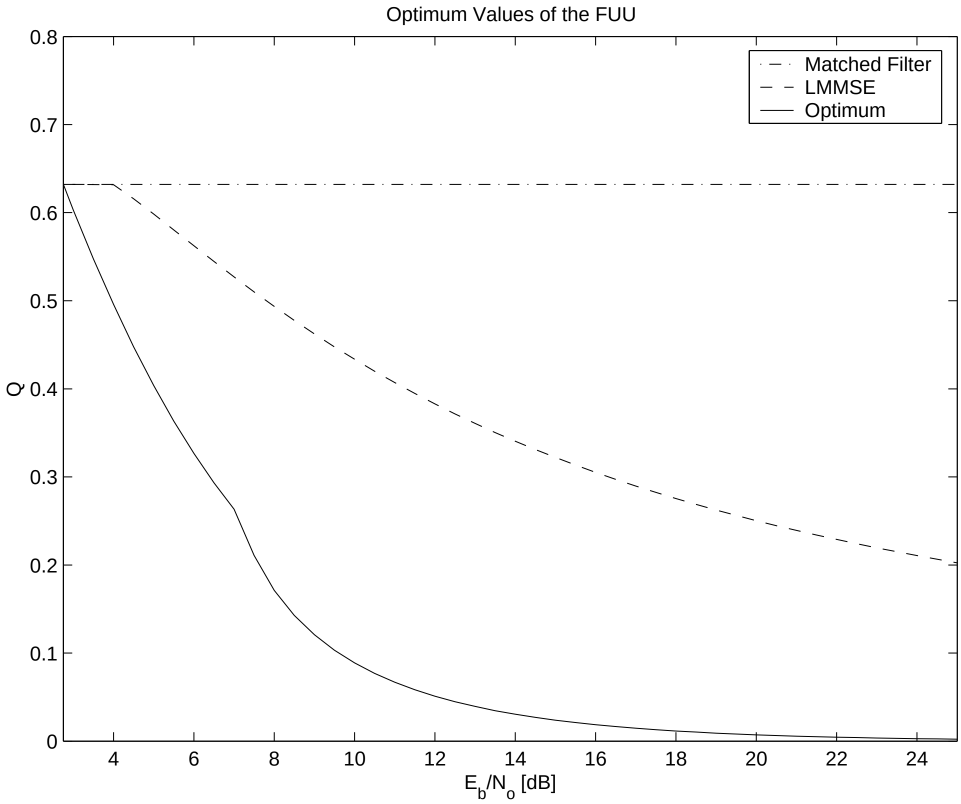

Figure 9: Optimum values of $\mathcal{Q}$ for the three multiuser receivers in a single-cell setting ($\alpha = 0$, Rayleigh fading).

and

$$\mathcal{C}_{opt}(\delta\,;\bar{P}) \triangleq \beta\Gamma_{opt}\left(\delta\,;\bar{P}\eta_{opt}(\delta\,;\bar{P})\right) - \log\eta_{opt}(\delta\,;\bar{P}) + (\eta_{opt}(\delta\,;\bar{P}) - 1)\log e\ . \tag{3-14}$$

Then, the outage constrained capacity of the "optimum" receiver, for a target FUU of $\mathcal{Q}$, is given by

$$\tilde{R}_{T\,opt}(\mathcal{Q}) = \inf_{0\leq x<1} \frac{\mathcal{C}_{opt}\left[(1-\mathcal{Q})(1-x)\,;\bar{P}\right] - \mathcal{C}_{opt}\left[(1-\mathcal{Q})\,;\bar{P}\right]}{x}\ . \tag{3-15}$$

Proof: See [49] or [60]. ∎

The outage constrained capacity of the "optimum" receiver, optimized with respect to both the cell load β and the FUU, is plotted in Fig. 8 together with the corresponding results of the matched filter and linear MMSE receivers. The optimum FUU of the receivers in this setting in plotted in Fig. 9. As can be seen, the "optimum" receiver is not interference limited, provided that *both* the cell load β and the FUU are appropriately chosen. Analysis of the result of Proposition 3.5 shows that it is optimum in terms of the outage constrained capacity to take $\beta \to \infty$ for all $\frac{E_b}{N_0}$ values. Evaluating the limit of (3-15) as $\beta \to \infty$, it is observed that the outage constrained

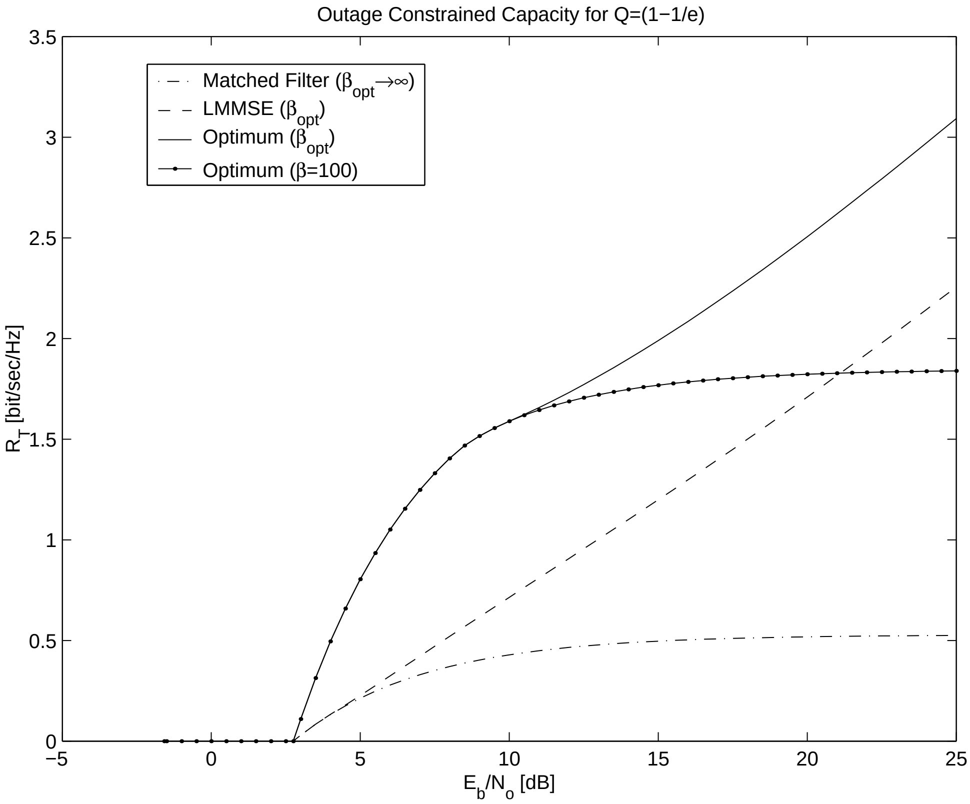

Figure 10: Outage constrained capacity of the multiuser receivers for $\mathcal{Q} = 1 - 1/e$ and $\alpha = 0$ (Rayleigh fading).

capacity of the "optimum" receiver, for Rayleigh fading channels, satisfies the following equation:

$$\lim_{\beta\to\infty} \tilde{R}_{T_{\rm opt}}(\mathcal{Q}) = \inf_{x\in[0,1)} \frac{1}{x} \log\left[1 + \frac{\beta\bar{P}(1-\mathcal{Q})\left[x\left[1-\ln(1-\mathcal{Q})\right] + (1-x)\ln(1-x)\right]}{1+\beta\bar{P}\left[\mathcal{Q}+(1-\mathcal{Q})\ln(1-\mathcal{Q})\right]}\right] , \tag{3-16}$$

which coincides with the analogous outage constrained capacity when no spreading is employed [48]. Considering this limiting regime, the outage constrained capacity, while optimized with respect to the FUU, approaches the *optimum spectral efficiency* (coinciding in this case with the *single user* AWGN channel capacity) at the high $\frac{E_b}{N_0}$ region, as observed in Fig. 8. Letting $\mathcal{Q}$ vanish as $\sqrt{\frac{2\epsilon}{\beta\bar{P}}}$ for some $\epsilon > 0$, the outage constrained capacity satisfies, as shown in [48],

$$\lim_{\beta\bar{P}\to\infty} \tilde{R}_{T_{\rm opt}}(\mathcal{Q}) = \log\left(1 + \frac{\beta\bar{P}}{1+\epsilon}\right) . \tag{3-17}$$

Comparing the outage constrained capacity of the "optimum" receiver to the optimum spectral efficiency at the *low* $\frac{E_b}{N_0}$ region, the strongest-users decoding scheme is observed to induce a severe penalty in system performance, emphasizing the crucial role of *rate*-adjustment feedback in this regime.

