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## Active Polarization Descattering

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#### Abstract

When imaging in scattering media, there is poor visibility which hiders both human assisted operations and computer vision. Most computer vision methods face significant difficulties if employed directly underwater. This is due to the particularly challenging environmental conditions, which complicate image matching and analysis. The problem is even more severe when using artificial illumination- strong backscatter veils the object signal. In this work we analyze image formation under wide-band wide-field artificial illumination. We suggest a visibility recovery approach. Our approach first estimates the backscatter component. Based on that component, it estimates a rough 3D scene structure. The method is simple and requires compact hardware, using active wide field polarized illumination. Two images of the scene are instantly taken, with different states of a camera-mounted polarizer. A recovery algorithm then follows. We demonstrate the approach in underwater field experiments and analyze limits concerns to acquisition noise.

### 1 Introduction

Scattering media exist in bad weather, liquids and biological tissue. Images taken in scattering media characterize in from poor visibility and loss of contrast. Light passing through undergoes absorption and scattering, causing changes in color and brightness. Moreover, light that is scattered back from the medium along the light of sight (*backscatter*) veils the object, degrading visibility and contrast. There-fore, applying traditional computer vision methods in such environments is difficult. Nevertheless, there is a strong need to perform vision tasks in these media. Examples include vision through biological tissues[12], underwater applications like port construction and inspections, measuring ecological systems, etc. [11], and navigation in bad weather [1].

Previous works tackled this challenge in various ways. Some recovered visibility as well as the three dimensional (3D) structure of underwater sites [33] under distant natural illumination. However, application fields operating in highly turbid media use *artificial illumination* sources at *short* distances, be it underwater or in the human body. However, artificial lighting usually causes a strong *backscatter*. Backscatter can be modulated and then compensated for in image post-processing. Such current methods require acquisition of long image sequences by structured light [17, 19, 27] or time-gating [5,

7, 10, 38, 40]. Ref. [28] required many frames as well, to achieve quality results. Such sequences may lengthen the overall acquisition time. Moreover, such systems are very complex and expensive.

To counter these problems, we look at widefield (not scanning) illumination with a small (or no) baseline, where the backscatter is modulated by polarization. Preliminary studies [8, 9, 20] indicated that backscatter can be reduced by polarization. However, we go further. By post-processing we remove residual backscatter that is not blocked by optical means. Moreover, a rough estimate of the 3D scene structure may be obtained from the acquired frames. The acquisition setup is a simple modification of instruments used routinely in such media: simply mounting two polarizers, one on the light source and another on the camera. The acquisition process is instantaneous, i.e., requiring only two frames, rather than scanning. In this paper, we describe and demonstrate each step separately.

The approach is based on several insights into the image formation process. We show that backscatter and attenuation of artificial illumination can be well approximated by simple closed-form parametric expressions. To incorporate polarization, we have performed empirical polarization measurements in real underwater scenes: in a temperate latitude sea (Mediterranean), a tropical sea (the Red Sea), in a murky lake (Sea of Galilee) and a swimming pool.

This paper first describes the scientific model of the imaging system, and sets the grounds for polarization imaging. The reconstruction is done in two steps: first, we recover the object signal. Then, we estimate the scene structure. Results follow each step. We conclude by analyzing the limits of our method related to imaging noise.

### 2 Statement of the Problem

Consider a perspective underwater camera (Fig. 1). Let  $\mathbf{X} = (X, Y, Z)$  be the world coordinates of a point in the water. We set the world system's axes (X, Y) to be parallel to the (x, y) coordinates at the image plane, while Z aligns with the camera's optical axis, and the system's origin is at the camera's center of projection. The projection of  $\mathbf{X}$  on the image plane is  $\mathbf{x} = (x, y)$ . In particular, an object point at  $\mathbf{X}_{obj}$  corresponds to an image point  $\mathbf{x}_{obj}$ . The line of sight (LOS) to the object is

$$LOS \equiv \{ \mathbf{X} : Z \in [0, Z_{obj}] , X = (Z/f) x_{obj} , Y = (Z/f) y_{obj} \} , \qquad (1)$$

where f is the focal length of the camera. The measured image is

$$I(\mathbf{x}_{\rm obj}) = S(\mathbf{x}_{\rm obj}) + B(\mathbf{x}_{\rm obj}) \quad , \tag{2}$$

where  $S(\mathbf{x}_{obj})$  is the object signal and  $B(\mathbf{x}_{obj})$  is the backscatter [13, 22, 24]. Before detailing these components, note that backscatter is the major cause of contrast deterioration [14], rather than signal blur. This was demonstrated in [33] using objective criteria. Interestingly, according to Ref. [43],



Figure 1: A camera inside a dome port with a radius r. The variables are detailed in the text.



Figure 2: Simulation of an underwater scene. The scene was assigned a linearly varying distance map ranging between [0.2m, 1m]. (a) A uniformly lit clear scene. (b) The simulated attenuated signal. (c) The backscatter component. (d) The sensed underwater scene, accounting for both scattering and attenuation.

human vision associates image quality mostly with contrast, rather than resolution. For these reasons, we do not focus here on image blur or deblurring. Rather, we consider the prime effects associated with turbidity to be backscatter and attenuation. Fig. 2 demonstrates these effects.

Define  $L_{obj}(\mathbf{x}_{obj})$  as the object radiance we would have sensed had no disturbances been caused by the medium along the LOS, and under uniform illumination. Propagation of light to the object and then to the camera via the medium yields an attenuated [13, 22] signal

$$S(\mathbf{x}_{\rm obj}) = L_{\rm obj}(\mathbf{x}_{\rm obj})F(\mathbf{x}_{\rm obj}) \quad , \tag{3}$$

where F is a falloff function described below.

A point **X** in the water is at total distance  $||\mathbf{X}||$  from the camera. If the camera is enclosed in a dome port as in [33], then the distance from the dome to **X** is

$$R_{\rm cam}(\mathbf{X}) = \|\mathbf{X}\| - r \quad , \tag{4}$$

where r is the dome's radius. Consider for the moment a single illumination point source. From

this source, light propagates a distance  $R_{\text{source}}$  to  $\mathbf{X}_{\text{obj}}$ . Free space propagation creates a  $1/R_{\text{source}}^2$  irradiance falloff. Yet, there is turbidity, characterized by an attenuation coefficient c. Hence

$$F(\mathbf{x}_{\rm obj}) = \frac{\exp\left\{-c\left[R_{\rm source}(\mathbf{X}_{\rm obj}) + \|\mathbf{X}_{\rm obj}\| - r\right]\right\}}{R_{\rm source}^2(\mathbf{X}_{\rm obj})}Q(\mathbf{X}_{\rm obj}) .$$
(5)

Here  $Q(\mathbf{X})$  expresses the non-uniformity of the scene irradiance, solely due to the inhomogeneity of the illumination. It similarly exists if the water is clear, i.e., c = 0, and can thus be pre-calibrated in clear water. For multiple illumination sources, or for a wide spread source Eq. (5) is derived for each point source, and then all F's are summed up. This can be generalized to include illumination due to multiple scattering [39].

