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Throughput of cellular systems with conferencing mobiles and cooperative base stations

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Abstract

This paper considers an enhancement to multi-cell processing for the uplink of a cellular system, whereby the mobile stations are allowed to exchange messages on orthogonal channels of fixed capacity (conferencing). Both conferencing among mobile stations in different cells and in the same cell (inter and intra-cell conferencing, respectively) are studied. For both cases, it is shown that a rate-splitting transmission strategy, where part of the message is exchanged on the conferencing channels and then transmitted cooperatively to the base stations, is capacity-achieving for sufficiently large conferencing capacity. In case of inter-cell conferencing, this strategy can be regarded as able to perform convolutional pre-equalization of the signal encoding the common messages in the spatial domain, where the number of taps of the finite-impulse response equalizer depends on the number of conferencing rounds. Analysis in the low signal-to-noise ratio regime and numerical results validate the advantages of conferencing as a complementary technology to multi-cell processing.

I. INTRODUCTION

Recent information-theoretic results have shown that high-rate transmission over networks without any infrastructure (ad hoc networks) is bound to be infeasible over a large scale [1][1]. Therefore, the solution of choice for providing broadband communications necessarily implies the support of an infrastructure made of base stations (BSs, or access points) connected by a high-capacity backbone. This class of solutions includes: conventional cellular systems, where BSs are regularly placed in the area of interest [3]; distributed antenna systems, which are characterized by a less regular (e.g., random) deployment [4]; and hybrid networks, where infrastructure nodes coexist with multi-hopping [5]. In all these networks, a solution that promises to greatly improve the overall throughput and that is gaining increasing interest in the community is *multi-cell processing*. This refers to the class of transmission/ reception technologies that exploit the high-capacity backbone among the BSs to perform joint encoding/ decoding at different cell-sites (see [6] [7] for a list of references).

[1] Notice that this is envisaged to be true even if recent results show that, under demanding assumptions on channel state information availability and by resorting to complex transmission schemes, high-scale transmission on ad hoc network can in principle be achieved effectively [2].

In this paper, we focus on the uplink of a cellular system and investigate a potential improvement to multicell processing. In particular, we consider a network where additional spectral resources allow nearby mobile stations (MSs) to exchange signals over finite-capacity channels, which are orthogonal to the main uplink channel. In other words, while with ordinary multi-cell processing only the BSs are enabled to cooperate (for joint decoding), in our setting MSs are allowed to collaborate as well, but only through *finite-capacity* and *localized* links. We should remark that the limitation and localization of the inter-MS channels contrast with the typical assumption of unlimited and global connectivity among the BSs via the high-capacity backbone [3] [6] [7], which is reasonable due to topological and infrastructure constraints[2]. Our goal is to bring insight into effective transmission strategies that exploit these additional system resources and into the performance gains that might be harnessed by deploying such technology.

A. Main contributions

In modelling the interaction among the terminals, we follow the framework of conferencing encoders first studied in [9] in the context of a two-user multiple access channel and then extended in a number of recent works to other scenarios (see, e.g., [10] [11] and references therein). Moreover, the topology of a cellular system is abstracted according to the linear version of the model introduced in [3] (see [6] [7] for a review of related papers). We will refer to this model in the following as the *linear Wyner model*. Under such assumptions, we consider two scenarios: in the first only one MS is active in each cell at any given time (*intra-cell Time Division Multiple Access, TDMA*) and conferencing channels exist between MSs belonging to adjacent cells (*inter-cell conferencing*); in the second, simultaneous uplink transmission by multiple MSs per cell is allowed and conferencing channels are present only among MSs sharing the same cell (*intra-cell conferencing*). These two scenarios conceivably correspond to limiting situations with either small cells, so as to enable inter-cell conferencing, or large

[2]However, see [8], for a recent work that considers multi-cell processing with limited backhaul capacity.

cells, where only connections among same-cell MSs are feasible. Our main contributions are as follows:

- An achievable rate for the linear Wyner model with conferencing MSs is presented for both cases of inter-cell conferencing with intra-cell TDMA and intra-cell conferencing (Propositions 3 and 5). The considered transmission scheme prescribes rate splitting at the MSs, where part of the message (the "common" message) is exchanged during the conference phase among neighboring (out-of-cell or in-cell) MSs and transmitted cooperatively to the BSs;
- In the case of inter-cell conferencing, we show that the considered transmission scheme can be regarded as being able to perform convolutional pre-equalization of the signal encoding the common messages in the spatial domain, where the equalizer is a finite-impulse response (FIR) filter whose number of taps depends on the number of conferencing rounds;
- For both inter and intra-cell conferencing, the considered transmission schemes are proved to be optimal as long as the conferencing capacity is large enough (Propositions 5 and 6);
- An approximate analysis in the low signal-to-noise ratio regime is presented that enables to gain further insight into the advantages of conferencing (Sec. II-E and III-E);
- It is shown that intra-cell TDMA is not optimal in the presence of intra-cell conference channels, as opposed to the basic scenario without conferencing studied in [3] (Sec. III).