It is interesting to note at this point, that taking $\beta \to \infty$ (in fact if it exceeds a threshold) is *no longer the optimum choice* for the "optimum" receiver in the case in which, due to some practical implementation considerations, the minimum FUU cannot go below some fixed positive value. As can be observed from (3-16), the "optimum" receiver becomes in fact *interference limited* for any *fixed* $\mathcal{Q}$. It can be therefore concluded, that in such case incorporating some level of spreading is *beneficial* in terms of system performance in the high $\frac{E_b}{N_0}$ region, and not all bandwidth should be made available for coding. This somewhat surprising behavior is in sheer contrast to that of the optimum spectral efficiency, for which no spreading is optimum, and it emanates from the inherent suboptimality of the equal-rate equal-transmit-power strongest-users decoding scheme. This phenomena is clearly demonstrated in Fig. 10 where the outage constrained capacities, for Rayleigh fading channels, are plotted for the optimum cell load β and with the FUU fixed to $\mathcal{Q} = 1 - 1/e$, the optimum value in the low $\frac{E_b}{N_0}$ regime (and in fact the optimum for all $\frac{E_b}{N_0}$ values for the matched-filter receiver). The high β behavior of the "optimum receiver" is demonstrated here by taking $\beta = 100$. The corresponding results for $\mathcal{Q} = 0.1$ are plotted in Fig. 11 on the following page. As can be observed, the performance of both the linear MMSE receiver and the "optimum receiver" is considerably enhanced in the latter example (as compared to Fig. 10), at the high $\frac{E_b}{N_0}$ region. In contrast, the performance of the matched-filter receiver is severely degraded for all $\frac{E_b}{N_0}$ values as compared to the optimum choice of $\mathcal{Q}$. Fig. 12 shows the optimum cell load β for the "optimum receiver" as a function of $\frac{E_b}{N_0}$, for $\mathcal{Q} = 0.1$. At low $\frac{E_b}{N_0}$ values it is optimum to take $\beta \to \infty$ (the maximum value in Fig. 12 was truncated at $\beta = 10$). However, beyond some critical value the optimum cell load is observed to be monotonically decreasing with $\frac{E_b}{N_0}$, reaching a value of $\beta \approx 1.78$ at $\frac{E_b}{N_0} = 30$dB.

We now turn to the general case in which $\alpha > 0$. Starting with the SCO receiver, the following result has been derived.

Proposition 3.6 *The asymptotic outage constrained capacity of the SCO receiver, for a target FUU of $\mathcal{Q}$, is obtained by substituting $\bar{P}_{eq}(\bar{P})$ instead of $\bar{P}$ into the result of Proposition 3.5, where $\bar{P}_{eq}(\bar{P})$ is given by* (2-16)*.*

Proof: The proposition can be proved by applying verbatim the observation used in the proof of Proposition 2.3 to the relation between the outage constraint capacity of the "optimum" receiver (under the strongest-users decoding scheme) in a *single-cell* setup, and that of the SCO receiver in the multi-cell setup considered here. That is, the outage constrained capacity of the SCO is obtained by substituting $\bar{P}_{\text{eq}}(\bar{P})$ instead of $\bar{P}$ in the corresponding outage constrained capacity expression for the "optimum" receiver in the single-cell setup. ■

Due to the treatment of out-of-cell interference as AWGN, the SCO receiver is interference limited, as observed in the spectral efficiency analysis of Subsection 2.2.2 (note again that $\lim_{E_b/N_0 \to \infty} \bar{P}_{\text{eq}} = \frac{1}{2\beta\alpha^2}$). The relation between the outage constrained capacity expressions, with and without the presence of out-of-cell interference, as stated by Proposition 3.6, dictates that while optimizing with respect to *both* β and $\mathcal{Q}$, it is again optimum in terms of the outage constrained capacity (for the SCO receiver) to take $\beta \to \infty$ for all $\frac{E_b}{N_0}$ values. The optimum $\mathcal{Q}$ monotonically decreases with

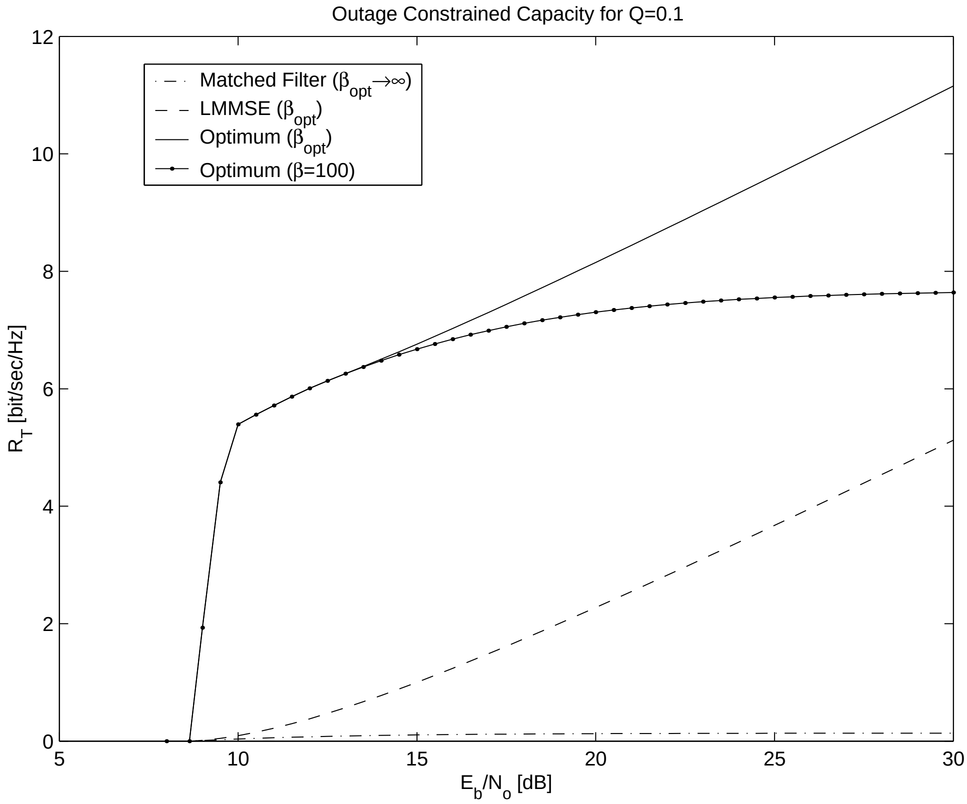

Figure 11: Outage constrained capacity of the multiuser receivers for $\mathcal{Q} = 0.1$ and $\alpha = 0$ (Rayleigh fading).

$\frac{E_b}{N_0}$ as can be observed in Fig. 7, eventually reaching for Rayleigh fading a limit of $\mathcal{Q}^* = 0.4795$, for $\alpha = \frac{1}{2}$, as $\frac{E_b}{N_0} \to \infty$ (this limiting optimum FUU is derived while investigating the derivative with respect to $\mathcal{Q}$ of the limiting outage constrained capacity as $\beta \to \infty$, and $E_b/N_0 \to \infty$). However, fixing the FUU, the optimum choice of cell load β (as a function of $\frac{E_b}{N_0}$) depends on the value of the adjacent-cell interference factor α, and may take on finite values in the high $\frac{E_b}{N_0}$ region (decreasing from infinity beyond some critical $\frac{E_b}{N_0}$), provided that the factor α is low enough. This behavior is in agreement with the behavior of the receiver in a *single-cell* setup discussed above. For the particular case of $\alpha = \frac{1}{2}$, however, the results show that taking $\beta \to \infty$ is optimum for *all* $\frac{E_b}{N_0}$ values.

Finally, the outage constrained capacity of the SCPO receiver is given by the following theorem.

Proposition 3.7 *Let* $\eta_{_{scpo}}(\delta\,;\bar{P})$ *be the unique solution to the equation*

$$1 - \beta(3-\delta) = \eta - \beta\Theta_{_{scpo}}(\delta\,;\bar{P}\eta) \ , \tag{3-18}$$

where

$$\Theta_{_{scpo}}(\omega\,;\varsigma) = \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} 1/(1+\varsigma\phi)d\mathcal{F}_\nu(\phi) + 2E_{\mathcal{F}_\nu}\left\{1/(1+\alpha^2\varsigma\nu)\right\} \ . \tag{3-19}$$

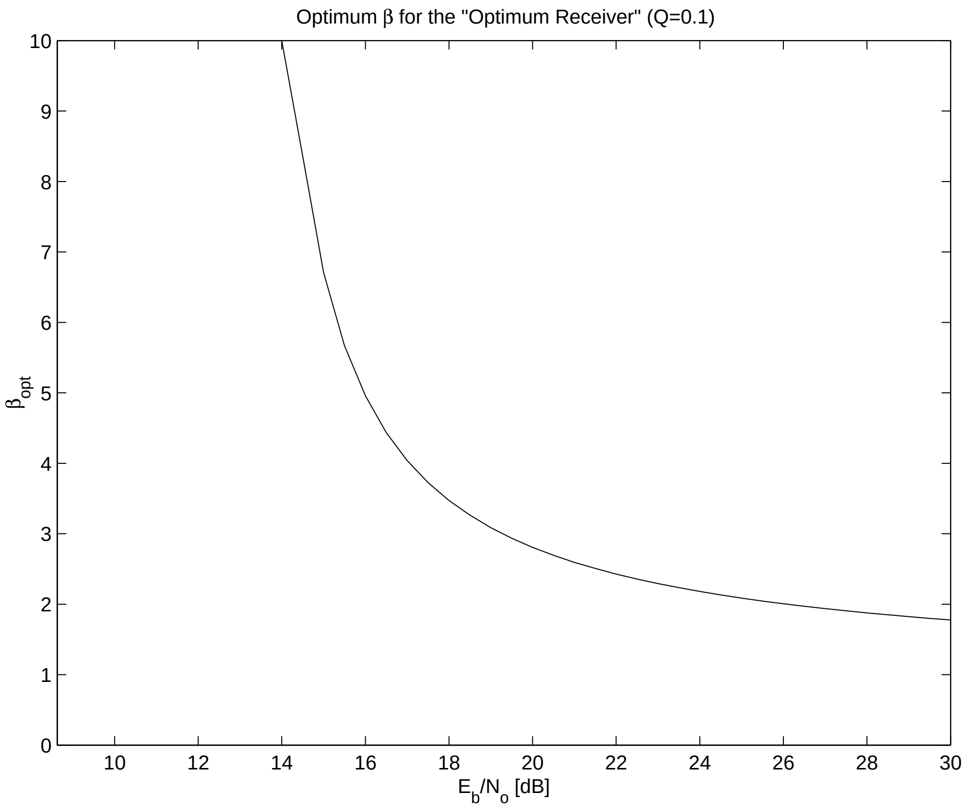

Figure 12: The optimum cell load β for the "optimum receiver" for $\mathcal{Q} = 0.1$ (Rayleigh fading). The maximum value is truncated at $\beta = 10$.