In order to calculate the backscatter that appears in Eq. (2), define first  $I^{\text{source}}$  as the irradiance of a point in the volume [13] by a small illumination source of radiance  $L^{\text{source}}$ :

$$I^{\text{source}}(\mathbf{X}) = L^{\text{source}}\left[1/R_{\text{source}}^2(\mathbf{X})\right] \exp[-cR_{\text{source}}(\mathbf{X})]Q(\mathbf{X}) \quad .$$
(6)

Then, the backscatter is given [13, 41] by integration along the LOS

$$B(\mathbf{x}_{\rm obj}) = \int_{R_{\rm cam}=0}^{R_{\rm cam}(\mathbf{X}_{\rm obj})} b[\theta(\mathbf{X})] I^{\rm source}(\mathbf{X}) \exp[-cR_{\rm cam}(\mathbf{X})] dR_{\rm cam} , \ \mathbf{X} \in \text{LOS}$$
(7)

where  $\theta \in [0, \pi]$  is the scattering angle, and b is the scattering coefficient<sup>1</sup> of the medium: it expresses the ability of an infinitesimal medium volume to scatter flux to  $\theta$ . Eq. (7) applies to each illumination source: accumulating the results yields the total backscatter. Note that the integration in Eq. (7) stops when it reaches the object in the LOS. Therefore, the backscatter accumulates (increases) with the object distance. If there is no object on the LOS, the integration in Eq. (7) continues to an infinite distance. The value of B then increases until it reaches a *saturation value*. We term the distance in which B effectively saturates as the *saturation distance*  $z_{sat}$ .

Our goal in this research is two-fold: first, to estimate the backscatter component, in order to remove it from the raw image and reveal the object signal. Second, to study the potential use of the backscatter component for extracting information about the distance map of the scene. Sec. 3 describes how we achieve the first goal by polarizing the light source.

### 3 Polarization Imaging

As mentioned earlier, we suggest modulating the light by polarizing the light source and imaging through a camera-mounted polarizer (*analyzer*) in two orthogonal polarization states. The system setup is depicted in Figs. 1 and 3. By mounting a polarizer (either linear or circular) on the light source, we polarize the illumination. The polarized light propagates to illuminate the scene and part

<sup>&</sup>lt;sup>1</sup>Note that  $b(\theta)$  and c depend on the wavelength. Thus each available wavelength band is analyzed independently.



Figure 3: A single-lamphead version of our system.

of it is scattered back by particles in the medium towards the camera. During this propagation, some energy of the light becomes unpolarized (a process termed *depolarization*). This process is complex and depends on the distribution of particle types and sizes [15, 21, 31]. Apparently, this process affects each polarization type differently: some studies suggest that depolarization during propagation is weaker in circular polarization [15, 21, 25, 31], while Refs. [15, 31] suggest weaker depolarization of linear polarization in dense tissues. An empirical study [37] has looked at the the rate of depolarization with distance in seawater. A preliminary empirical study [9] done decades ago has shown that if the illumination is circularly polarized, then it flips handedness upon backscattering. Thus, Ref. [9] achieved significant improvement in image contrast in an optical method, where it used an analyzer having the same handedness as the illumination polarizer.

That said, despite the scientific efforts that have been invested by various researchers (see for example [29]). The known art has not supplied a clear answer as to which polarization type is preferable in the true environments we worked in, and how the depolarization rate can be determined by the scattering and attenuation coefficients in those environments. Therefore, we tested our method with either linear or circular polarization in different locations. In the case of linear polarization we mount a linear polarizer on the light source and a linear analyzer on the camera. Then, an orthogonal image pair is taken by either rotating the polarizer or the analyzer. Specifically, we chose to rotate the analyzer, as it was easier in our setup. When using circular polarization, orthogonal states result from switching handedness rather than rotating the polarizers. As a consequence, linear polarization is easier to use. Moreover, wideband and widefield circular polarization is difficult to create. In any case, raw polarization data still contains significant backscatter. Therefore, there is a need for post processing, as we describe in Sec. 4. The post processing we perform does not depend on the polarization type used.

### 4 Backscatter Removal by Polarization

This section describes and demonstrates through experiments visibility enhancement by active polarization imaging. This is done by separating the signal and the backscatter components. Later, in Sec. 5.1, we explain how the estimated backscatter may be used for estimating the 3D structure of the scene.

#### 4.1 Model and Algorithm

Former studies have used polarized illumination for backscatter removal. Ref. [41] assumed that objects back-reflect unpolarized light to the camera. On the other hand, studies using polarization difference imaging (PDI) assume the contrary- that the light reflected from the objects is polarized and that the backscatter is almost unpolarized. Here we give a more general model. Fortunately, if the object yields polarized specular reflection, it behaves similarly to the backscatter: out of the two frames, generally, the one in which the backscatter is brighter is also the one in which the object back-reflection is brighter.<sup>2</sup>

As described in Sec. 3, we take two images of the same scene using two orthogonal polarization states of the polarizer. Had the backscattered light completely retained its polarization, it could have been optically eliminated by the analyzer. We discovered that a substantial degree of polarization (DOP) is maintained upon backscattering. We exploit this phenomenon.<sup>3</sup> Consequently, placing an analyzer in the orthogonal state to the backscatter's polarization state yields an image with minimum visible backscatter. We denote this image as  $I^{\min}$ . Imaging with the opposite orthogonal state, denoted  $I^{\max}$ , maximizes the backscatter.

We expand Eq. (2) to the polarized components  $B_{\text{max}}, B_{\text{min}}, S_{\text{max}}, S_{\text{min}}$ . Thus, the raw images are:

$$I_{\max}(\mathbf{x}_{obj}) = S_{\max}(\mathbf{x}_{obj}) + B_{\max}(\mathbf{x}_{obj}) \quad , \quad I_{\min}(\mathbf{x}_{obj}) = S_{\min}(\mathbf{x}_{obj}) + B_{\min}(\mathbf{x}_{obj}) \quad . \tag{8}$$

The DOP of the signal  $p_{obj}$  and the DOP of the backscatter  $p_{scat}$  are defined as:

$$p_{\rm obj}(\mathbf{x}_{\rm obj}) = \frac{S_{\rm max}(\mathbf{x}_{\rm obj}) - S_{\rm min}(\mathbf{x}_{\rm obj})}{S_{\rm max}(\mathbf{x}_{\rm obj}) + S_{\rm min}(\mathbf{x}_{\rm obj})}, \qquad p_{\rm scat}(\mathbf{x}_{\rm obj}) = \frac{B_{\rm max}(\mathbf{x}_{\rm obj}) - B_{\rm min}(\mathbf{x}_{\rm obj})}{B_{\rm max}(\mathbf{x}_{\rm obj}) + B_{\rm min}(\mathbf{x}_{\rm obj})}$$
(9)

In the following  $(\mathbf{x}_{obj})$  is omitted for simplicity. We end up with two equations for the two unknown fields - S and B:

$$\mathbf{I}_{\max} + \mathbf{I}_{\min} = \mathbf{B} + \mathbf{S} \tag{10}$$

$$\mathbf{I}_{\max} - \mathbf{I}_{\min} = p_{\text{scat}} \mathbf{B} + p_{\text{obj}} \mathbf{S} \quad . \tag{11}$$

<sup>&</sup>lt;sup>2</sup>Empirically, we never encountered a reversed polarization of the signal relative to the backscatter.

