

Odd mode separation in concentric resonators with bi-prism like element

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ABSTRACT

New methods of laser metrology (interferometry and microscopy) based on applications of beams with special structures provide increased resolution and efficiency. To generate a beam with linear singularity (dark beam) we recently proposed a beam shaping method using a bi-prism-like element within the laser resonator. There we have studied resonators that are traditionally designed to oscillate on the fundamental mode designed within the range of configuration parameters, $0.5 \leq G \leq 1$.

In the present work we extend the approach and show that the choice of specific configurations, outside the above range of configuration parameters, can lead to much better results for our application. This is the case in particular for an approximately semi-concentric resonator ($G \sim -1$). The optimal dark beam is obtained for a bi-prism angle about twice that obtained for the earlier configurations. For this case the difference between the losses of the first odd mode and other modes is 0.12-0.15, which is adequate for oscillation on this mode in lasers with any type of active media.

Keywords: mode selection, beam shaping, dark beam, semi-concentric resonator, bi-prism.

1. INTRODUCTION

Laser beam shaping is a well-studied subject and is usually implemented by some optical system, external to the laser cavity. In most cases such an external system consists of one or more phase elements and diffractive optical elements^{1,2}. The disadvantages of such a solution include limited light efficiency, optical noise and intricate alignment procedures. One approach to mitigate these disadvantages is to shape the laser beam by a dedicated design of the laser resonator^{3,4}. Our specific interest here is the generation of laser beams with a line singularity employed for high-resolution optical metrology. In this application, generally termed as Singular Beam Microscopy, a laser beam structured by one or more singularities scans the object and the scattered light is analyzed to derive nanoscale information about the object⁵⁻⁷.

In recent work^{8,9} we have studied the possibilities of employing a Bi-Prism like Element (BPE) to replace one of the laser mirrors for constructing a cavity that will primarily support oscillation on the first odd mode, which contains the desired line singularity. In that work we have studied resonators that are traditionally used in lasers to generate the fundamental even mode. These lasers belong to a family that possesses effective configuration parameters, G , (see Eq. 1 below) within the range $0 \leq G \leq 1$ and we have demonstrate efficient ways to achieve the desired beam shape. The main purpose of the present paper is to demonstrate that using resonators outside this family can lead to even better solutions for obtaining our desired beam structure. We show that resonators with $-1.2 \leq G \leq 0$ in combination with a BPE are extremely effective in oscillating on the first odd mode. Focusing on the neighborhood of the semi-concentric configuration we show that the intracavity BPE implements a physical process similar to splitting the resonator into two separate partial resonators with strong diffraction coupling.

In the following section we review relations and methods used for the numerical study. In Sec. 3 we present a numerical study to demonstrate the advantages of using resonators from the extended regime of configuration parameters and a concluding section completes the paper.

2. THE RESONATOR SCHEME AND ITS MATHEMATICAL MODEL FOR CALCULATION AND SIMULATION

In this section we review the definitions and relations as already presented in earlier publications^{8,9} and expand it to encompass the present work. In the schematic diagram of the resonator of Fig. 1, M_1 is a BPE with a variable angle, β , and we define the normalized angle by the relation,

$$\alpha = a_1 \beta / \lambda$$

With this definition we obtain a linear phase variation from the center to the edge of the BPE reaching a maximal value of $\varphi = 2\pi\alpha$.

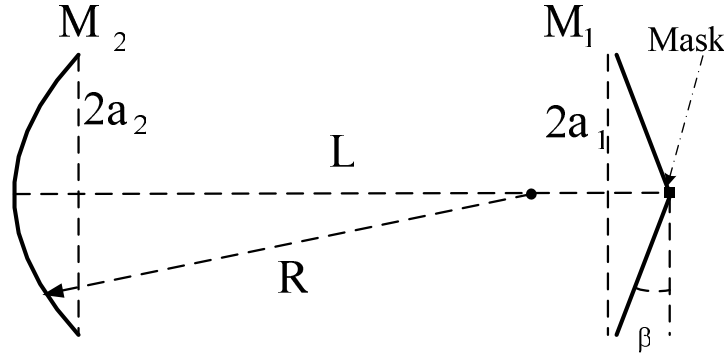


Fig.1 Definition of the resonator parameters: M_1 is the BPE and M_2 is a cylindrical mirror with radius of curvature R . The distance between the mirrors is L while a_1 and a_2 are the half widths of mirrors M_1 and M_2 , respectively. As marked, the angle β has negative value. An optional mask over the BPE vertex is also shown.