Finally, numerical results validate the relevant advantages of inter-cell and intra-cell conferencing (Sec. II-F and III-F).

B. Related work

In addition to the quickly growing body of work on multi-cell processing for cellular systems [6] [7], there has recently been some activity around the basic idea of complementing and comparing the advantages of cooperation between BSs with some form of collaboration at the MS level as well. In references [12] [13] [14], the basic linear Wyner model was extended by including a layer of dedicated relay terminals, one for each cell, that forward traffic from MSs to BSs (uplink). Focusing on intra-cell TDMA, different transmission schemes were considered, namely half-duplex and full-duplex amplify-and-forward in [12] and [14], respectively, and decode-and-forward in [13], and the respective merits of multi-cell processing and MS cooperative transmission technologies, and combinations thereof, were discussed. Another related work is [15], where the linear Wyner model with intra-cell TDMA and single-cell processing was modified by assuming that the active MS in a given cell knows (anti-causally) the messages to be sent by a number of its neighbors (as might be the case in some implementations of the principle of cognitive radio).

Notation: Throughout the paper, bold letters denote either vectors or matrices; capital letters are used for random variables while lower-case letters indicate specific realizations of the corresponding random variable.

II. Inter-cell conferencing with intra-cell TDMA

In this section, we consider the first scenario of interest, which consists of a modification of the linear Wyner model with intra-cell TDMA where inter-cell conferencing channels are present.

A. System model

We consider the uplink of a cellular system, abstracted according to the linear Wyner model, as sketched in the upper part of Fig. 1. M cells are arranged into a linear array, where each cell contains J MSs ($J = 1$ in the figure). Following [3], the signal transmitted by each MS is received only by the same-cell BS, with unitary gain, and by the two adjacent BSs with inter-cell gain α. As anticipated, we consider at first the case where only one MS transmits in each cell at any give time in a TDMA fashion (intra-cell TDMA). It should be remarked that this choice does not entail any loss of optimality in a basic Wyner model with no conferencing, as shown in [3]. Overall, defining as X_m the input symbol of the MS active in the mth cell, the signal received by the mth BS reads ($X_m = 0$ for $m > M$ and $m < 1$)

$$Y_m = X_m + \alpha(X_{m-1} + X_{m+1}) + N_m,\ m = 1, ..., M, \tag{1}$$

with $N_m \sim \mathcal{CN}(0,1)$ being an i.i.d. sequence of complex Gaussian noise samples. Notice that we assume full (symbol and codeword) synchronization among the cell. We focus on multi-cell processing, that is we assume that the signals received by the BSs, $\{Y_m\}_{m=1}^{M}$, are jointly processed by a central unit that detects the transmitted signals. Finally, each MS has an average power constraint of P so that the available power per cell is $\tilde{P} = JP$. With intra-cell TDMA, each MS is active for a fraction $1/J$ of the time, wherein it can transmit with power $\tilde{P}$, still satisfying the average power constraint. The power constraint then is given by $E[|X_m|^2] = \tilde{P}$, which can be interpreted as the signal-to-noise-ratio (SNR) for the system at hand. We remark at this point that in the following we will be interested in limiting results for a very large number of cells ($M \rightarrow \infty$): edge effects can be handled as in [3] and we will neglect them in the presentation below, unless explicitly stated otherwise. We refer the reader to [7] for a discussion of the relevance of this asymptotic regime in practical scenarios with a limited number of cells.