<sup>&</sup>lt;sup>3</sup>Polarization has also aided other computer vision aspects [2, 4, 6, 23, 34, 44].

The last equation is derived from plugging Eq. (9) into Eq. (8). The solution to this equation set is:

$$\hat{\mathbf{S}} = \frac{1}{p_{\text{scat}} - p_{\text{obj}}} [\mathbf{I}_{\min}(1 + p_{\text{scat}}) - \mathbf{I}_{\max}(1 - p_{\text{scat}})]$$
(12)

$$\hat{B} = \frac{1}{p_{\text{scat}} - p_{\text{obj}}} [\mathbf{I}_{\text{max}}(1 - p_{\text{obj}}) - \mathbf{I}_{\text{min}}(1 + p_{\text{obj}})] \quad .$$
(13)

This is a general result, enabling separation of **B** and **S** from the two raw images, given the DOPs  $p_{obj}$  and  $p_{scat}$ .

A very important property of Eq. (12) is that  $p_{obj}$  contributes only a scale factor to the signal reconstruction  $\hat{\mathbf{S}}$ . Suppose that  $p_{obj}$  is approximately constant across the scene, but it is unknown. Then, the signal estimation (12) is consistent with the true  $\mathbf{S}$  up to a scale. For purposes of visibility enhancement, the scaled  $\hat{\mathbf{S}}$  is sufficient: the backscatter is removed, and missing parts are revealed. Furthermore, the backscatter is usually not uniform across the image; some regions have high intensity backscatter, and others have low intensity backscatter (see Fig. 2). This hampers standard image enhancement techniques. Therefore, removing separating the backscatter results in a signal estimation  $\hat{\mathbf{S}}$  with a more uniform intensity. Thus, further image improvement may be obtained by applying standard image enhancement techniques to  $\hat{\mathbf{S}}$ , rather than applying them to  $\mathbf{I}$  or  $\mathbf{I}_{min}$ .

As  $p_{obj}$  changes only the scale of  $\hat{\mathbf{S}}$ , good results can be achieved [33, 35, 41] based on the assumption that  $p_{obj} = 0$ . In this case, Eqs. (12,13) reduce to:

$$\hat{\mathbf{S}} = [\mathbf{I}_{\min}(1 + p_{\text{scat}}) - \mathbf{I}_{\max}(1 - p_{\text{scat}})]/p_{\text{scat}}$$
(14)

$$\mathbf{B}(x,y) = (\mathbf{I}_{\max} - \mathbf{I}_{\min})/p_{\text{scat}} \quad . \tag{15}$$

Note, that in this case,

$$\mathbf{I}_{\min} = [\mathbf{B}(1 - p_{\text{scat}}) + \mathbf{S}]/2 , \quad \mathbf{I}_{\max} = [\mathbf{B}(1 + p_{\text{scat}}) + \mathbf{S}]/2 .$$
 (16)

Now, let us examine what is the consequence of using the assumption  $p_{obj} = 0$  in Eq. (13), when image creation (Eq. 8) experienced  $p_{obj} \neq 0$ . This case yields a false estimation of the backscatter,  $\tilde{\mathbf{B}}$ :

$$\tilde{\mathbf{B}} = \frac{\mathbf{I}_{\max} - \mathbf{I}_{\min}}{p_{\text{scat}}} = \hat{\mathbf{B}} + \frac{\mathbf{S}_{\max} - \mathbf{S}_{\min}}{p_{\text{scat}}} = \hat{\mathbf{B}} + \frac{p_{\text{obj}}}{p_{\text{scat}}} \mathbf{S} \quad .$$
(17)

The last equality results from plugging in the DOP  $p_{obj}$  from Eq. (9). As discussed in Sec. 2, **B** increases with the distance. From Eq. (5), when the camera and the light sources are on the same side of the object (a common scenario), S decreases with the distance. In that case, a result of Eq. (17) is that  $\tilde{\mathbf{B}}$  is no longer monotonic with  $Z_{obj}$ .

As opposed to the assumption  $p_{obj} = 0$ , methods based on PDI [42] assume that  $p_{scat}/p_{obj} \rightarrow 0$ . Plugging  $p_{scat}/p_{obj} \rightarrow 0$  to Eqs. (12,13) results in:

$$\hat{\mathbf{S}} = \frac{1}{p_{\rm obj}} [\mathbf{I}_{\rm max} - \mathbf{I}_{\rm min}] \quad , \tag{18}$$

$$\hat{\mathbf{B}} = \frac{1}{p_{\rm obj}} [\mathbf{I}_{\rm min}(1+p_{\rm obj}) - \mathbf{I}_{\rm max}(1-p_{\rm obj})] \quad .$$
(19)

Note that in this case, Eq. (18) is a scaled version of the polarization difference image. Here we see that Eqs. (12,13) unify both the dehazing methods [33, 35, 41], in which  $p_{obj} = 0$ , and the PDI methods where  $p_{scat}/p_{obj} \rightarrow 0$ .

Using Eqs. (12,13) without such approximations requires the estimation of the DOPs. Sec. 7 describes how the DOPs are estimated in the general case. First, however, we demonstrate backscatter removal in experiments.

#### 4.2 Experiments

The method described above is general and it does not assume a specific medium. However, as discussed in Sec. 3, depolarization depends on the medium [18]. Therefore, in order to demonstrate the effectiveness of the method in real world situations, we embarked on underwater dives rather than using indoor tanks. Particles in substances (like milk, lipids, etc.) used for diluting water in indoor tanks are usually homogeneous and sometimes symmetric [15, 30] while oceanic particles are heterogeneous [24]. Therefore, we were concerned that polarization experiments done with diluted substances would not represent correctly the properties and the variety of the media in the field, e.g., seawater. We have done experiments while scuba diving at night in various environments, in a pool, the Red Sea, the Mediterranean and the Sea of Galilee (Fig. 4).

#### 4.3 Equipment

The system we used is shown in Fig. 3. It consists of two main parts:

- An SLR camera with an underwater housing. We use a Nikon D100 camera, which has a linear response [33]. The camera is placed in a Sealux underwater housing with a mounted polarizer. The considerations for choosing a camera, an underwater housing and mounted polarizers are detailed in [33].
- Underwater AquaVideo light sources, with 80W Halogen bulbs. A polarizer is mounted on the lighthead. We had special consideration behind the selection of the lighting setup, as detailed in [41].

We used standard off-the-shelf polarizers of Schneider and Tiffen. The camera was mounted on a tripod. To safely transport this amount of equipment while diving, we used a 50kg lift-bag (Fig. 4). The tripod was set to resist swell by attaching weights on its lower part.