The second mirror, M_2 , is cylindrical with radius of curvature, R . For the calculations we use the configuration parameter of this mirror,

$$g_2 = 1 - L/R,$$

the effective aperture parameters,

$$b_j = \sqrt{2\pi a_j^2 / (\lambda L)},$$

where a_j is the half-width of mirror M_j ($j=1,2$), and L is the length of the resonator. For our study it is convenient to introduce the equivalent configuration parameter of the resonator without a BPE ($\alpha = 0$) by the relation,

$$G = 2g_2g_1 - 1 = 2g_2 - 1, \quad (1)$$

since $g_1 = 1$. We shall study resonators with negative G in the range

$$-1.2 \leq G \leq 0,$$

which contains the region of stability (from semi-confocal up to semi-concentric) and an adjoining region of instability ($-1.2 \leq G \leq -1$).

The integral equation for the field, $u(x)$, close to the first mirror is given by,

$$\Lambda u_1(x_1) = \int_{-a_1}^{a_1} P_1(x_1)P_1(x_3)K(x_1, x_3)u_1(x_3)dx_3, \quad (2)$$

where

$$K(x_1, x_3) = \int_{-a_2}^{a_2} G(x_2, x_1)G(x_2, x_3)P_2^2(x_2)dx_2$$

with $G(x_1, x_2)$ being the Green function defining the distribution of the field between the mirrors⁴ and $P_j(x)$ ($j=1,2$) are phase correctors of the respective mirrors, M_j . The solutions of this equation are the mode fields on the first mirror $\{u_{1n}(x), n=0,1,\dots\}$ and the corresponding eigenvalues, $\{\Lambda_n\}$. The integral equation (2) was approximated by a matrix with its eigenvalues and eigenfunctions calculated numerically using the QR or LR algorithms¹⁰.

It is convenient to define a selection parameter, S , which defines the advantage of the first odd mode, TEM₁₀, as the main oscillating laser mode by the relation,

$$S = |\Lambda_1|^2 - |\Lambda_{\max}|^2 \quad (3)$$

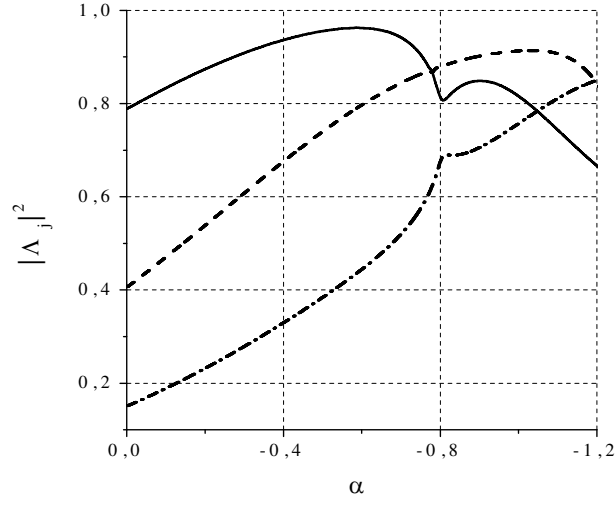
where Λ_1 is the eigenvalue of the TEM₁₀ mode, while $\Lambda_{\max}$ is the eigenvalue of a mode with the minimal losses apart from those of the TEM₁₀ mode (in the case under investigation these were the even modes TEM₀₀ and TEM₂₀). For a preferential oscillation on the first odd mode the region of interest for us corresponds to $S > 0$.