We now extend the basic linear Wyner model discussed above to include conferencing among the active MSs in adjacent cells (inter-cell conferencing). A different variation of the Wyner model where intra-cell conferencing is enabled is discussed in Sec. III. As shown in the lower part of Fig. 1, with inter-cell conferencing, $2M-2$ orthogonal channels with capacity C (bits/symbol) are assumed to exist, each linking the MS currently active in any mth cell to the active MS in an adjacent cell (i.e., the $m+1$ or $m-1$th cell, unless $m = 1$ or $m = M$). We assume block-transmission as shown in Fig. 2. Within any tth block and in any mth cell, the MS currently active generates a message $W_m(t) \in \mathcal{W} = \{1, 2, ..., 2^{NR/J}\}$ meant to be decoded by the central processor connecting the BSs, where N is the number of channel uses per block and R is the *per-cell rate* (bits/ channel use). According to standard information-theoretic assumption, we will consider large block length $N \rightarrow \infty$. Transmission of a given set of messages $\{W_m(t)\}_{m=1}^M$ takes place in two successive phases (or slots). In the first phase (*conferencing phase*), during the tth block, the MSs exchange information on the conferencing channels during K rounds (see further details below). This information collected during the conferencing phase by each active MS is then leveraged by the latter to encode the local message $W_m(t)$ for transmission to the BSs in the $(t+1)$th block (*transmission phase*). Notice that, as shown in Fig. 2, the phase transmission corresponding $\{W_m(t)\}_{m=1}^M$ can be carried out at the same time as the transmission phase for messages $\{W_m(t-1)\}_{m=1}^M$ given the orthogonality of conferencing channels and uplink channel.

To formalize the model discussed in the previous paragraph, we need to specify the coding/ decoding operations allowed at different terminals. Given our intra-cell TDMA assumption, each set of M active MSs uses both the conferencing channels and the uplink channels for a fraction $1/J$ of the time. In the following, we focus on a specific set of M active MSs and, furthermore, we drop the dependence on the block index t for simplicity of notation. For the *conferencing phase,* following [9], we consider K rounds of conference. In each kth round ($k = 1, 2, ..., K$), any active mth MS transmits a message $c_{k,m\to m+i}$ to the adjacent MSs $m+i$ with $i = -1, 1$. This depends on the messages received by the mth MS during the previous rounds ($c_{1:k-1,m-1\to m}$ and $c_{1:k-1,m+1\to m}$) as

$$c_{k,m\to m+i} = h_{k,m\to m+i}(c_{1:k-1,m-1\to m}, c_{1:k-1,m+1\to m}) \in \mathcal{C}_{k,m\to m+i}, \tag{2}$$

where $h_{k,m\to m+i}(\cdot)$ is a given deterministic function and $\mathcal{C}_{k,m\to m+i}$ a given alphabet. For convenience of notation, the K messages transmitted on each link are collected in $K\times 1$ vectors $\mathbf{c}_{m\to m+i}$. The finite capacity of the conferencing links imposes the following constraint on the alphabets [9][3]:

$$\frac{1}{N}\sum_{k=1}^{K}\log|\mathcal{C}_{k,m\to m+i}| \leq \frac{C}{J}. \tag{3}$$

For the *transmission phase,* encoding at each mth MS takes place according to a determin-

[3] All the logarithms are to be assumed base-2 in keeping with our measure of information in bits/symbol.

istic mapping $f_m(\cdot)$ from the message set and the received conferencing messages to sequences of N (complex) channel symbols $\mathbf{x}_m \in \mathbb{C}^N$ (codewords) as $\mathbf{x}_m = f_m(w_m, \mathbf{c}_{m-1\to m}, \mathbf{c}_{m+1\to m})$ for $w_m \in \mathcal{W}$. Decoding at the central processor is based on the $N \times M$ signal $\mathbf{y} = [\mathbf{y}_1 \cdots \mathbf{y}_M]$ received by the M BSs over the N channel uses according to the deterministic mapping $g(\cdot)$: $\mathbb{C}^{N\times M} \to \mathcal{W}^M$ as $\hat{\mathbf{w}} = [\hat{w}_1 \cdots \hat{w}_M]^T = g(\mathbf{y})$. Following standard definitions, a *per-cell rate* R is said to be achievable if there exists a sequence of encoders and decoders such that the probability of error $P[\hat{\mathbf{W}} \neq \mathbf{W}]$ tends to zero for block length $N \to \infty$.

B. Reference results

In this section, we discuss lower and upper bounds on the per-cell achievable rate R in the presence of inter-cell conferencing. The first result is due to [3] and does not assume a priori intra-cell TDMA.