#### 4.4 Real World Results

Fig. 5 shows the results of applying Eqs. (14,15) on images taken during four different experiments we have performed. We tested the method using different light source locations. The left column presents



Figure 4: [Left] Scuba diving with a lift-bag, towards night experiments in the Red Sea. [Right] Preparations for an experiment in the Sea of Galilee.

the raw images I. The center column shows  $\hat{\mathbf{S}}$  (where the estimated backscatter is removed). The right column shows the estimated backscatter component  $\ddot{\mathbf{B}}$ . The experiments in the three top rows were performed in the Mediterranean in three different occasions. In all these three cases, using linear polarizers have yielded a DOP of  $p_{\text{scat}} \approx 65\%$ . In experiment 1 we used two light sources, shining from above and below the camera. Here,  $Z_{obj} < 3m$ . Notice the revealed rock in the upper left part, the sand in the right side, the rocks on the bottom and the distant part of the tube. In experiment 2,  $Z_{\rm obj} \in [0.5m, 6m]$ . Here, we used a single light source, coming from the top right. Notice the revealed rectangle rock in the background. The revealed objects in the background are dark, as at this distance they receive only dim irradiance from the sources. **Experiment 3** shows a scene illuminated from the bottom right. As a consequence, the lower parts have a lot of backscatter, hence poor visibility. Our method enhanced the visibility in this part. **Experiment 4** shows a result of an experiment done in the Sea of Galilee, a very murky lake. The light source is placed above the camera. Here,  $Z_{\rm obj} \approx 0.5m$ , which was the maximum visibility distance. Here, circular polarization yielded  $p_{\text{scat}} \approx 9\%$  while linear polarization yielded  $p_{\text{scat}} \approx 5\%$ . Despite the difficult conditions, the method revealed the imaged object, its rough contour and its colors. Notice that in both experiments 2 and 4, the upper part of the raw frame is very bright, due to backscatter. This may cause the viewer to falsely assume there is a bright object in that part of the scene. After removing the backscatter, these areas become dark, as there is actually no light reflecting from objects there. After removing the backscatter, we expect the scene radiance to act according to Eq. (5). Indeed, in experiment 2, the brightest part of  $\hat{\mathbf{S}}$  is the lower, close sand.

In the field experiments we did, both types of polarization (linear and circular) yielded good results. When visibility was moderate (in the Mediterranean), linear polarization retained  $p_{\text{scat}} \approx 60\% - 70\%$ , higher than circular polarization, for which  $p_{\text{scat}} \approx 50\%$ . In the murky Sea of Galilee, on the other hand, circular DOP was higher than the linear one. There, hardly any perceptual difference existed between the raw frames, due to the low DOP value. Nevertheless, our method still enhanced  $\hat{\mathbf{S}}$  significantly.

#### The Mediterranean



Figure 5: Results of four different experiments. [Left] The raw images I. [Middle] The recovered signals  $\hat{\mathbf{S}}$ . [Right] The estimated backscatter field  $\hat{\mathbf{B}}$ .

## 5 Range and Falloff

#### 5.1 Range

Having an estimation for the backscatter  $\hat{\mathbf{B}}$  in a scene, we now wish to know if we may leverage it to estimate the 3D structure of the scene. We now present a general approach for exploiting  $\hat{\mathbf{B}}$  to estimate  $Z_{\text{obj}}$ . It does not depend on the algorithm used for extracting  $\hat{\mathbf{B}}$  itself.

Similarly to [3, 26, 33, 35], the backscatter *B* increases with the distance  $Z_{obj}$ , hence it can indicate the distance. Previously [33, 35], this principle was developed in the simple special case of distant



Figure 6: The backscatter given by Eq. (7) as approximated by Eq. (21), with  $c = 0.1m^{-1}$ .

illumination sources (natural light), where the following relation holds:

$$B = B_{\infty} \{ 1 - \exp[-cR_{\text{cam}}(x, y, Z_{\text{obj}})] \} \approx B_{\infty} \{ 1 - \exp[-cZ_{\text{obj}}(x, y)] \} \quad .$$
(20)

Such an estimation can be generalized to the use of sources close to the camera. We found numerically [41] that in widefield lighting, Eq. (7) can be approximated as

$$B(\mathbf{x}_{\rm obj}) \approx B_{\infty}(\mathbf{x}_{\rm obj}) \left(1 - \exp\left\{-k(\mathbf{x}_{\rm obj})[Z_{\rm obj}(\mathbf{x}_{\rm obj}) - Z_0(\mathbf{x}_{\rm obj})]\right\}\right) , \qquad (21)$$

resembling Eq. (20). Fig. 6 presents an approximation done for a particular setup. A major difference between Eqs. (20) and (21) is that in Eq. (21)  $B_{\infty}$  is space variant. Eq. (21) introduces two new space-variant parameters,  $Z_0(\mathbf{x}_{obj})$  and  $k(\mathbf{x}_{obj})$ . These parameters  $(B_{\infty}, Z_0, k)$  depend on the lighting geometry, the non-uniformity (anisotropy)  $Q(\mathbf{X}_{obj})$  of the illumination sources and on the medium parameters c and b (described in Sec. 2). They do *not* depend on  $Z_{obj}$ .

Eq. (21) is easy to invert, deriving an estimate  $\hat{Z}_{obj}(\mathbf{x}_{obj})$  as a function of  $\hat{B}(\mathbf{x}_{obj})$ :

$$\hat{Z}_{\rm obj}(\mathbf{x}_{\rm obj}) = Z_0(\mathbf{x}_{\rm obj}) - \left[\ln\left(1 - \frac{\hat{B}(\mathbf{x}_{\rm obj})}{B_{\infty}(\mathbf{x}_{\rm obj})}\right)\right] \frac{1}{k(\mathbf{x}_{\rm obj})}$$
(22)

This, of course, requires calibration of the spatially varying parameter fields  $\mathbf{B}_{\infty}$ ,  $\mathbf{k}$  and  $\mathbf{Z}_0$ . An important parameter is  $B_{\infty}$ . It expresses the backscatter at  $\mathbf{x}_{obj}$ , had there been no object in the LOS. Therefore, the relation

$$B_{\rm rel}(\mathbf{x}_{\rm obj}) = \frac{\hat{B}(\mathbf{x}_{\rm obj})}{B_{\infty}(\mathbf{x}_{\rm obj})} \quad , \tag{23}$$

indicates how much the backscatter has reached its saturation value  $B_{\infty}$ . Thus,  $B_{\text{rel}}$  is monotonic with  $Z_{\text{obj}}$ . The parameters k and  $Z_0$  function as scaling factors in Eq. (22). It is easy [41] to determine the field  $\mathbf{B}_{\infty}$  by taking a photograph in the medium, where the camera is pointing "no-where" (to infinity). By approximating that  $\mathbf{k}$  and  $\mathbf{Z}_0$  to be uniform and plugging in typical values for them in Eq. (22), a rough distance map can be estimated.

We simulated similar setups to those we used in our experiments. To simplify the analysis, let us assume that the backscatter coefficient  $b(\theta)$  is uniform in the range of angles we use. This assumption is



Figure 7: Estimation of a distance map. [Top] The different components of Eqs. (21,23) in an underwater experiment. The image  $\mathbf{B}_{rel}$  is scaled to yield an estimation of the distance map. [Bottom] Views from different elevations of the reconstructed 3D scene composed of the recovered signal and the estimated distance map.

supported by [18], which shows that in oceanic water the function  $b(\theta)$  is insensitive to  $\theta$  at backscatter angles ( $\theta \ge \pi/2$ ). Fig. 7 shows a distance map we derived by applying Eqs. (22,23) on an underwater scene. For Eq. (22) we used the values  $Z_0 = 20cm$  and k = 0.6. Those values were chosen based on a numerical analysis of setups where the light source was in proximity to the camera. This analysis showed that  $Z_0$  ranges between 10cm - 30cm and k ranges between 0.4 - 5.