3. THE RESULTS OF CALCULATION ANALYSIS

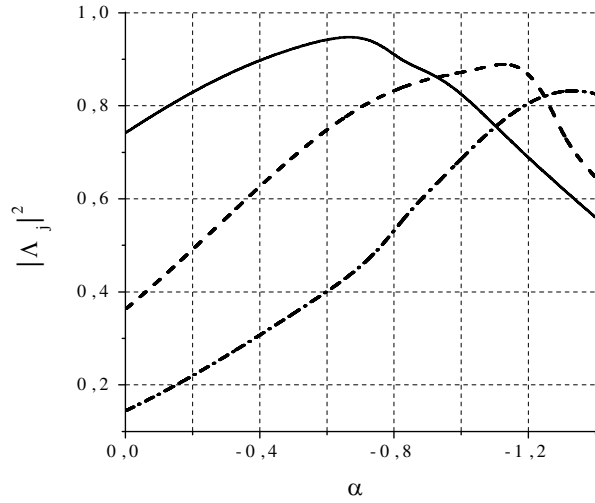
We have studied the dependence of the eigenvalues, the mode fields and the selection parameter on α , the mirror sizes and the configuration parameter, G , within an extended range. The most important results of the investigation are shown in Figs. 2-7.

Fig. 2 illustrates diagrams of $|\Lambda_j|^2$ for the first three modes in the instability region for $G = -1.01$ (Fig. 2a) and $G = -1.2$ (Fig. 2b) as a function of the bi-prism angle, α . Depending on the configuration parameter, we may observe regions with $S > 0$ around different angles of the BPE (for example, around $\alpha \sim -1.1$ in Fig. 2b). The results have the same nature as those derived earlier^{8,9} where a configuration parameter from another region was used, but they differ in the details. The increase in the angle α is accompanied by a local maximum of the fundamental mode losses along with a minimum of losses for the first odd mode at significantly different angles. The contribution of the second even mode increases (its losses decreases) up to quite large values of α .

The physical interpretation of the behavior of the curves is that the bi-prism fulfills the role of a focusing mirror with strong aberrations. Therefore, the minima (maxima) of the diffraction losses of different oscillating modes are attained for different angles, α , as their positions strongly depend on the corresponding mode field distributions. Thus, there is a region with $S > 0$ where the TEM₁₀ mode has its minimal losses while the TEM₀₀ mode radiation does not attain its maximum.



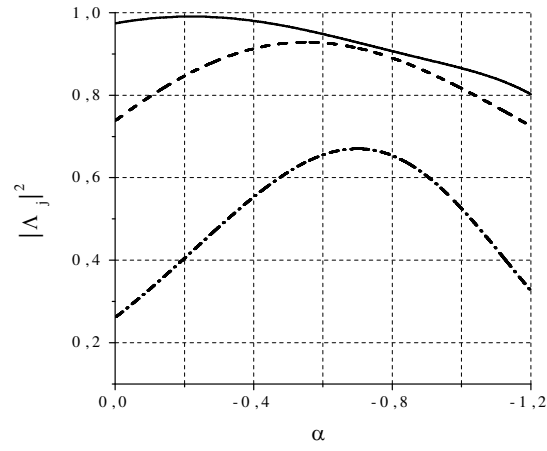
a



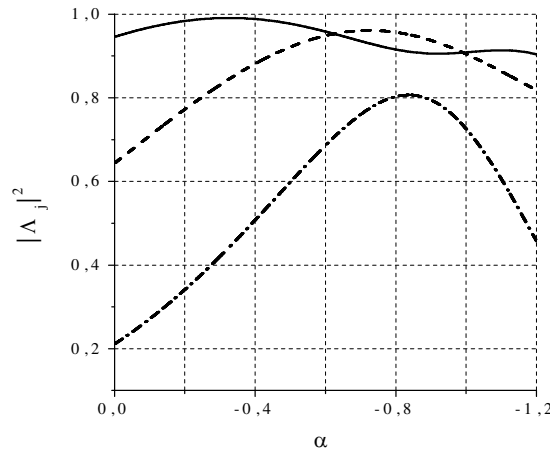
b

Fig.2. Dependence of $|\Lambda_j|^2$ on the angle α for modes TEM_{00} (solid line), TEM_{10} (dashed line) and TEM_{20} (dot-dash line) for the unstable resonator with parameters: $b_1=b_2=2.5$, $G=-1.01$ (a) and $G=-1.2$ (b).