Define

$$\Gamma_{scpo}(w\,;\varsigma) = \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} \log(1+\varsigma\phi)d\mathcal{F}_\nu(\phi) + 2E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\varsigma\nu\right)\right\} \ , \tag{3-20}$$

and

$$\mathcal{C}_{scpo}(\delta\,;\bar{P}) = \beta\Gamma_{scpo}\left(\delta\,;\bar{P}\eta_{scpo}(\delta\,;\bar{P})\right) - \log\eta_{scpo}(\delta\,;\bar{P}) + (\eta_{scpo}(\delta\,;\bar{P})-1)\log e \ . \tag{3-21}$$

Then, the asymptotic outage constrained capacity of the SCPO receiver, for a target FUU of $\mathcal{Q}$, equals

$$\tilde{R}_{scpo}(\mathcal{Q}) = \inf_{0\le x<1} \frac{\mathcal{C}_{scpo}\left[(1-\mathcal{Q})(1-x)\,;\bar{P}\right] - \mathcal{C}_{scpo}\left[(1-\mathcal{Q})\,;\bar{P}\right]}{x} \ . \tag{3-22}$$

Proof: See Appendix D, where explicit expressions for Rayleigh fading channels are also presented. ∎

The SCPO receiver is not interference limited, which should have been expected, provided that *both* the cell load β and the FUU are appropriately chosen, as can be observed from Fig. 6. With this receiver, unlike the single-cell setup discussed above, it is no longer optimum in terms of the outage constrained capacity to take $\beta \to \infty$ for *all* $\frac{E_b}{N_0}$ values. Although for low $\frac{E_b}{N_0}$ values it is still optimum to take $\beta \to \infty$, beyond some critical $\frac{E_b}{N_0}$ the optimum cell load starts to decrease monotonically with $\frac{E_b}{N_0}$, while the outage constrained capacity grows without bound. With $\beta \to \infty$ the outage constrained capacity of the SCPO receiver coincides with that of the SCO receiver, and the receiver becomes *interference limited*, regardless of the FUU. Also, as can be observed from Fig. 6, when undecodable out-of-cell interference is introduced, a clear performance degradation is evident as compared to the ultimate spectral efficiency of the receiver. This behavior is in sheer contrast to the asymptotic approach (with $\frac{E_b}{N_0}$) of the optimized outage constrained capacity to the optimum spectral efficiency, in the single-cell setting. This contrast emphasizes the role of rate-adjustment feedback in the presence of undecodable out-of-cell interference.

Examining the optimum FUU of the receiver, as plotted in Fig. 7, it is observed that in the low $\frac{E_b}{N_0}$ region where it is optimum to take $\beta \to \infty$, the optimum FUU of the SCPO receiver coincides with that of the SCO receiver. However, beyond a critical $\frac{E_b}{N_0}$, which corresponds to the point beyond which the optimum cell load decreases to finite values, the optimum FUU of the SCPO receiver starts to decrease more rapidly with $\frac{E_b}{N_0}$ as compared to that of the SCO receiver, which goes to a limit.

4 Summary and Conclusions

This paper studies the performance of four multiuser receiver strategies in a simple multi-cell cellular model with flat fading channels. Focusing first on the ultimate performance, specified by the spectral efficiency of the receivers, the analysis demonstrates the dramatic effect of information about interfering signals on system performance. The effect is most clearly seen by comparing the linear MMSE receiver and the SCO receiver, that represent a tradeoff between the processing complexity of intra-cell transmissions, and having additional information at the receiver on adjacent-cell interference. It was shown that one can gain even without trying to decode the transmissions of the interfering users in adjacent cells (which enables interference *cancellation*), or treating them optimally in the setting of an interference channel (see [41]). The gain emerges by the very fact that the linear MMSE filter accounts for the reduction of interference, provided that the signatures of interfering users are known not only at the intended cell-site, but at those cell-sites where they cause interference. It may be concluded that for high data rates, inherently demanding high $\frac{E_b}{N_0}$, it is advantageous to mitigate out-of-cell interference through linear MMSE processing.

Assuming *equal* transmit powers, it was shown that the matched-filter and SCO receivers are asymptotically unaffected by the presence of fading, in the large (optimum) cell load region. In contrast, the linear MMSE receiver and the SCPO receiver experience performance degradation, when fading is present, at the high $\frac{E_b}{N_0}$ region where the optimum cell load β is lower than $\frac{1}{3}$. However, when *fixing* $\beta > \frac{1}{3}$, both receivers *benefit* from the presence of fading due to its "population control" effect, and attain a higher spectral efficiency, as compared to the non-fading case, beyond a critical $\frac{E_b}{N_0}$ value.

In the second part of this paper, a practically appealing transmission and decoding strategy that assumes no rate-adjustment feedback from the receiver has been considered. Accordingly, all users are assumed to employ equal and constant transmit powers and rates, the receiver ranks all intra-cell users according to their *received* powers, and decodes only a subset of the strongest intra-cell users. The fraction of undecodable users (FUU) is assumed to be a system design parameter.

Considering the outage constrained capacity of the corresponding four multiuser receivers, as analyzed in terms of spectral efficiency, the strongest-users decoding scheme was shown to induce a severe penalty in the minimum receive $\frac{E_b}{N_0}$ allowing reliable communications. For Rayleigh fading the penalty is of at least $10\log_{10} e \approx 4.34$dB. The above minimum penalty is attained by setting the FUU to $\mathcal{Q}^\star = 1 - 1/e$, the optimum choice at the low-SNR regime.

Both the matched filter and the linear MMSE receivers suffer a significant performance degradation for all $\frac{E_b}{N_0}$ values, as compared to their spectral efficiency, regardless of whether out-of-cell interference is present (representing a multi-cell system) or not (representing a single-cell system). In contrast, with no out-of-cell interference the outage constrained capacity of the "optimum" receiver (equivalently either the SCO or SCPO receiver) is shown to asymptotically approach the *optimum spectral efficiency* in the high $\frac{E_b}{N_0}$ region, when the cell load is made large ($\beta \to \infty$), and when the FUU vanishes at an appropriate rate as $\frac{E_b}{N_0} \to \infty$. This asymptotic approach emphasizes the crucial role of *rate-adjustment feedback* at the *low*-SNR regime, where the strongest-users decoding scheme induces a *severe degradation* in system performance. An interesting observation is that when the FUU is *fixed*, the "optimum" receiver becomes *interference limited* in the large cell load region. In such case, the use of some amount of spreading ($\beta < \infty$) turns out beneficial beyond some critical $\frac{E_b}{N_0}$, and the outage constrained capacity of the receiver grows without bound with $\frac{E_b}{N_0}$ for an appropriately chosen cell load. This behavior is due to the inherent suboptimality of the strongest-users decoding scheme. The asymptotic approach to the ultimate performance is no longer observed when out-of-cell interference is introduced, and the SCO receiver becomes in fact interference limited regardless of the FUU. The contrast between the behavior of the SCO and SCPO receivers in the single- and multi-cell setups emphasizes the importance of rate-adjustment feedback in the presence of undecodable out-of-cell interference.

Finally, it is noted that both analyses apply verbatim (rescaling rates by N) to a multiple users multi-receive antennas system, operating with different received powers (say, due to shadowing or power-control).

A Proof of Proposition 2.7

The proof of Proposition 2.7 follows a similar path to that of the proof of Theorem III.4 in [17]. However, before proceeding with the proof, it useful to introduce the notion of average *received* SNR which is defined as

$$\bar{P}_{\text{rec}} = E_{\mathcal{F}_\nu}\left\{\nu\right\}\bar{P} \ . \tag{A-1}$$

Although a normalized fading distribution ($E_{\mathcal{F}_\nu}\{\nu\} = 1$) was assumed throughout this paper, implying equality of the average *transmit* and *receive* SNRs, this restriction is lifted for the purpose of proving Proposition 2.7 in order to produce a more fundamental result.