#### 5.2 Falloff

Sec. 5.1 described the estimation of  $\hat{Z}_{obj}(\mathbf{x}_{obj})$ . Based on  $\hat{Z}_{obj}(\mathbf{x}_{obj})$ , we may now estimate the falloff, using Eq. (5). Here we need three additional parameters. First is the attenuation coefficient c, which can be measured by a transmissiometer. Second, we need  $Q(\mathbf{X}_{obj})$ . This can be pre-calibrated once per light source. In addition, there is a need to know  $R_{\text{source}}$ . It is derived based on *a-priori* knowledge about the system baseline [39]: it is sufficient to know the camera-light-source baseline  $R_{\text{sc}}$ , and the angle between this source and the LOS,  $\gamma$  (See Fig. 1). Then,

$$R_{\rm source} = \sqrt{R_{\rm sc}^2 + R_{\rm cam}^2 - 2R_{\rm cam}R_{\rm sc}\cos\gamma} \quad . \tag{24}$$

The value of  $\hat{R}_{\text{cam}}$  is estimated by setting  $z = \hat{Z}_{\text{obj}}$  in Eq. (4). Then Eq. (24) derives  $\hat{R}_{\text{source}}$ . The use of  $\hat{Z}_{\text{obj}}$  and  $\hat{R}_{\text{source}}$  in Eq. (5), derives an estimate for the falloff  $\hat{F}(\mathbf{x}_{\text{obj}})$ . Compensating for the falloff by inverting Eq. (3) yields

$$\hat{L}_{\text{object}}(\mathbf{x}_{\text{obj}}) = \hat{S}(\mathbf{x}_{\text{obj}}) / \hat{F}(\mathbf{x}_{\text{obj}}) \quad .$$
(25)

To illustrate this, Fig. 8 shows a simulation of the entire recovery method. A simulated object was assigned a non-trivial distance map and artificial noise was added with standard deviation of



Figure 8: Simulated backscatter removal, 3D recovery and falloff compensation of a noisy object. (a) An object was assigned a distance map varying linearly to 1m with a sticking rectangle at a distance of 0.3m. (b) The simulated underwater raw frame **I**, with added noise. (c) The estimated distance map  $\hat{\mathbf{Z}}_{obj}$ . (d) The recovered object radiance  $\hat{\mathbf{L}}_{object}$ . In (c) and (d) the noise is amplified in the distant parts.



Figure 9: The relative backscatter  $\mathbf{B}_{rel}$  as a function of the object distance. The values for *b* and *c* are taken from [24]. The backscatter saturates within a range of 1.5*m*. Moreover, the saturation distance  $z_{sat}$  is similar in all three different water types.

 $\sigma_{I_{\min}} = \sigma_{I_{\max}} = 1$  grey level (out of 256 gray levels in the raw frames  $\mathbf{I}_{\min}$ ,  $\mathbf{I}_{\max}$ ). Fig. 8d shows  $\hat{L}_{object}(\mathbf{x}_{obj})$  after both removal of the estimated backscatter and falloff compensation. While the image is enhanced relative to the simulated I, there is noise amplification in the distant parts [16, 32].

### 6 Effectiveness Under Noise

An important question to ask is how distant can objects be, and still be recovered? Even in a nonscattering medium, widefield illumination is limited by the free-space falloff term  $1/R_{source}^2$ . This poses an inherent limit on all approaches that use widefield illumination. Objects at long distances, which are not lit effectively, cannot be reconstructed. Moreover, no imaging system is free of noise. As a consequence, when the signal is in the order of the noise, reconstruction is limited. For example, in our system, the recorded intensity of objects further than 6-7[m] was too low to be recovered by removing the backscatter component. As for distance recovery, a major concern is the resolution of the function  $\hat{B}(Z_{obj})$ . The function in Eq. (21) is approximately linear at short distances, yielding good distance resolution. However, very quickly Eq. (21) saturates, thus losing the capacity for proper recovery. Again, when the resolution is in the magnitude of the noise, the reconstruction may become fruitless. What are the typical saturation distances? Fig. 9 depicts  $\mathbf{B}_{rel}$  as a function of the object distance. It is a result of simulations based on three classes of values for b and c, taken from [24], which are typical to seawater at different environments. The light source was placed 15cm above and to the left of the camera. We can see that  $z_{sat}$  does not vary much with the water properties. In any case, after  $\approx 1.5[m]$  the backscatter is already saturated and is thus uninformative with respect to  $Z_{obj}$ . Therefore, exact distance reconstruction based on backscatter is limited to the close distances. Sections 6.1 and 6.2 analyze the limits as a function of various medium and imaging parameters.

### **6.1** $\hat{S}$ and $\hat{B}$

Suppose we have two statistically independent intensity measurements,  $I_{\text{max}}$  and  $I_{\text{min}}$  with noise variances  $\sigma_{I_{\text{max}}}$  and  $\sigma_{I_{\text{min}}}$  respectively. Let variable v be a function of  $I_{\text{max}}$  and  $I_{\text{min}}$ . Then, its noise variance is given by:

$$\sigma_v^2 = \left(\frac{\partial v}{\partial I_{\min}}\right)^2 \sigma_{I_{\min}}^2 + \left(\frac{\partial v}{\partial I_{\max}}\right)^2 \sigma_{I_{\max}}^2 \quad . \tag{26}$$

Using Eqs. (12,13), the noise variances in **S** and **B** are:

$$\sigma_{\hat{S}}^2 = \left(\frac{1+p_{\text{scat}}}{p_{\text{obj}} - p_{\text{scat}}}\right)^2 \sigma_{I_{\min}}^2 + \left(\frac{1-p_{\text{scat}}}{p_{\text{obj}} - p_{\text{scat}}}\right)^2 \sigma_{I_{\max}}^2 \tag{27}$$

$$\sigma_{\hat{B}}^2 = \left(\frac{1+p_{\rm obj}}{p_{\rm obj}-p_{\rm scat}}\right)^2 \sigma_{I_{\rm min}}^2 + \left(\frac{1-p_{\rm obj}}{p_{\rm obj}-p_{\rm scat}}\right)^2 \sigma_{I_{\rm max}}^2 \quad . \tag{28}$$

It is obvious that if  $p_{obj} \approx p_{scat}$ ,  $\{\sigma_{\hat{S}}, \sigma_{\hat{B}}\} \to \infty$ , hence the reconstruction is unstable. Thus, the method will work best if the medium and object differ significantly in their DOPs. Specifically, in a medium where  $p_{scat}$  is relatively high (usually in good visibility), the method is stronger with depolarizing objects. While in a strongly depolarizing medium (low  $p_{scat}$ ) the objects will be reconstructed better if they are polarizing. Note that in Eqs. (27,28), the noise component due to  $I_{min}$  is amplified more than that of  $I_{max}$ . For example, consider  $\sigma_{\hat{S}}^2$ . If  $p_{scat} = 0.5$ , then  $\sigma_{I_{min}}^2$  is amplified 9 times more than  $\sigma_{I_{max}}^2$ .