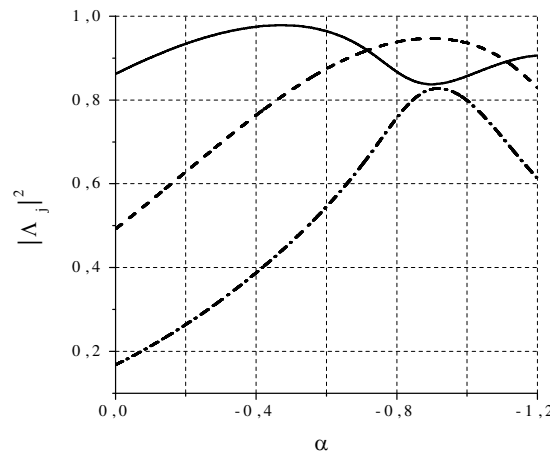
Fig. 3 illustrates similar dependence in the stability region with the three configuration parameters, $G=0$ (Fig. 3a); $G=-0.5$ (Fig. 3b); $G=-0.9$ (Fig. 3c). In this case the main new observation is that the points of extreme are located around the same value of α for all three modes: the first even, TEM_{00} mode (fundamental mode), the first odd, TEM_{10} mode, and the second even, TEM_{20} mode. The selection parameter also depends strongly on the configuration parameter. While $S < 0$ for the whole range of angles in the semi-confocal resonator (Fig. 3a), it becomes positive for the other two cases, reaching a maximum for the last case (Fig. 3c) around $\alpha \sim -0.9$.



a



b



c

Fig.3. Dependence of $|\Lambda_j|^2$ on the angle α for mode TEM_{00} (solid line), TEM_{10} (dashed line) and TEM_{20} (dot-dash line); in a stable resonator with parameters: $b_1=b_2=2.5$, $G=0$ (a), $G=-0.5$ (b) and $G=-0.9$ (c).

A more significant deviation from the earlier results can be observed for resonators near the semi-concentric configuration as shown in Fig. 4 for the limiting case: $G = -1$, $b_1 = b_2 = 2.5$. The local extreme points of three modes practically coincide, and the selection parameter becomes maximal.

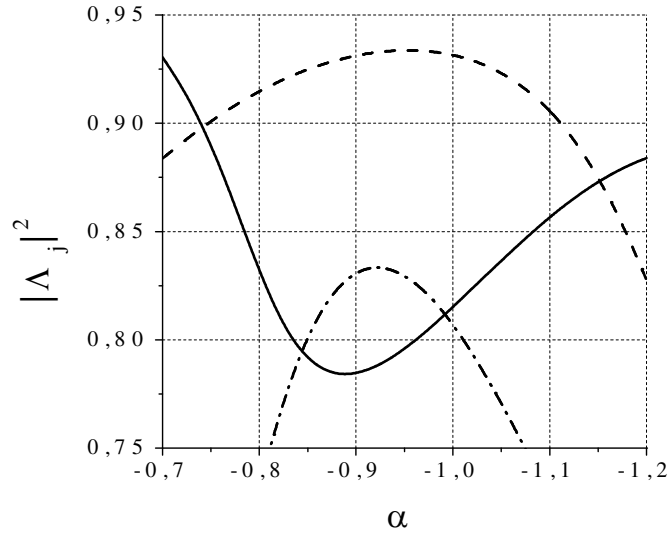


Fig.4. Dependence of $|\Lambda_j|^2$ on angle α for mode TEM_{00} (solid line), TEM_{10} (dashed line), TEM_{20} (dot-dash line) for the semi-concentric resonator ($G = -1$) with $b_1 = b_2 = 2.5$.