The underlying idea in the proof is that one can tell whether the optimum β is finite or not for a given (received) $\frac{E_b}{N_0}$, by examining the behavior of the spectral efficiency as $\beta \to \infty$, while $\bar{P}_{\text{rec}}$ is adjusted to comply with the given $\frac{E_b}{N_0}$. To that end, as in [17], we fix an arbitrary $d \triangleq \beta\bar{P}_{\text{rec}} \geq 0$. As discussed in Subsection 2.2.3, the spectral efficiency of the linear MMSE receiver coincides as $\beta \to \infty$ with the spectral efficiency of the matched filter receiver, and satisfies

$$\lim_{\beta\to\infty} \tilde{C}_{\text{ms}} = \frac{1}{1+2\alpha^2}\left[\log e - \left(\frac{E_b}{N_0}\right)^{-1}\right] . \tag{A-2}$$

Therefore, using the fact that

$$d = \beta\bar{P}_{\text{rec}} = \tilde{C}_{\text{ms}}\frac{E_b}{N_0} , \tag{A-3}$$

it is immediately observed that the constant d uniquely determines $\frac{E_b}{N_0}$, at the limit as $\beta \to \infty$, through the relation

$$d = \frac{1}{1+2\alpha^2}\left(\frac{E_b}{N_0}\log e - 1\right) . \tag{A-4}$$

The next step is to examine the derivative of the spectral efficiency, as given in (2-2), with respect to $\frac{1}{\beta}$, or equivalently, with respect to $\bar{P}_{\text{rec}}$, and find the value of d for which the derivative becomes 0 at $\bar{P}_{\text{rec}} = 0$. The rationale behind the above investigation is as follows. For a fixed $\frac{E_b}{N_0} > \frac{E_b}{N_0}_{\text{cr}}$ the spectral efficiency attains a maximum at some finite value of β, while for $\frac{E_b}{N_0} < \frac{E_b}{N_0}_{\text{cr}}$ the spectral efficiency monotonically grows with β. Therefore, relying on the continuity of the spectral efficiency with respect to β, investigating its derivative, as discussed above, reveals the critical $\frac{E_b}{N_0}$ at which the maximum is *"attained"* for $\frac{1}{\beta} = 0$, which implies $\beta \to \infty$. With that in mind, the aim is to find the value of d for which the following holds:

$$\frac{\partial}{\partial\bar{P}_{\text{rec}}}\left[\frac{d}{\bar{P}_{\text{rec}}}E_{\mathcal{F}_\nu}\left\{\log\left(1+\nu\eta(\bar{P}_{\text{rec}})\frac{\bar{P}_{\text{rec}}}{E_{\mathcal{F}_\nu}\{\nu\}}\right)\right\}\right]\Bigg|_{\bar{P}_{\text{rec}}=0} = 0 , \tag{A-5}$$

where the notation $\eta(\bar{P}_{\text{rec}})$ is used to emphasize the dependency of the multiuser efficiency in $\bar{P}_{\text{rec}}$. Equation (A-5) is obtained by substituting $\beta = \frac{d}{\bar{P}_{\text{rec}}}$ and $\bar{P} = \frac{\bar{P}_{\text{rec}}}{E_{\mathcal{F}_\nu}\{\nu\}}$ in (2-2). Now following some algebra, it can be shown that for (A-5) to hold it is required that

$$\dot{\eta}(\bar{P}_{\text{rec}})|_{\bar{P}_{\text{rec}}=0} = \frac{1}{2}\kappa_\nu\eta^2(\bar{P}_{\text{rec}})|_{\bar{P}_{\text{rec}}=0} , \tag{A-6}$$

where $\dot{\eta}$ denotes the derivative of η with respect to $\bar{P}_{\text{rec}}$, and κ_ν denotes the kurtosis of the fading distribution [17] defined as

$$\kappa_\nu = \frac{E_{\mathcal{F}_\nu}\{\nu^2\}}{\left(E_{\mathcal{F}_\nu}\{\nu\}\right)^2} . \tag{A-7}$$

In order to proceed from here, it is necessary to investigate the behavior of $\eta(\bar{P}_{\text{rec}})$, as given in Theorem 2.5, for $\bar{P}_{\text{rec}} \to 0$. The result of this investigation is summarized in the following Lemma, which is straightforwardly derived by carefully applying Taylor expansions to the expressions in (2-22). The Lemma is the multi-cell analogy of [[17], Property 9].

Lemma A.1 *Fix $d = \beta \bar{P}_{rec} \geq 0$, then the behavior of the multiuser efficiency of the linear MMSE receiver for $\bar{P}_{rec} \to 0$ is*

$$\eta(\bar{P}_{rec}) = \frac{1}{1 + d(1 + 2\alpha^2)} + \frac{d\kappa_\nu(1 + 2\alpha^4)}{[1 + d(1 + 2\alpha^2)]^3} \bar{P}_{rec} + o(\bar{P}_{rec}) \ . \tag{A-8}$$

Using Lemma A.1 in (A-6), and solving for d, it follows after some algebra that

$$d = \frac{1}{1 - 2\alpha^2 + 4\alpha^4} \ . \tag{A-9}$$

Finally, substituting (A-9) in (A-4) yields (2-26). One can also observe that substituting $\alpha = 0$ into (2-26) yields the corresponding single-cell result of [17].

B Some Useful Order Statistics Results

Let i_k denote the index of the kth strongest user at the receiver, i.e.,

$$\nu_{i_1} \geq \nu_{i_2} \geq \cdots \geq \nu_{i_K} \ . \tag{B-1}$$

Using $\mathcal{F}_\nu$ to denote the cumulative probability distribution (cdf) of each of the i.i.d. fading levels $\{\nu_m\}$, the distribution of ν_{i_k} is given by (e.g., [51] pp. 388)

$$\mathcal{F}_{\nu_{i_k}}(x) = \int_0^x \frac{K!}{(K-k)!(k-1)!} \left(1 - \mathcal{F}_\nu(z)\right)^{k-1} \mathcal{F}_\nu(z)^{K-k} d\mathcal{F}_\nu(z) \ , \tag{B-2}$$

which for $K \to \infty$ yields [48]

$$\lim_{K\to\infty} \mathcal{F}_{\nu_{i_k}}(x) = \begin{cases} 0 & 1 - \mathcal{F}_\nu(x) > \frac{k-1}{K-1}, \\ 1 & 1 - \mathcal{F}_\nu(x) \leq \frac{k-1}{K-1}. \end{cases} \xrightarrow[K,k\to\infty \frac{k}{K}\to\delta\in(0,1]]{} \begin{cases} 0 & \mathcal{F}_\nu(x) < 1 - \delta, \\ 1 & \mathcal{F}_\nu(x) \geq 1 - \delta. \end{cases} \tag{B-3}$$

The asymptotic expectation of the kth strongest fading (power) level is given by [49]

$$\begin{aligned} E\{\nu_{i_k}\} &= \int_0^\infty \left(1 - \mathcal{F}_{\nu_{i_k}}(x)\right) dx \\ &\xrightarrow[K,k\to\infty,\ \frac{k}{K}\to\delta\in(0,1]]{} \int_0^\infty 1\{1 - \mathcal{F}_\nu(x) > \delta\}\ dx + o(1) \\ &= \mathcal{F}_\nu^{-1}(1-\delta) + o(1) \ , \end{aligned} \tag{B-4}$$

where $1\{\cdot\}$ designates the indicator function, and $\mathcal{F}_\nu^{-1}(\cdot)$ denotes the inverse function of the cdf $\mathcal{F}_\nu(x)$. Finally, for a continuous and differentiable function $g(x)$ it follows that [49]

$$\frac{1}{K}\sum_{k=k_L}^{k_H} g(\nu_{i_k}) \xrightarrow[\frac{k_L}{K}\to\omega_L,\frac{k_H}{K}\to\omega_H]{K,k_L,k_H\to\infty,} \int_{\omega_L}^{\omega_H} g\left(\mathcal{F}_\nu^{-1}(1-x)\right)\,dx = \int_{\mathcal{F}_\nu^{-1}(1-\omega_H)}^{\mathcal{F}_\nu^{-1}(1-\omega_L)} g(y)\,d\mathcal{F}_\nu(y)\ . \tag{B-5}$$

C Proof of Proposition 3.4

The proof of Proposition 3.4 follows the same lines as the proof of Proposition 2.7, and will therefore be presented in short. Again we fix an arbitrary $d \triangleq \beta\bar{P}_{\text{rec}} \geq 0$. As discussed in Section 3.2, the outage constrained capacity of the linear MMSE receiver coincides as $\beta\to\infty$ with that of the matched filter receiver, and admits the following limiting exprssion

$$\lim_{\beta\to\infty}\tilde{R}_{T_{\text{ms}}}(\mathcal{Q}) = \frac{1}{1+2\alpha^2}\left[(1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})\log e - \left(\frac{E_b}{N_0}\right)^{-1}\right]\ . \tag{C-1}$$

Using the relation (see (3-4))

$$d = \beta\bar{P}_{\text{rec}} = \tilde{R}_{T_{\text{ms}}}(\mathcal{Q})\frac{E_b}{N_0}\ , \tag{C-2}$$

the constant d uniquely determines $\frac{E_b}{N_0}$ (fixing $\mathcal{Q}$), at the limit as $\beta\to\infty$, through the relation

$$d = \frac{1}{1+2\alpha^2}\left[(1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})\log e\frac{E_b}{N_0} - 1\right]\ . \tag{C-3}$$

The next step is to find the value of d for which the following holds:

$$\frac{\partial}{\partial\bar{P}_{\text{rec}}}\left[\frac{d}{\bar{P}_{\text{rec}}}(1-\mathcal{Q})\log\left(1+\mathcal{F}_\nu^{-1}(\mathcal{Q})\eta(\bar{P}_{\text{rec}})\frac{\bar{P}_{\text{rec}}}{E_{\mathcal{F}_\nu}\{\nu\}}\right)\right]\Bigg|_{\bar{P}_{\text{rec}}=0} = 0\ . \tag{C-4}$$

After some algebra (C-4) yields

$$\dot{\eta}(\bar{P}_{\text{rec}})\Big|_{\bar{P}_{\text{rec}}=0} = \frac{1}{2}\frac{\mathcal{F}_\nu^{-1}(\mathcal{Q})}{E_{\mathcal{F}_\nu}\{\nu\}}\eta^2(\bar{P}_{\text{rec}})\Big|_{\bar{P}_{\text{rec}}=0}\ , \tag{C-5}$$

from which the following expression for d is obtained using Lemma A.1

$$d = \frac{\mathcal{F}_\nu^{-1}(\mathcal{Q})}{2\kappa_\nu E_{\mathcal{F}_\nu}\{\nu\}(1+2\alpha^4) - \mathcal{F}_\nu^{-1}(\mathcal{Q})(1+2\alpha^2)}\ . \tag{C-6}$$

The proof of the proposition is completed by substituting (C-6) and (3-5) into in (C-3).