Let us look for a moment on a case when signal-independent noise dominates. Then,  $\sigma_{I_{\text{max}}} = \sigma_{I_{\text{min}}} = \sigma_0$ , and

$$\sigma_{\hat{S}}^2 = 2\sigma_0^2 \left[ \frac{1 + p_{\text{scat}}^2}{(p_{\text{obj}} - p_{\text{scat}})^2} \right] \quad , \quad \sigma_{\hat{B}}^2 = 2\sigma_0^2 \left[ \frac{1 + p_{\text{obj}}^2}{(p_{\text{obj}} - p_{\text{scat}})^2} \right] \quad . \tag{29}$$

Fig. 10 depicts  $\sigma_{\hat{S}}/\sigma_0$  and  $\sigma_{\hat{B}}/\sigma_0$  from Eq. (29). The cases  $[p_{\text{scat}}, p_{\text{obj}}] = [0, 1]$  and  $[p_{\text{scat}}, p_{\text{obj}}] = [1, 0]$  are two local minima. In other words, it is preferable that polarization of either the backscatter or



Figure 10: The noise standard deviations  $\sigma_{\hat{B}}$  and  $\sigma_{\hat{S}}$  as a function of  $\sigma_0$ ,  $p_{obj}$  and  $p_{scat}$ . The diagonal  $p_{obj} = p_{scat}$  is unstable and therefore cut for illustration.

the backreflection would be high and exclusive. In any case,  $\{\sigma_{\hat{B}}, \sigma_{\hat{S}}\} > 1$ , i.e., even in the best case scenario, the noise is amplified.

In reality,  $\sigma_{I_{\text{max}}} \neq \sigma_{I_{\text{min}}}$  due to imaging noise. Define  $g_{\text{electr}}$  as the number of photo-generated electrons required to change a unit gray-level. Following [36], the noise variance of a pixel gray level in an image I can be modeled as:

$$\sigma_I^2 = \rho^2 / g_{\text{electr}}^2 + Dt / g_{\text{electr}}^2 + \frac{I(\mathbf{X}_{\text{obj}})}{g_{\text{electr}}} \quad . \tag{30}$$

The electronic readout noise,  $\rho$ , is induced by electronic circuity in the camera system. The dark current noise Dt is related to the exposure time, t, and the detector dark current D. In Eq. 30, the first two terms are *signal-independent*. The third term is photon noise, which is *signal-dependent*. As in [36], we encompass the signal independent components as:

$$\kappa_{\rm gray}^2 = \rho^2 / g_{\rm electr}^2 + Dt / g_{\rm electr}^2 \quad . \tag{31}$$

assuming the same acquisition time for all frames. Plugging Eqs. (30,31) into Eq. (27,28) yields:

$$\sigma_{\hat{S}}^{2}(\mathbf{x}_{\rm obj}) = \left(\frac{1+p_{\rm scat}}{p_{\rm obj}-p_{\rm scat}}\right)^{2} \left[\kappa_{\rm gray}^{2} + \frac{I_{\rm min}(\mathbf{x}_{\rm obj})}{g_{\rm electr}}\right] + \left(\frac{1-p_{\rm scat}}{p_{\rm obj}-p_{\rm scat}}\right)^{2} \left[\kappa_{\rm gray}^{2} + \frac{I_{\rm max}(\mathbf{x}_{\rm obj})}{g_{\rm electr}}\right]$$
(32)

$$\sigma_{\hat{B}}^2(\mathbf{x}_{\rm obj}) = \left(\frac{1+p_{\rm obj}}{p_{\rm obj}-p_{\rm scat}}\right)^2 \left[\kappa_{\rm gray}^2 + \frac{I_{\rm min}(\mathbf{x}_{\rm obj})}{g_{\rm electr}}\right] + \left(\frac{1-p_{\rm obj}}{p_{\rm obj}-p_{\rm scat}}\right)^2 \left[\kappa_{\rm gray}^2 + \frac{I_{\rm max}(\mathbf{x}_{\rm obj})}{g_{\rm electr}}\right] \quad . \tag{33}$$

Let us look at the simple case of  $p_{obj} = 0$ . From Eq. (16), Eq. (32) then becomes,

$$\sigma_{\hat{S}}^2(\mathbf{x}_{\rm obj}) = \frac{1}{p_{\rm scat}^2} \left\{ \left[ 2\kappa_{\rm gray}^2 + \frac{S(\mathbf{x}_{\rm obj})}{g_{\rm electr}} \right] (1+p_{\rm scat}^2) + \frac{B(\mathbf{x}_{\rm obj})}{g_{\rm electr}} (1-p_{\rm scat}^2) \right\} \quad , \tag{34}$$

while Eq. (33) becomes,

$$\sigma_{\hat{B}}^2(\mathbf{x}_{\rm obj}) = \frac{1}{p_{\rm scat}^2} \left[ 2\kappa_{\rm gray}^2 + \frac{I(\mathbf{x}_{\rm obj})}{g_{\rm electr}} \right] \quad . \tag{35}$$

It is interesting to see that  $\sigma_{\hat{S}}^2$  depends also on the backscatter component  $\hat{\mathbf{B}}$ . Therefore, it is beneficial to reduce  $\hat{\mathbf{B}}$  during acquisition.



Figure 11: Standard deviations of the reconstruction noise of  $\hat{S}$  for different values of k. The effective distances are in the range of a few meters. The value of k is higher for pixels closer to the light source.

We can further use Eqs. (5,21) to calculate the dependency of  $\sigma_{\hat{S}}$  on  $Z_{obj}$ :

$$\sigma_{\hat{S}}^{2}(\mathbf{x}_{\rm obj}) = \frac{1}{p_{\rm scat}^{2}} \left[ \{ 2\kappa_{\rm gray}^{2} + \frac{L_{\rm obj}(\mathbf{x}_{\rm obj})e^{-2cZ_{\rm obj}}/Z_{\rm obj}^{2}}{g_{\rm electr}} \} (1+p_{\rm scat}^{2}) + \frac{B_{\infty}[1-e^{-k(Z_{\rm obj}-Z_{0})}]}{g_{\rm electr}} (1-p_{\rm scat}^{2}) \right]. (36)$$

To gain insight, in Fig. 11 we plot Eq. (36) in two cases. The first takes into consideration only signalindependent noise  $\kappa_{\text{gray}}$ . The second case accounts for all noise effects, using Eq. (30). Let  $B_{\infty} = 255$ , a saturated backscatter value. We chose a moderate DOP  $p_{\text{scat}} = 0.5$ , attenuation coefficient  $c = 0.2m^{-1}$ and  $Z_0 = 0.2m$ . For acquisition noise we use typical values from [36]:  $\kappa_{\text{gray}} = 0.4$ ,  $g_{\text{electr}} = 50$ . For the falloff calculation we assumed the simple case of collinear camera and light source, and a uniform light source.