Proposition 1 (lower bound, no conferencing) [3]: The per-cell capacity (i.e., maximum achievable per-cell rate) in a basic linear Wyner model with no conferencing ($C = 0$) and $M \to \infty$ is achieved with intra-cell TDMA and is given by

$$R_{lower} = \int_0^1 \log\left(1 + \tilde{P} \cdot H(f)^2\right) df, \tag{4}$$

with

$$H(f) = 1 + 2\alpha\cos(2\pi f). \tag{5}$$

It should be noted that the rate (4) can be understood by regarding the Wyner model of Fig. 1 as an inter-symbol interference (ISI) channel in the spatial domain, characterized by the channel impulse response $h_m = \delta_m + \alpha\delta_{m-1} + \alpha\delta_{m+1}$ (δ_m denotes the Kronecker delta function) and corresponding transfer function $H(f)$ in (5). Moreover, we emphasize that the rate (4) clearly sets a lower bound on the performance achievable with inter-cell conferencing since it assumes $C = 0$.

The following proposition defines a useful upper bound on the performance attainable with inter-cell conferencing and intra-cell TDMA.

Proposition 2 (upper bound, perfect conferencing): An upper bound on the rate achievable with inter-cell conferencing and intra-cell TDMA in the linear Wyner model (with $M \rightarrow \infty$) is given by

$$R_{upper} = \int_0^1 \log\left(1 + \tilde{P} \cdot H(f)^2 S(f)\right) df \quad (6)$$

with

$$S(f) = \left(\mu - \frac{1}{\tilde{P} H(f)^2}\right)^+ \quad (7a)$$

$$\int_0^1 S(f) df = 1. \quad (7b)$$

This results follows by considering the cut-set bound [21] applied to the cut that divides MSs and BSs, or equivalently by assuming a perfect conferencing phase ($C \rightarrow \infty$) where each mth active MS is able to exchange the local message W_m with all the other active MSs in other cells. In fact, in such an asymptotic regime, joint encoding of the set of messages $\{W_m\}_{m=1}^M$ by all the M MSs is feasible, and, recalling the equivalence of (1) with an ISI channel, we can conclude that the optimal transmission strategy is defined by the waterfilling solution (7) [16]. Notice that the waterfilling solution is obtained for a sum-power constraint over the MSs but, given the symmetry of our setting, it also applies to the considered per-MS power constraint. It should also be remarked that this results shows that, in the limit $C \rightarrow \infty,$ a stationary input in the spatial domain with power spectral density $S(f)$ is capacity-achieving. This conclusion will be used in the next section to bring insight into the performance of inter-cell conferencing with finite capacity. While for convenience and generality the upper bound (6)-(7) is reported here in integral form, in Appendix-A we present a closed-form expression for (6) that holds in a specific regime of interest (see Appendix-A for details).

C. An achievable rate

In this section, we derive an achievable rate for the Wyner model with inter-cell conferencing and intra-cell TDMA and discuss some of the implications of this result.

Proposition 3 (achievable rate): The following per-cell rate is achievable for the linear Wyner model with inter-cell conferencing and intra-cell TDMA for $M \to \infty$ and any $K \geq 1$:

$$
\begin{aligned}
R &= \max_{P_c, P_p, \mathbf{h}_c} \min \left\{ \int_0^1 \log\left(1 + P_p H(f)^2 + P_c H(f)^2 |H_c(f)|^2\right) df, \right. \\
&\left. \int_0^1 \log\left(1 + P_p H(f)^2\right) df + \frac{C}{K} \right\},
\end{aligned}
\tag{8}
$$

with constraints

$$P_c + P_p = \tilde{P} \tag{9a}$$

$$\|\mathbf{h}_c\|_2^2 = 1 \tag{9b}$$

and definitions: $\mathbf{h}_c = [h_{c,-K} \cdots h_{c,K}]^T \in \mathbb{C}^{2K+1}$ and