D Proof of Proposition 3.7

The following notation shall be used throughout this proof. Let the subset of indices of the J strongest intra-cell users be denoted by

$$\mathcal{D}_J = \{i_1, \ldots, i_J\} \ . \tag{D-1}$$

The set of channel inputs due to the users in a subset $\mathcal{S}$ is denoted by $\overline{\mathcal{X}}_{\mathcal{S}}$. Considering in particular the subset of J strongest intra-cell users, it follows that

$$\overline{\mathcal{X}}_{\mathcal{D}_J} = \{x_{i_1}, \ldots, x_{i_J}\} \ . \tag{D-2}$$

The set of channel inputs due to *all* received users *other* than those in the subset $\mathcal{S}$ is denoted by $\underline{\mathcal{X}}_S$. In particular, it follows that

$$\underline{\mathcal{X}}_{\mathcal{D}_J} = \left\{x_{i_{J+1}}, \ldots, x_{i_K}, \mathcal{X}^{(\pm)}\right\} \ , \tag{D-3}$$

where $\mathcal{X}^{(\pm)} = \left\{x_1^+, \ldots, x_K^+, x_1^-, \ldots, x_K^-\right\}$ (i.e., the set of all inputs due to other-cell users). The corresponding channel fading gain matrices (see (3-1), and note that with some abuse of notation the inter-cell interference factor α is incorporated here, for convenience, within the matrices), and signature matrices are denoted, respectively, by

$$\overline{\mathbf{H}}_{\mathcal{D}_J} = \mathrm{diag}\left(h_{i_1}, \ldots, h_{i_J}\right) \ , \tag{D-4}$$

$$\overline{\mathbf{S}}_{\mathcal{D}_J} = \left[\boldsymbol{s}_{i_1} \ \cdots \ \boldsymbol{s}_{i_J}\right] \ , \tag{D-5}$$

$$\underline{\mathbf{H}}_{\mathcal{D}_J} = \mathrm{diag}\left(h_{i_{J+1}}, \ldots, h_{i_K}, \alpha h_1^+, \ldots, \alpha h_K^+, \alpha h_1^-, \ldots, \alpha h_K^-\right) \ , \tag{D-6}$$

$$\underline{\mathbf{S}}_{\mathcal{D}_J} = \left[\boldsymbol{s}_{i_{J+1}} \ \cdots \ \boldsymbol{s}_{i_K} \ \boldsymbol{S}^+ \ \boldsymbol{S}^-\right] \ , \tag{D-7}$$

where $\boldsymbol{s}_{i_k}$ denotes the spreading sequence (signature) of the kth strongest intra-cell user. For consistency, the set of channel outputs is denoted by $\mathcal{Y}$, and the total set of channel inputs (due to both intra-cell and other-cell users) is denoted by $\mathcal{X}^{\mathrm{T}}$. Finally, the (ordered) channel fading gains and signature matrices corresponding to *all* channel inputs are denoted by

$$\widetilde{\mathbf{H}} = \mathrm{diag}\left(h_{i_1}, \ldots, h_{i_K}, \alpha h_1^+, \ldots, \alpha h_K^+, \alpha h_1^-, \ldots, \alpha h_K^-\right) \ , \tag{D-8}$$

$$\widetilde{\mathbf{S}} = \left[\boldsymbol{s}_{i_1} \ \cdots \ \boldsymbol{s}_{i_K} \ \boldsymbol{S}^+ \ \boldsymbol{S}^-\right] . \tag{D-9}$$

The first step in the proof is the derivation of the nonasymptotic capacity region. Towards this end, it is assumed that the spreading sequences are chosen randomly, and independently, *on a per symbol basis* (this setup differs for example from the case in which the signatures are randomly chosen *once* at the beginning of the transmission, and stay *unchanged* during the whole transmission period). The above assumption is introduced for technical reasons in order to simplify the derivations for the nonasysmptotic setup. It is emphasized however that this assumption has no effect on the final asymptotic result of Proposition 3.7, as both signature selection approaches yield the same limiting result as the system size becomes asymptotically large.

In principle, the fact that some users are decoded, whereas the code structure of some users is not exploited, could lead to analytical difficulties. However, it is possible to sidestep those difficulties by using Kolmogorov's identity. Conditioned on the channel fading gains, the mutual information between the channel output, and the signals transmitted by the users to be decoded at the receiver can be decomposed as (omitting for simplicity of notation the conditioning on the channel fading gains)

$$\begin{aligned}
I(\overline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}) &= I(\overline{\mathcal{X}}_{\mathcal{D}_J},\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}) - I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) && \text{(D-10)}\\
&= I(\mathcal{X}^{\mathrm{T}}\,;\mathcal{Y}) - I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) && \text{(D-11)}\\
&= E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\widetilde{\mathbf{H}}^{\dagger}\widetilde{\mathbf{S}}^{\dagger}\widetilde{\mathbf{S}}\widetilde{\mathbf{H}}\right)\right\} - E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}_{\mathcal{D}_J}^{\dagger}\underline{\mathbf{S}}_{\mathcal{D}_J}^{\dagger}\underline{\mathbf{S}}_{\mathcal{D}_J}\underline{\mathbf{H}}_{\mathcal{D}_J}\right)\right\}\ , && \text{(D-12)}
\end{aligned}$$

where (D-12) follows from [61], and the independence of the information transmitted by different users, and $E_{\boldsymbol{S}}\{\cdot\}$ denotes expectation with respect to the realizations of the random signatures. It is emphasized that the fact that the signatures are known also for the users that are not decoded (both intra-cell and in other cells), is fully exploited in (D-11)–(D-12). Also, when spreading is used, unlike [48], it would be suboptimal to have a front end of J matched filters that neglects the $K-J$ weakest intra-cell users, and the other-cells' users.

Conditioned on the channel fades, the capacity region (bits/sec) determining the rates achievable by the J strongest intra-cell users is given by:

$$C = \bigcap_{\mathcal{D}\subset\mathcal{D}_J}\left\{\sum_{i\in\mathcal{D}} R_i \le I(\overline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J-\mathcal{D}})\right\}\ . \tag{D-13}$$

As can be seen from the following result, due to the particular structure of the considered problem, only J out of the 2^J-1 rate constraint equations in (D-13) are sufficient in order to describe the capacity region.

Proposition D.1 *The capacity region achievable by the J strongest intra-cell users is equal to (bits/sec)*

$$C = \bigcap_{j=1}^{J}\left\{\sum_{\ell=j}^{J} R_{i_\ell} \le I(\underline{\mathcal{X}}_{\mathcal{D}_{j-1}}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_{j-1}}) - I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J})\right\}\ . \tag{D-14}$$

Proof: First it is required to show that the following J equations are sufficient:

$$C = \bigcap_{j=1}^{J}\left\{\sum_{\ell=j}^{J} R_{i_\ell} \le I(x_{i_j},\ldots,x_{i_J}\,;\mathcal{Y}\,|\,x_{i_1},\ldots,x_{i_{j-1}})\right\}\ . \tag{D-15}$$

To that end, proceeding as in (D-11), note that all the mutual informations arising in (D-13) take the form

$$I(\overline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J-\mathcal{D}}) = I(\overline{\mathcal{X}}_{\mathcal{D}}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J-\mathcal{D}}) \tag{D-16}$$

$$= I(\overline{\mathcal{X}}_{\mathcal{D}},\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J-\mathcal{D}}) - I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) \tag{D-17}$$

$$= E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}^{\dagger}_{\mathcal{D}_J-\mathcal{D}}\underline{\mathbf{S}}^{\dagger}_{\mathcal{D}_J-\mathcal{D}}\underline{\mathbf{S}}_{\mathcal{D}_J-\mathcal{D}}\underline{\mathbf{H}}_{\mathcal{D}_J-\mathcal{D}}\right)\right\} - I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) \;. \tag{D-18}$$

The right hand side of (D-18) is always lower bounded by the choice of $\mathcal{D}$ corresponding to the ℓ-weakest users among the J strongest intra-cell users, i.e., the users of indices $i_{J-\ell+1},\ldots,i_{J-1},i_J$. Thus, the corresponding equation dominates. Note that the expression $I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\overline{\mathcal{X}}_{\mathcal{D}_J})$ in (D-12) is fixed for a fixed J, and thus it is not affected by the specific choice of $\mathcal{D}\subset\mathcal{D}_J$.