To better understand the unusual behavior of resonators in this region of the configuration parameter we studied the field structure of the modes. The most demonstrative results are shown in Figs. 5-7, indicating a cardinal reorganization of the mode fields. The fundamental mode, E_{00} , becomes flat around $\alpha = -0.8$ (Fig. 5), which splits when the BPE angle increases. In the neighborhood of the maximal diffraction losses the mode field E_{00} has a double-peak shape similar to the field shape of the first odd mode (Fig. 6) but the two peaks are in phase rather than the opposite phase possessed by the E_{10} mode, the shape of which remains unchanged. At the same time, in the second even mode, E_{20} (Fig. 7), the maximum of the mode radiation is in the side lobes that eventually merge into a central peak. The field distribution of this mode becomes similar to the distribution of the fundamental mode of a resonator without the bi-prism.

The configuration of the modes shown in Figs. 5-7 suggests a simple method to increase the selection parameter. As already discussed in Ref. 9, inserting a narrow strip with a half-width, w , over the vertex of the bi-prism (see Fig. 1) does not influence the losses of the first odd mode that has its field minimum at the center. However, it strongly suppresses the TEM_{20} mode with a marginal effect on the fundamental mode. Fig. 8 shows the dependence of the shape of the function $S(\alpha)$ on the relative half-width of the strip, w/a_1 . The maximal value, $S \sim 0.15$, was obtained for $w/a_1 \geq 10^{-2}$. It is interesting to note that such a strip hides any imperfection along the vertex of the bi-prism providing a significant technological advantage in relaxing the fabrication tolerances.

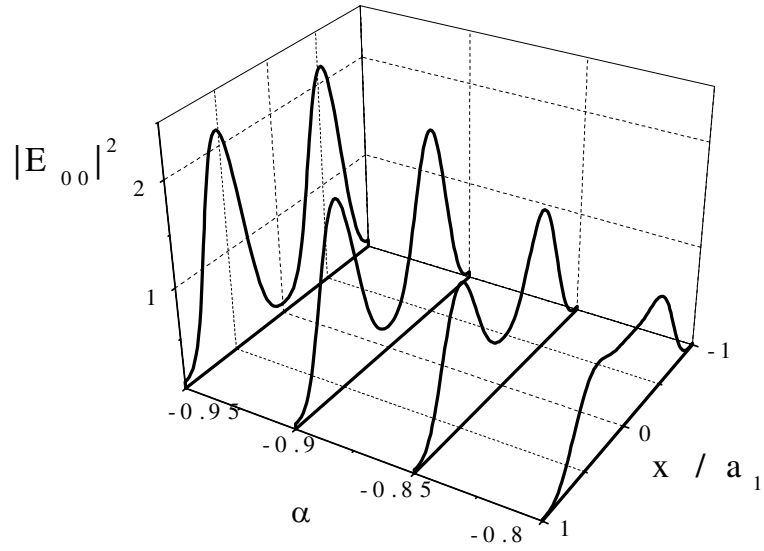


Fig.5. Dependence of the TEM_{00} mode shape on the angle α in the semi-concentric resonator.

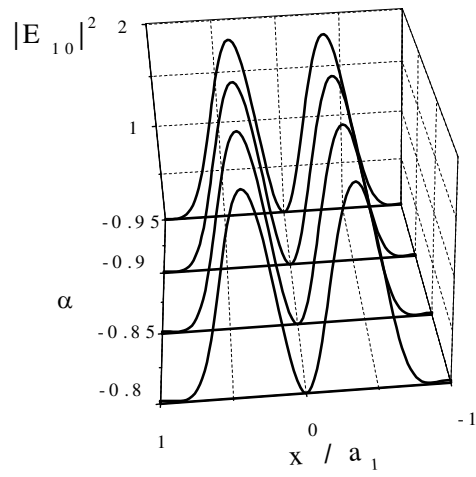


Fig.6. Dependence of the TEM_{10} mode shape on the angle α in the semi-concentric resonator.