As expected, taking into consideration photon noise, the effective distances shorten. There is a small difference between the effective distance for different values of k. The value of k changes between different illumination-camera setups. It also changes spatially in the image, pixels closer to the light source having a higher k. Therefore, for these pixels, the reconstruction will be noisier.

### 6.2 Noise in $\hat{Z}_{obj}$

The estimated  $\hat{\mathbf{B}}$  does not statistically depend on  $\mathbf{B}_{\infty}$ , as both are based on different measurements. Therefore, as for  $\hat{Z}_{obj}$ , in analogy to Eq. (26),

$$\sigma_{\hat{Z}_{\rm obj}}^2 = \left(\frac{\partial \hat{Z}_{\rm obj}}{\partial \hat{B}}\right)^2 \sigma_{\hat{B}}^2 + \left(\frac{\partial \hat{Z}_{\rm obj}}{\partial B_{\infty}}\right)^2 \sigma_{B_{\infty}}^2 \quad . \tag{37}$$

From Eq. (22):

$$\frac{\partial \hat{Z}_{\rm obj}}{\partial \hat{B}} = \left[\frac{1}{k(1-B_{\rm rel})}\right] \frac{1}{B_{\infty}} \quad , \quad \frac{\partial \hat{Z}_{\rm obj}}{\partial B_{\infty}} = \left[-\frac{B_{\rm rel}}{k(1-B_{\rm rel})}\right] \frac{1}{B_{\infty}} \quad . \tag{38}$$



Figure 12: Standard deviations of the reconstruction noise of  $\hat{Z}_{obj}$  for different values of k. The effective distances are in the range of a few meters. The value of k is higher for pixels close to the light source.

Thus,

$$\sigma_{\hat{Z}_{\rm obj}}^2 = \left[\frac{1}{k(1-B_{\rm rel})B_{\infty}}\right]^2 (\sigma_{\hat{B}}^2 + B_{\rm rel}^2 \cdot \sigma_{B_{\infty}}^2) = \left[\frac{1}{kB_{\infty}}e^{k(Z_{\rm obj}-Z_0)}\right]^2 \left\{\sigma_{\hat{B}}^2 + [1-e^{-k(Z_{\rm obj}-Z_0)}]^2 \sigma_{B_{\infty}}^2\right\}.$$
(39)

As expected,  $\sigma_{\hat{Z}_{obj}} \to \infty$  the noise is amplified as  $B_{rel} \to 1$ , i.e. when  $Z_{obj} \gg Z_0$ , destabilizing the reconstruction.

We plot in Fig. 12 the dependency of  $\sigma_{\hat{Z}_{obj}}$  in  $Z_{obj}$  in the cases of signal-independent and signaldependent noise. We use the same values used for Fig. 11. In addition, we used  $I_{\max}, I_{\min} \in [0, 255]$ . For the signal independent case we take the best case scenario following Sec. 6.1,  $\sigma_{\hat{B}} = 2\kappa_{\text{gray}}$  (Fig. 10). As in Fig. 11, taking into consideration photon noise, the effective distances shorten. This time, changing k significantly changes the effective distance. Therefore, if reconstruction is in mind, it is preferable to design the imaging setup according to the location of the object in the scene.

### 7 Estimation of the DOPs

In Sec. 4 we use the parameters  $p_{\text{scat}}$  and  $p_{\text{obj}}$  to reconstruct **S** and **B**. This section describes ways for estimating these parameters.

### 7.1 Extraction of $p_{\text{scat}}$

We found empirically that the value of  $p_{\text{scat}}$  is practically constant across the field of view (FOV) in seawater.<sup>4</sup> This makes it is easier to estimate. Note that light depolarizes as it propagates [37]. Therefore, it is reasonable to expect  $p_{\text{scat}}$  to be non uniform. A possible explanation to our experience with the contrary, is that the backscatter usually saturates fast and therefore maybe the light depolarization

<sup>&</sup>lt;sup>4</sup>We found it is constant up to  $\approx 24^{\circ}$  relative to the optical axis.

does not influence our measurements.

The value of  $p_{\text{scat}}$  can be retrieved in one of two ways:

- 1. Measure it from an area in the FOV in which there is no signal, or,
- 2. Rigidly shift the camera/illuminator system, to point to a void region in the medium (where no object is in sight) like in Sec. 5.1. Then, take an image pair  $I_{max}, I_{min}$ . Measure the DOP out that image pair.

In both cases, there is no object in the region of interest at  $(\mathbf{x}_{obj})$ . Therefore,  $\mathbf{I} = \mathbf{B}$  and

$$\hat{p}_{\text{scat}} = \frac{I_{\text{max}}(\mathbf{x}_{\text{obj}}) - I_{\text{min}}(\mathbf{x}_{\text{obj}})}{I_{\text{max}}(\mathbf{x}_{\text{obj}}) + I_{\text{min}}(\mathbf{x}_{\text{obj}})} \quad .$$

$$(40)$$

The above methods rely on the assumption that  $p_{\text{scat}}$  is uniform across the scene. Nevertheless, if a spatially varying  $p_{\text{scat}}$  is experienced, it can be calibrated exactly. This can be done by taking an image pair  $\mathbf{I}_{\text{max}}, \mathbf{I}_{\text{min}}$  of a void region (like  $B_{\infty}$ ) and calculating  $p_{\text{scat}}$  according to Eq. (9).

Let us analyze the consequences of a mistake in the estimation of  $p_{\text{scat}}$ , i.e.  $p_{\text{scat}}^{\text{true}} = \psi \hat{p}_{\text{scat}}$ . From Eq. (15), in the case where  $p_{obj} = 0$  the expression for  $\hat{B}$  will then be

$$\tilde{\mathbf{B}} = \frac{\mathbf{I}_{\max} - \mathbf{I}_{\min}}{\psi \hat{p}_{\infty}} = \frac{1}{\psi} \mathbf{B} \quad , \tag{41}$$

yielding the erroneous signal:

$$\tilde{\mathbf{S}} = \mathbf{I} - \frac{1}{\psi} \mathbf{B} = \mathbf{S} + \left(1 - \frac{1}{\psi}\right) \mathbf{B} \quad .$$
(42)

The errors in the estimated components will be:

$$\frac{\mathbf{E}_B}{\mathbf{B}} = \frac{\tilde{\mathbf{B}} - \mathbf{B}}{\mathbf{B}} = \frac{1}{\psi} - 1 \tag{43}$$
$$\frac{\mathbf{E}_S}{\mathbf{E}} = \frac{\tilde{\mathbf{S}} - \mathbf{B}}{\mathbf{B}} = 1 - \frac{1}{\psi} \tag{44}$$

$$\frac{\mathbf{E}_S}{\mathbf{S}} = \frac{\mathbf{S} - \mathbf{B}}{\mathbf{B}} = 1 - \frac{1}{\psi} \quad . \tag{44}$$

Fig. 13 depicts  $|\frac{\mathbf{E}_B}{\mathbf{B}}| = |\frac{\mathbf{E}_S}{\mathbf{B}}|$ , the absolute value of the relative error. The error in the estimation of  $p_{\text{scat}}$  can be either up  $\psi > 1$  or down  $\psi < 1$ . From Fig. 13 the error is smaller when  $(\psi > 1)$ . Therefore, it is better to overestimate  $p_{\text{scat}}$  rather than to underestimate it.