Finally, generalizing the decomposition in (D-11), every mutual information appearing in (D-15) can be written as:

$$\begin{aligned} I(x_{i_j},\ldots,x_{i_J}\,;\mathcal{Y}\,|\,x_{i_1},\ldots,x_{i_{j-1}}) &= I(x_{i_j},\ldots,x_{i_J},\underline{\mathcal{X}}_{D_J}\,;\mathcal{Y}\,|\,x_{i_1},\ldots,x_{i_{j-1}}) \\ &\quad - I(\underline{\mathcal{X}}_{D_J}\,;\mathcal{Y}\,|\,x_{i_1},\ldots,x_{i_{j-1}},x_{i_j},\ldots,x_{i_J}) \\ &= I(\underline{\mathcal{X}}_{\mathcal{D}_{j-1}}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_{j-1}}) - I(\underline{\mathcal{X}}_{D_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) \end{aligned} \tag{D-19}$$

for $j=1,\ldots,J$. It also is noted that both mutual informations in the right hand side of (D-19) can be evaluated using the $E_{\boldsymbol{S}}\{\log\det(\cdot)\}$ expressions as in (D-12), yielding

$$\begin{aligned} I(\underline{\mathcal{X}}_{\mathcal{D}_{j-1}}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_{j-1}}) - I(\underline{\mathcal{X}}_{D_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J}) &= E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}^{\dagger}_{\mathcal{D}_{j-1}}\underline{\mathbf{S}}^{\dagger}_{\mathcal{D}_{j-1}}\underline{\mathbf{S}}_{\mathcal{D}_{j-1}}\underline{\mathbf{H}}_{\mathcal{D}_{j-1}}\right)\right\} \\ &\quad - E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}^{\dagger}_{\mathcal{D}_J}\underline{\mathbf{S}}^{\dagger}_{\mathcal{D}_J}\underline{\mathbf{S}}_{\mathcal{D}_J}\underline{\mathbf{H}}_{\mathcal{D}_J}\right)\right\} \;. \end{aligned} \tag{D-20}$$

■

Following the derivation of the capacity region of the J strongest users, in the non-asymptotic regime, it is required to examine the behavior of the $E_{\boldsymbol{S}}\{\log\det(\cdot)\}$ expressions in (D-20) as the system size becomes large (i.e., $K,N\to\infty$, $\frac{K}{N}\to\beta<\infty$). In particular, it is of interest to examine the expression

$$E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}^{\dagger}_{\mathcal{D}_L}\underline{\mathbf{S}}^{\dagger}_{\mathcal{D}_L}\underline{\mathbf{S}}_{\mathcal{D}_L}\underline{\mathbf{H}}_{\mathcal{D}_L}\right)\right\} \;, \tag{D-21}$$

where $\frac{L}{K}\to\delta$ $(0\le\delta\le1)$, in the asymptotic regime. Defining $\mathcal{C}_{\text{scpo}}(L\,;\bar{P})$ as

$$\mathcal{C}_{\text{scpo}}(L\,;\bar{P}) \triangleq \frac{1}{N}E_{\boldsymbol{S}}\left\{\log\det\left(\boldsymbol{I}+\bar{P}\underline{\mathbf{H}}^{\dagger}_{\mathcal{D}_L}\underline{\mathbf{S}}^{\dagger}_{\mathcal{D}_L}\underline{\mathbf{S}}_{\mathcal{D}_L}\underline{\mathbf{H}}_{\mathcal{D}_L}\right)\right\} \;, \tag{D-22}$$

it is observed that $\mathcal{C}_{\text{scpo}}(L\,;\bar{P})$ equals the spectral efficiency of the optimum multiuser receiver in a single-cell setting, where the cell-site receiver *receives and decodes* only the transmissions of the intra-cell users *not* included in the set $\mathcal{D}_L$, *and* the transmissions of other-cell users (alternatively one can assume that the channel inputs due to the users within the set $\mathcal{D}_L$ are known at the receiver). Hence, it follows from Lemma 2.4 that

$$
\begin{aligned}
\mathcal{C}_{\text{scpo}}(\delta\,;\bar{P}) \triangleq & \lim_{N,K\to\infty,\,\frac{K}{N}\to\beta,\,\frac{L}{K}\to\delta} \mathcal{C}_{\text{scpo}}(L\,;\bar{P}) \\
= & \lim_{N,K\to\infty,\,\frac{K}{N}\to\beta,\,\frac{L}{K}\to\delta} \frac{1}{N}\sum_{m=L+1}^{K} \log\left(1+\bar{P}\nu_{i_m}\eta_{\text{scpo}}(L\,;\bar{P})\right) \\
& + \frac{1}{N}\sum_{m=1}^{K} \log\left(1+\alpha^2\bar{P}\nu_m^{+}\eta_{\text{scpo}}(L\,;\bar{P})\right) \\
& + \frac{1}{N}\sum_{m=1}^{K} \log\left(1+\alpha^2\bar{P}\nu_m^{-}\eta_{\text{scpo}}(L\,;\bar{P})\right) \\
& - \log\eta_{\text{scpo}}(L\,;\bar{P}) + \left(\eta_{\text{scpo}}(L\,;\bar{P})-1\right)\log e\ ,
\end{aligned}
\tag{D-23}
$$

where $\eta_{\text{scpo}}(L\,;\bar{P})$ is the unique solution to the equation

$$
1 = \eta + \frac{1}{N}\sum_{m=L+1}^{K}\frac{\bar{P}\nu_{i_m}\eta}{1+\bar{P}\nu_{i_m}\eta} + \frac{1}{N}\sum_{m=1}^{K}\frac{\alpha^2\bar{P}\nu_m^{+}\eta}{1+\alpha^2\bar{P}\nu_m^{+}\eta} + \frac{1}{N}\sum_{m=1}^{K}\frac{\alpha^2\bar{P}\nu_m^{-}\eta}{1+\alpha^2\bar{P}\nu_m^{-}\eta}\ . \tag{D-24}
$$

Using (B-5) and some algebra, it can be shown that

$$
\lim_{N,K\to\infty,\,\frac{K}{N}\to\beta,\,\frac{L}{K}\to\delta} \eta_{\text{scpo}}(L\,;\bar{P}) = \eta_{\text{scpo}}(\delta\,;\bar{P})\ , \tag{D-25}
$$

where $\eta_{\text{scpo}}(\delta\,;\bar{P})$ is the unique solution to the equation

$$
1-\beta(3-\delta) = \eta - \beta\Theta_{\text{scpo}}(\delta\,;\bar{P}\eta)\ , \tag{D-26}
$$

and

$$
\begin{aligned}
\Theta_{\text{scpo}}(\omega\,;\varsigma) = & \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} \frac{1}{1+\varsigma\phi}\,d\mathcal{F}_\nu(\phi) + 2\int_0^\infty \frac{1}{1+\alpha^2\varsigma\phi}\,d\mathcal{F}_\nu(\phi) \\
\underset{\text{Rayleigh}}{=} & \int_0^{-\ln\omega} \frac{1}{1+\varsigma\phi}\,e^{-\phi}\,d\phi + 2\int_0^\infty \frac{1}{1+\alpha^2\varsigma\phi}\,e^{-\phi}\,d\phi \\
= & \frac{1}{\varsigma}\,e^{\frac{1}{\varsigma}}\left[\mathcal{E}_1\!\left(\frac{1}{\varsigma}\right) - \mathcal{E}_1\!\left(\frac{1}{\varsigma}-\ln\omega\right)\right] + \frac{2}{\alpha^2\varsigma}\,e^{\frac{1}{\alpha^2\varsigma}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\varsigma}\right)\ .
\end{aligned}
\tag{D-27}
$$

The sum expressions in (D-23) can be handled in an analogous manner, and defining

$$
\begin{aligned}
\Gamma_{\text{scpo}}(w\,;\varsigma) = & \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} \log\left(1+\varsigma\phi\right)\,d\mathcal{F}_\nu(\phi) + 2\int_0^\infty \log\left(1+\alpha^2\varsigma\phi\right)\,d\mathcal{F}_\nu(\phi) \\
\underset{\text{Rayleigh}}{=} & \int_0^{-\ln\omega} \log\left(1+\varsigma\phi\right)e^{-\phi}\,d\phi + 2\int_0^\infty \log\left(1+\alpha^2\varsigma\phi\right)e^{-\phi}\,d\phi \\
= & -\omega\log(1-\varsigma\ln\omega) + e^{\frac{1}{\varsigma}}\left[\mathcal{E}_1\!\left(\frac{1}{\varsigma}\right) - \mathcal{E}_1\!\left(\frac{1}{\varsigma}-\ln\omega\right)\right] + 2e^{\frac{1}{\alpha^2\varsigma}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\varsigma}\right)\log e\ ,
\end{aligned}
\tag{D-28}
$$

it follows that

$$\mathcal{C}_{\text{scpo}}(\delta\,;\bar{P}) = \beta\Gamma_{\text{scpo}}\left(\delta\,;\bar{P}\eta_{\text{scpo}}(\delta\,;\bar{P})\right) - \log\eta_{\text{scpo}}(\delta\,;\bar{P}) + (\eta_{\text{scpo}}(\delta\,;\bar{P}) - 1)\log e\ . \tag{D-29}$$

The final step in the proof of Proposition 3.7 is to plug the above asymptotic expressions in the capacity region (D-14). Recalling the underlying equal-rates assumption and (D-20), it follows that the non-asymptotic capacity region (*bits/sec/Hz*) becomes

$$\begin{aligned}\tilde{C} &= \bigcap_{j=1}^{J}\left\{(J-j+1)\tilde{R} \le \frac{1}{N}I(\underline{\mathcal{X}}_{\mathcal{D}_{j-1}}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_{j-1}}) - \frac{1}{N}I(\underline{\mathcal{X}}_{\mathcal{D}_J}\,;\mathcal{Y}\,|\,\overline{\mathcal{X}}_{\mathcal{D}_J})\right\} \\ &= \bigcap_{j=1}^{J}\left\{(J-j+1)\tilde{R} \le \mathcal{C}_{\text{scpo}}(j-1\,;\bar{P}) - \mathcal{C}_{\text{scpo}}(J\,;\bar{P}))\right\}\ .\end{aligned} \tag{D-30}$$

Letting $1-\mathcal{Q}$ and $1-x$ take the asymptotic role of $\frac{J}{K}$ and $\frac{j}{J}$, respectively, and taking the limits in (D-30), the outage constrained capacity $J\tilde{R}$, for an FUU of $\mathcal{Q}$, is given by

$$\tilde{R}_{T_{\text{scpo}}}(\mathcal{Q}) = \inf_{0<x\le 1}\frac{\mathcal{C}_{\text{scpo}}\left[(1-\mathcal{Q})(1-x)\,;\bar{P}\right] - \mathcal{C}_{\text{scpo}}\left[(1-\mathcal{Q})\,;\bar{P}\right]}{x}\ , \tag{D-31}$$

where $\mathcal{C}_{\text{scpo}}$ is given by (D-29). This completes the proof of Proposition 3.7. ■