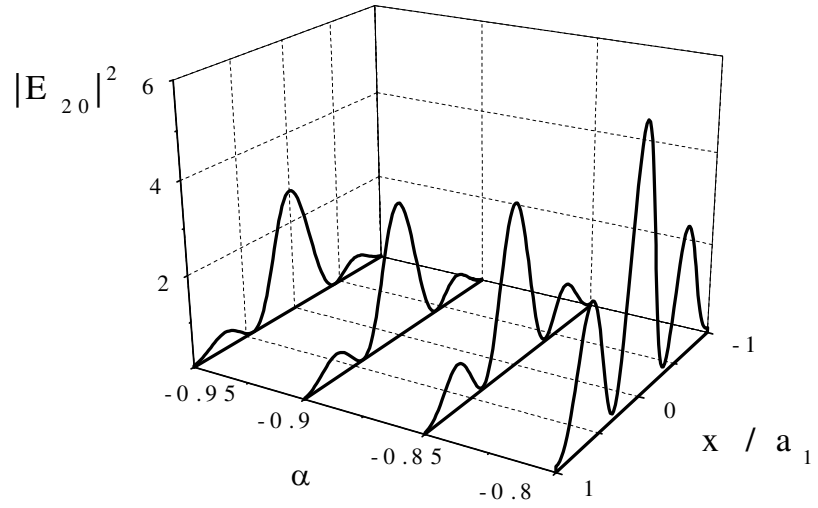


Fig.7. Dependence of the TEM_{20} mode shape on the angle α in the semi-concentric resonator.

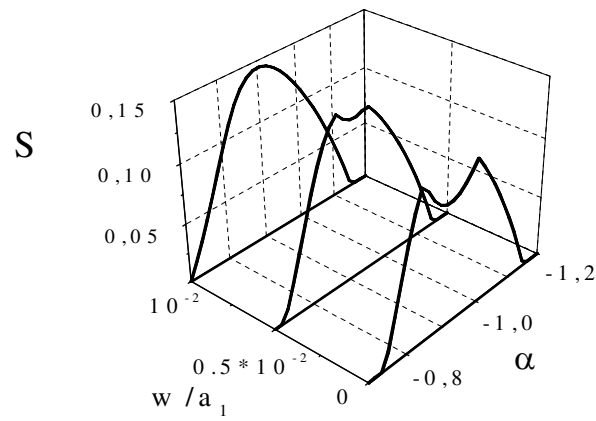


Fig.8. Dependence of S on the relative half-width of the strip, w/a_1 , and α .

The above results represent new solutions that have the following physical interpretation. Each face of the BPE together with the concentric mirror creates a partial resonator with aperture $b < 1.25$. The field, which diffracts toward the second face of the BPE is deflected into the direction of the second partial resonator creating a diffraction coupling between the two partial resonators.

As the center of curvature of the concentric mirror is located on the bi-prism center and the distance between the mirrors is much larger than their width, the configuration of each partial resonator practically does not depend on α . As a consequence, a change in the BPE angle affects only the phase relation between fields coupled from the two partial resonator modes. It should be reiterated that these considerations hold only for configurations close to the semi-concentric one but they quickly fail in the instability region (see Fig. 2a). In the stability region this structure is maintained in a wider range of G (see Fig. 3c), but the extreme value of S is gradually decreased.

4. CONCLUSION

This paper extends earlier work dedicated to laser mode selection by replacing one of the laser mirrors by a BPE with its vertex masked by a narrow strip. The main new result of this work is that our present goal of high-efficiency oscillation on the first odd laser mode can be achieved more effectively if the laser resonator is designed around configuration parameters outside the regime of traditional lasers that are designed to preferentially oscillate on the fundamental mode. Focusing on the limiting configuration of semi-concentric resonator, a mode selectivity of around 0.15 was obtained, which is sufficient for pure oscillation on the first odd mode for the majority of laser types.

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