#### 7.2Estimating $p_{obi}$

Sec. 4 shows that for purposes of signal reconstruction, it is possible to assume that  $p_{obj} = 0$ . But, from Eq. (13), this assumption damages the estimation of **B**. As a consequence, it damages the estimation of the object distances  $\mathbf{Z}_{obj}$  out of  $\mathbf{B}$ , as described in Sec. 5.1. Methods using polarized light under natural illumination [33, 35] assumed that  $p_{obj} = 0$  also for distance estimation. In the case of artificial illumination, however, the light source is polarized and the objects are closer. Therefore, significant



Figure 13: Influence of wrongly estimating  $p_{\text{scat}}$ . The error is smaller when the estimated  $p_{\text{scat}}$  is higher than the real one, rather than lower than the real value.



Figure 14: An image of  $\mathbf{B}_{rel}$  for an underwater scene. (a) Assuming  $p_{obj} = 0$ . (b) Using an estimated  $p_{obj}$ . On the left image areas in proximity to the camera (lower part of the image) are falsely assigned a high value unlike the correct low values in the right image. Assuming  $p_{obj}$  is constant across the scene, areas that do not comply to this assumption damage the monotonicity of  $B_{rel}(Z_{obj})$  (blue ellipses).

values for  $p_{obj}$  can be expected. For example, in a scene which we present in the following, the rocks were  $\approx 30\%$  polarized. Failing to estimate  $p_{obj}$  correctly damages the monotonic relation between the estimated backscatter and the object distance from Eq. (21). Fig. 14 demonstrates that. In Fig. 14(a)  $\hat{\mathbf{B}}_{rel}$  is estimated under the assumption that  $p_{obj} = 0$ . Here  $\hat{\mathbf{B}}_{rel}$  is uniform, despite variations of  $Z_{obj}$ . On the other hand, when taking into consideration  $p_{obj}$ , Fig. 14(b) has a strong dependency on  $Z_{obj}$ .

There are a few cases when  $p_{obj}$  can be sampled directly from the scene. When the light source lights from one side of the FOV to another, the objects in the far end are lit but no backscatter reaches the camera (like in [27]). For example, the area like the upper left part of the scene in Fig. 15. When sampling areas like this one  $\mathbf{I} = \mathbf{S}$  and similarly to Eq. (40),

$$p_{\rm obj} = \frac{I_{\rm max}(\text{clear area}) - I_{\rm min}(\text{clear area})}{I_{\rm max}(\text{clear area}) + I_{\rm min}(\text{clear area})} .$$
(45)

For example, in the scene presented in Fig. 15, the measured values were  $p_{obj}[Red, Green, Blue] = [0.22, 0.27, 0.34]$ .



Figure 15: (a) A raw image I of an underwater scene. (b) Estimated  $\hat{\mathbf{B}}$  with the assumption of  $p_{obj} = 0$ . (c)  $\mathbf{B}_{\infty}$  of that setup. (d)  $\hat{B}$  using an estimation for  $p_{obj}$ . (e) MI calculated between  $\hat{\mathbf{B}}$  and  $\hat{\mathbf{S}}$  using different values of  $p_{obj} = 0$ . Minimum values of the MI in each color channel correspond to the  $p_{obj}$  values of the scene.

#### 7.2.1 Automatic estimation

We present here a general automatic approach for the estimation of  $p_{obj}$ . It is based on the observation that using a wrong value for  $p_{obj}$  results in a high mutual information (MI) between the backscatter  $\hat{\mathbf{B}}$ and the signal component  $\hat{\mathbf{S}}$ . The MI is a quantity that measures mutual dependency of two variables  $\mathbf{V}_1$ ,  $\mathbf{V}_2$ . It is defined as:

$$MI(\mathbf{V}_1, \mathbf{V}_2) = \sum_{v_1 \in \mathbf{V}_1} \sum_{v_2 \in \mathbf{V}_2} pr(v_1, v_2) log \left[ \frac{pr(v_1, v_2)}{pr(v_1)pr(v_2)} \right]$$
(46)

where  $pr(v_1, v_2)$  is the joint probability distribution function of  $\mathbf{V}_1$  and  $\mathbf{V}_2$ . The marginal distribution functions of  $\mathbf{V}_1$  and  $\mathbf{V}_2$  are defined as  $pr(v_1)$  and  $pr(v_2)$  respectively. Thus, the value of  $p_{obj}$  that minimizes the mutual information between  $\hat{\mathbf{B}}$  and  $\hat{\mathbf{S}}$  is the value of  $p_{obj}$  we are looking for.

$$\hat{p}_{\text{obj}} = \arg\max_{p} \left\{ MI[\hat{\mathbf{B}}(p), \hat{\mathbf{S}}(p)] \right\}, p \in [0, 1] \quad , \tag{47}$$

For example, Fig. 15(a) shows a raw image  $\mathbf{I}$  of an underwater scene. Notice that the rock in the top left part of the image (circled) is clearly lit but has no backscatter. Fig. 15(b) shows the estimated backscatter  $\hat{\mathbf{B}}$  calculated with the assumption that  $p_{obj} = 0$  (Eq. 15). Note that the value of  $\hat{\mathbf{B}}$  in the circled area is high. In fact, a rock from  $\mathbf{I}$  can be seen there. Fig. 15(c) shows  $\mathbf{B}_{\infty}$  for this setup. The value of the circled part in  $\hat{\mathbf{B}}$  is almost as high as its value in  $\mathbf{B}_{\infty}$ . This falsely indicates a far object. Fig. 15(d) shows  $\hat{\mathbf{B}}$  with an estimation of  $p_{obj}$  (Eqs. 13, 47). Now the circled part has a low value, as expected for a close object. Fig. 15(e) depicts the mutual information calculated between  $\hat{\mathbf{S}}$  and  $\hat{\mathbf{B}}$  of this scene, for different values of  $p_{obj}$ . In each color channel there is one value of  $p_{obj}$  which yields the minimal MI. These values were used in Eq. (13) to calculate  $\hat{\mathbf{B}}$  in Fig. 15(d). Note, that these values are *almost identical* to the values acquired by sampling (Eq. 45).

The problem becomes more complicated when the DOP of the objects varies across the scene. In Fig. 14 we can see (in blue ellipses) two objects with a significantly different DOP than the rest of the scene. It causes distortions in backscatter image. In this case, we assigned these objects their exact DOP (0) in order to get an estimation of the distance map shown in Fig. 7.

#### 8 Summary

We presented a polarization-based method for visibility enhancement and distance estimation in scattering media. We demonstrated the method in real-life experiments. Our method uses two frames taken with widefield polarized illumination. Therefore, it is fast and simple. We use wide band light sources, enabling colorful results. The visibility enhancement range depends on the range of the light source. However, the distance reconstruction is effective only in a range of 1 - 2m in water. While we performed experiments in the underwater domain, the formulation of most of our problems is general and is thus applicable to other media. Hence, we hope to perform tests in at least another medium (fog or biological tissue). In the future, it would be beneficial to expand the work to deal with objects having reflectance with spatially varying  $p_{obj}$ .

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