E On the Minimum Receive $\frac{E_b}{N_0}$ Required for Reliable Communication

Following (3-4), the minimum required receive $\frac{E_b}{N_0}$ for reliable communication is given by [17] [52]

$$\frac{E_b}{N_0}_{\min} = \lim_{\bar{P}\to 0}\frac{\beta\bar{P}}{\tilde{R}_T(\beta,\bar{P})}\ , \tag{E-1}$$

where the notation $\tilde{R}_T(\beta,\bar{P})$ was introduced to emphasize that the outage constrained capacity is a function of $\bar{P}$ (the SNR), and the cell load β. Starting with the linear receivers, it follows from (3-3) that

$$\tilde{R}_T \underset{\bar{P}\to 0}{=} \beta\bar{P}(1-\mathcal{Q})\mathcal{F}_{\nu}^{-1}(\mathcal{Q})\eta\log e + o(\bar{P})\ , \tag{E-2}$$

yielding

$$\frac{E_b}{N_0}_{\min} = \lim_{\bar{P}\to 0}\frac{\ln 2}{(1-\mathcal{Q})\mathcal{F}_{\nu}^{-1}(\mathcal{Q})\eta}\ . \tag{E-3}$$

Observing (2-12) and (2-22), it follows that for both linear receivers $\lim_{\bar{P}\to 0}\eta = 1$, and hence

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{(1-\mathcal{Q})\mathcal{F}_{\nu}^{-1}(\mathcal{Q})}\ , \tag{E-4}$$

which establishes (3-5).

Showing that the same result holds also for the SCO and the SCPO receiver is less trivial, and the derivation follows the proof of Proposition 3.7 as given in Appendix D. Again the focus is on the asymptotic setup in which

$$K, N, L \to \infty,\ \frac{K}{N} \to \beta < \infty,\ \frac{L}{K} \to \delta\ , \tag{E-5}$$

however empirical averages and nonasymptotic notation shall be used whenever necessary for purposes of the derivation (the validity of this approach follows form the underlying assumption of a.s. convergence of the limiting empirical distribution of the fade power levels, as stated in Subsection 2.1). Also, the rest of the proof shall focus on the particular case of $\alpha = 0$ (i.e., a single-cell setting) for simplicity. It is emphasized however that the proof can be straightforwardly extended to the general case of $\alpha \neq 0$ as well (leading to the same result for the minimum $\frac{E_b}{N_0}$). Since both SCO and SCPO receivers are identical in the single-cell setting, they shall be referred to as the "optimum" receiver, following the discussion of Subsection 3.3. Considering $\mathcal{C}_{\text{opt}}(L\,;\bar{P})$, as defined in (D-22) it follows from (D-23) and (D-24) that

$$\mathcal{C}_{\text{opt}}(L\,;\bar{P}) \triangleq \frac{1}{N}\sum_{m=L+1}^{K}\log\left(1+\bar{P}\nu_{i_m}\eta_{\text{opt}}(L\,;\bar{P})\right) - \log\eta_{\text{opt}}(L\,;\bar{P}) + \left(\eta_{\text{opt}}(L\,;\bar{P})-1\right)\log e\ , \tag{E-6}$$

where $\eta_{\text{opt}}(L\,;\bar{P})$ is the unique solution to the equation

$$1 = \eta + \frac{1}{N}\sum_{m=L+1}^{K}\frac{\bar{P}\nu_{i_m}\eta}{1+\bar{P}\nu_{i_m}\eta}\ . \tag{E-7}$$

As can be observed, the constant $\eta_{\text{opt}}(L\,;\bar{P})$ satisfies

$$\eta \underset{\bar{P}\to 0}{=} \frac{1}{1+\bar{P}\ \frac{1}{N}\sum_{m=L+1}^{K}\nu_{i_m}} + o(\bar{P})\ . \tag{E-8}$$

Similarly, it follows that

$$\frac{1}{N}\sum_{m=L+1}^{K}\log\left(1+\bar{P}\nu_{i_m}\eta_{\text{opt}}(L\,;\bar{P})\right) \underset{\bar{P}\to 0}{=} \bar{P}\eta_{\text{opt}}(L\,;\bar{P})\log e\frac{1}{N}\sum_{m=L+1}^{K}\nu_{i_m} + o(\bar{P})\ . \tag{E-9}$$

Substituting the above two limiting expressions in (E-6) yields

$$\mathcal{C}_{\text{opt}}(L\,;\bar{P}) \underset{\bar{P}\to 0}{=} \beta\bar{P}\frac{1}{K}\sum_{m=L+1}^{K}\nu_{i_m} + o(\bar{P})\ , \tag{E-10}$$

where the notation $\frac{K}{N} = \beta$ was used for simplicity (and in agreement with the underlying assumptions regarding the asymptotic regime).

Now as shown in Appendix D (see (D-30)), the capacity region of the "optimum" receiver is given by

$$C = \bigcap_{j=1}^{J}\left\{(J-j+1)\tilde{R} \le \mathcal{C}_{\text{opt}}(j-1\,;\bar{P}) - \mathcal{C}_{\text{opt}}(J\,;\bar{P}))\right\}\ . \tag{E-11}$$

Observing (E-10), the dominant inequality in (E-11) is clearly the one for which $j = J$, and the maximum attainable rate per user (under outage constraint) must satisfy

$$\tilde{R} = \mathcal{C}_{\rm opt}(J-1\,;\bar{P}) - \mathcal{C}_{\rm opt}(J\,;\bar{P}) \ . \tag{E-12}$$

Hence, recalling that $\tilde{R}_T = J\tilde{R}$, the outage constrained capacity of the "optimum" receiver satisfies

$$\tilde{R}_{T_{\rm opt}} \underset{\bar{P}\to 0}{=} \beta\bar{P}\frac{J}{K}\nu_{i_J}\log e + o(\bar{P}) \ , \tag{E-13}$$

which in the asymptotic setup of (E-5) (recalling also that $\frac{J}{K} \to 1 - \mathcal{Q}$ in this case) satisfies

$$\tilde{R}_{T_{\rm opt}} \underset{\bar{P}\to 0}{=} (1-\mathcal{Q})\mathcal{F}_\nu^{-1}(\mathcal{Q})\beta\bar{P}\log e + o(\bar{P}) \ . \tag{E-14}$$

(E-4) then follows immediately from (E-1).

References

[1] R. R. Müller, *Power and Bandwidth Efficiency of Multiuser Systems with Random Spreading*, Ph.D. thesis, Universtät Erlangen-Nürnberg, Erlangen, Germany, Nov. 1998.

[2] S. Verdú and S. Shamai (Shitz), "Spectral efficiency of CDMA with random spreading," *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 622–640, Mar. 1999.

[3] D. Tse and S. Hanly, "Linear multiuser receivers: Effective interference, effective bandwidth and user capacity," *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 641–657, Mar. 1999.

[4] P. Schramm and R. R. Müller, "Spectral efficiency of CDMA systems with linear MMSE interference suppression," *IEEE Transactions on Communications*, vol. 47, no. 5, pp. 722–731, May 1999.

[5] P. D. Alexander, M. C. Reed, J. A. Asenstorfer, and C. B. Schlegel, "Iterative multiuser interference reduction: turbo CDMA," *IEEE Transactions on Communications*, vol. 47, no. 7, pp. 1008–1014, July 1999.

[6] P. Viswanath, V. Anatharam, and D. N. C. Tse, "Optimal sequences, power control, and user capacity of synchronous CDMA systems with linear MMSE multiuser receivers," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1968–1983, Sept. 1999.

[7] P. Viswanath and V. Anatharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1984–1991, Sept. 1999.

[8] U. Madhow and M. Honig, "On the average near-far resistance for MMSE detection of direct sequence CDMA signals with random spreading," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 2039–2045, Sept. 1999.

[9] D. Tse and O. Zeitouni, "Performance of linear multiuser receivers in random environments," *IEEE Transactions on Information Theory*, vol. 46, no. 1, pp. 171–188, Jan. 2000.

[10] Kiran and D. N. C. Tse, "Effective interference and effective bandwidth of linear multiuser receivers in asynchronous systems," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1426–1447, 2000.

[11] J. Evans and D. N. C. Tse, "Large system performance of linear multiuser receivers in multipath fading channels," *IEEE Transactions on Information Theory*, vol. 46, no. 6, pp. 2059–2078, Sept. 2000.

[12] E. Biglieri, G. Caire, and G. Taricco, "CDMA system design through asymptotic analysis," *IEEE Transactions on Communications*, vol. 48, no. 11, pp. 1882–1896, Nov. 2000.

[13] D. N. C. Tse and S. Verdú, "Optimum asymptotic multiuser efficiency of randomly spread CDMA," *IEEE Transactions on Information Theory*, vol. 46, no. 7, pp. 2718–2722, Nov. 2000.

[14] P. Viswanath, D. N. C. Tse, and V. Anantharam, "Asymptotically optimal waterfilling in vector multiple access channels," *IEEE Transactions on Information Theory*, vol. 47, no. 1, pp. 241–267, Jan. 2001.

[15] R. R. Müller, "Multiuser receivers for randomly spread signals: Fundamental limits with and without decision-feedback," *IEEE Transactions on Information Theory*, vol. 47, no. 1, pp. 268–283, Jan. 2001.

[16] J. Zhang, E. K. P. Chong, and D. N. C. Tse, "Output MAI distributions of linear MMSE multiuser receivers in CDMA systems," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 1128–1144, Mar. 2001.

[17] S. Shamai (Shitz) and S. Verdú, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Transactions on Information Theory*, vol. 47, no. 4, pp. 1302–1327, May 2001.

[18] S. V. Hanly and D. N. C. Tse, "Resource pooling and effective bandwidths in CDMA networks with multiuser receivers and spatial diversity," *IEEE Transactions on Information Theory*, vol. 47, no. 4, pp. 1328–1351, May 2001.

[19] A. M. Tulino and S. Verdú, "Asymptotic analysis of improved linear receivers for BPSK-CDMA subject to fading," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1544–1555, Aug. 2001.

[20] R. R. Müller and S. Verdú, “Design and analysis of low-complexity interference mitigation on vector channels,” *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1429–1441, Aug. 2001.

[21] B. M. Zaidel, S. Shamai (Shitz), and S. Verdú, “Multi-cell uplink spectral efficiency of coded DS-CDMA with random signatures,” *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1556–1569, Aug. 2001, See also: — ”Spectral Efficiency of Randomly Spread DS-CDMA in a Multi-Cell Model,” Proceedings of the 37th Annual Allerton Conference on Communication, Control and Computing, Monticello, IL, pp. 841–850, Sept. 1999.

[22] K. Tang, P. H. Siegel, and L. B. Milstein, “A comparison of long versus short spreading sequences in coded asynchronous DS-CDMA systems,” *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1614–1624, Aug. 2001.

[23] V. V. Veeravalli and A. Mantravadi, “The coding-spreading tradeoff in CDMA systems,” *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 2, pp. 396–408, Feb. 2002.

[24] S. Bhashyam and B. Aazhang, “Multiuser channel estimation and tracking for long-code CDMA systems,” *IEEE Transactions on Communications*, vol. 50, no. 7, pp. 1081–1090, July 2002.

[25] T. Tanaka, “A statistical-mechanics approach to large-system analysis of CDMA multiuser detectors,” *IEEE Transactions on Information Theory*, vol. 48, no. 11, pp. 2888–2910, Nov. 2002.

[26] A. Mantravadi and V. V. Veeravalli, “MMSE detection in asynchronous CDMA systems: an equivalence result,” *IEEE Transactions on Information Theory*, vol. 48, no. 12, pp. 3128–3137, Dec. 2002.

[27] G. Caire, S. Guemghar, A.Roumy, and S. Verdú, “Maximizing the spectral efficiency of coded CDMA under successive decoding,” *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 152–164, Jan. 2004.

[28] S. Ulukus and R. D. Yates, “User capacity of asynchronous CDMA systems with matched filter receivers and optimum signature sequences,” *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 903–909, May 2004.

[29] L. Li, A. M. Tulino, and S. Verdú, “Design of reduced-rank MMSE multiuser detectors using random matrix methods,” *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 986–1008, June 2004.

[30] R. R. Müller and W. H. Gerstacker, “On the capacity loss due to separation of detection and decoding,” *IEEE Transactions on Information Theory*, vol. 50, no. 8, pp. 1769–1778, Aug. 2004.

[31] A. Tarable, G. Montorsi, and S. Benedetto, "Analysis and design of interleavers for iterative multiuser receivers in coded CDMA systems," *IEEE Transactions on Information Theory*, vol. 51, no. 5, pp. 1650–1666, May 2005.

[32] D. Guo and S. Verdú, "Randomly spread CDMA: asymptotics via statistical physics," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 1983–2010, June 2005.

[33] L. A. Imhof and R. Mathar, "Capacity regions and optimal power allocation for CDMA cellular radio," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2011–2019, June 2005.

[34] O. Somekh, B. M. Zaidel, and S. Shamai (Shitz), "Spectral efficiency of joint multiple cell-site processors for randomly spread DS-CDMA systems," Submitted to the IEEE Transactions on Information Theory, 2004.

[35] R. R. Müller, "On the asymptotic eigenvalue distribution of concatenated vector-valued fading channels," *IEEE Transactions on Information Theory*, vol. 48, no. 7, pp. 2086–2091, July 2002.

[36] A. M. Tulino and S. Verdú, "Random matrix theory and wireless communications," *Trends in Communications and Information Theory*, vol. 1, no. 1, 2004, Now Publishers Inc. Hanover, USA.

[37] E. Biglieri, G. Taricco, and A. Tulino, "How far away is infinity? Using asymptotic analyses in multiple-antenna systems," in *Proceedings of the International Symposium on Spread Spectrum Techniques and Applications (ISSSTA)*, Prague, Czech Republic, Sept. 2 – 5, 2002, vol. 1, pp. 1–6.

[38] A. D. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Transactions on Information Theory*, vol. 40, no. 6, pp. 1713–1727, Nov. 1994.

[39] O. Somekh and S. Shamai (Shitz), "Shannon-theoretic approach to a Gaussian cellular multi-access channel with fading," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1401–1425, July 2000.

[40] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.

[41] S. Shamai (Shitz) and A. D. Wyner, "Information-theoretic considerations for symmetric, cellular, multiple-access fading channels - Parts I & II," *IEEE Transactions on Information Theory*, vol. 43, no. 6, pp. 1877–1911, Nov. 1997.

[42] B. M. Zaidel, S. Shamai (Shitz), and S. Verdú, "Multi-cell uplink spectral efficiency of randomly spread DS-CDMA in Rayleigh fading channels," in *Proceedings of the 6th International Symposium on Communication Techniques and Applications (ISCTA'01)*, Ambleside,

UK, July 15 – 20, 2001, pp. 499–504, See also: —, “Random CDMA in the multiple cell uplink environment: the effect of fading on various receivers,” Proceedings of the 2001 IEEE Information Theory Workshop, Cairns, Australia, Sept. 2–7, 2001, pp. 42–45.

[43] E. Biglieri, G. Caire, G. Taricco, and E. Viterbo, “How fading affects CDMA: An asymptotic analysis with linear receivers,” *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 2, pp. 191–201, Feb. 2001.

[44] H. Li and H. V. Poor, “Power allocation and spectral efficiency of DS-CDMA systems in fading channels with fixed QoS - Part I: Single-rate case,” Technical report, Princeton University, 2004.

[45] H. Li and H. V. Poor, “Power allocation and spectral efficiency of DS-CDMA systems in fading channels with fixed QoS - Part II: Multiple-rate case,” Technical report, Princeton University, 2004.

[46] P. Viswanath, D. N. C. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas,” *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[47] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channel with partial side information,” *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[48] S. Shamai (Shitz) and I. Bettesh, “Outages, expected rates and delays in multiple-user fading channels,” in *Proceedings of the 2000 Conference on Information Sciences and Systems (CISS'2000)*, Princeton University, Princeton, NJ, Mar. 2000, pp. WA4.7–WA4.15.

[49] S. Shamai (Shitz) and S. Verdú, “Decoding only the strongest CDMA users,” in *Codes, Graphs, and Systems: A Celebration of the Life and Career of G. David Forney Jr. on the Occasion of his Sixtieth Birthday*, R. E. Blahut and R. Kotter, Eds., pp. 217–228. Kluwer, 2002.

[50] S. Shamai (Shitz) and A. Steiner, “A broadcast approach for a single-user slowly fading MIMO channel,” *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2617–2635, Oct. 2003.

[51] S. Verdú, *Multiuser Detection*, Cambridge University Press, Cambridge, UK, 1998.

[52] S. Verdú, “Spectral efficiency in the wideband regime,” *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1329–1343, June 2002.

[53] A. Lozano, A. M. Tulino, and S. Verdú, “High-SNR power offset in multi-antenna communications,” Preprint, July 2004.

[54] A. Lapidoth, “Nearest neighbor decoding for additive non-Gaussian noise channels,” *IEEE Transactions on Information Theory*, vol. 42, no. 5, pp. 1520–1529, Sept. 1996.

[55] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "Gaussian codes and the scaled nearest neighbor decoding in fading multi-antenna channels," *IEEE Transactions on Information Theory*, vol. 50, no. 8, pp. 1665–1686, Aug. 2004.

[56] D. Guo, S. Shamai (Shitz), and S. Verdú, "Mutual information and minimum mean-square error in Gaussian channels," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1261–1282, Apr. 2005.

[57] A. M. Tulino, A. Lozano, and S. Verdú, "Impact of antenna correlation on the capacity of multiantenna channels," *IEEE Transactions on Information Theory*, vol. 51, no. 7, pp. 2491–2509, July 2005.

[58] M. K. Varanasi and T. Guess, "Optimum decision feedback multiuser equalization with successive decoding achieves the total capacity of the Gaussian multiple-access channel," in *Conference Record of the 31th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2–5, 1997, vol. 2, pp. 1405–1409.

[59] S. Shamai (Shitz), "A broadcast strategy for the Gaussian slowly fading channel," in *Proceedings of the 1997 IEEE International Symposium on Information Theory*, Ulm, Germany, June 29 – July 4, 1997, p. 150.

[60] S. Shamai (Shitz), B. M. Zaidel, and S. Verdú, "Analysis of strongest-users-only detectors for randomly spread CDMA," Technical Report CCIT 354, Technion - IIT, Oct. 2002.

[61] S. Verdú, "Capacity region of Gaussian CDMA channels: The symbol synchronous case," in *Proceedings of the 24th Annual Allerton Conference on Communication, Control and Computing*, Monticello, IL, Oct. 1986, pp. 1025–1034.