$$H_c(f) = \sum_{m=-K}^{K} h_{c,m} \exp(-j 2\pi f m). \tag{10}$$

Here we would like to briefly discuss the transmission scheme that attains the rate (8) and to point out some implications of this result, leaving the details of the proof of achievability to Appendix-B. Again, to fix the ideas, consider the set of M active MSs at a given time, one per each cell, which employ a fraction of time $1/J$ of both the uplink and the conferencing channels. The proposed scheme works as follows. In the *conference phase*, each mth MS first splits its message W_m into two parts, say *private* ($W_{p,m}$) and *common* ($W_{c,m}$). Then, it shares the common part $W_{c,m}$ with the $2K$ neighboring MSs in cells $m+i$ with $i = -K, -K+1, ..., -1, 1, ..., K$, during K conferencing rounds. More precisely, in the first round, the mth MS transmits its local common information $W_{c,m}$ to the two adjacent MSs $m-1$ and $m+1$, which then propagate the information towards the two edges of the network, and so on. Notice that, after the conference phase, each mth MS is aware of the $2K+1$ common messages $\{W_{c,m+k}\}_{k=-K}^{K}$. During the *transmission phase*, each common message $W_{c,m}$ can be then transmitted cooperatively by all the $2K+1$ MSs that have acquired the information on $W_{c,m}$ in the conferencing phase. On top of the cooperative signal encoding common information, each MS jointly encodes the private message $W_{p,m}$. Gaussian codebooks are employed and the total power $\tilde{P}$ is divided as (9a) between the common (P_c) and private (P_p) parts.

As shown by Proposition 3, the impact of inter-cell conferencing, according to the scheme discussed above, is equivalent to that of allowing *precoding* (*pre-equalization*) of the common information by a $2K \times 1$ FIR filter $\mathbf{h}_c$ with frequency response $H_c(f)$ (10). The equivalent channel seen by the input symbols encoding the common information (say Z_m) is shown for illustration in Fig. 3 for $K = 1$ conference rounds. We emphasize that, while the number of taps increases with the number of conference rounds, the overall achievable rate may suffer according to (8). We further explore this trade-off in Sec. II-F with a numerical example.

D. Asymptotic optimality of the considered scheme

From Proposition 3, it is easy to see that the proposed scheme is optimal under a specific asymptotic regime, as stated in the following Proposition.

Proposition 4 (asymptotic optimality): The transmission scheme achieving the rate (8) is optimal for $C \to \infty$, $K \to \infty$ and $\frac{C}{K} \geq R_{upper}$.

Proof: It is enough to prove that the rate (8) equals the upper bound (6) under the conditions in the proposition above. This follows easily by setting $P_c = \tilde{P}$ (and $P_p = 0$) and recalling that the optimal power spectral density $S(f)$ (7) can be approximated arbitrarily well by the frequency response $|H_c(f)|^2$ in (10) as the number of taps $2K + 1$ increases unboundedly [17].

Remark 1: The argument in the proof above shows that, under the asymptotic conditions stated in Proposition 4, it is optimal to allocate all the power to the common messages, $P_c = \tilde{P}$ (and $P_p = 0$), and to select the filter $\mathbf{h}_c$ so that $|H_c(f)|^2 = S(f)$.

E. Discussion: the low-SNR regime

In this and in the next section, we elaborate on the performance of the considered scheme that exploits inter-cell conferencing. Here, this goal is pursued via an (approximate) analytical approach that focuses on the low-SNR regime according to the framework in [18], whereas in the next section we resort to numerical simulations to study the case of arbitrary SNR. The attention to the low-SNR regime is justified by the fact that, as discussed above, the advantages of inter-cell conferencing are (asymptotically) related to the opportunity of performing waterfilling power allocation, which is known to provide relevant gains only for low to moderate SNRs (see, e.g., [19]).

According to [18], for low SNRs, the rate R of a given transmission scheme can be described by the minimum transmit energy per bit required for reliable communication (normalized to the background noise level) $E_b/N_0|_{\min}$ (which is obtained for $\tilde{P} \rightarrow 0$) and by the slope $\mathcal{S}_0$ at $E_b/N_0|_{\min}$ (measured in $bit/s/Hz/(3dB)$). In the following, we focus for simplicity on the minimum energy per bit $E_b/N_0|_{\min}$, and use this criterion to compare the performance of inter-cell conferencing with the lower and upper bounds (4) and (6) in the low-SNR regime. Starting with the bounds, the minimum energy per bit is given by :

$$\left.\frac{E_b}{N_0}\right|_{\min,lower} = \frac{\ln 2}{1+2\alpha^2} \tag{11}$$

for the lower bound (4) (see [7]) and

$$\left.\frac{E_b}{N_0}\right|_{\min,upper} = \frac{\ln 2}{(1+2\alpha^2)^2} \tag{12}$$

for the upper bound (6). The latter can be proved by noticing, similarly to [18], that when the SNR tends to zero ($\tilde{P} \rightarrow 0$), it is optimal to allocate all the available power around the maximum value of the channel transfer function, $\max_f H(f)^2 = (1+2\alpha)^2$, which occurs at $f = 0$. In other words, the optimal waterfilling power allocation is $S(f) = \delta(f)$, where $\delta(f)$ is a Dirac delta function. Plugging $S(f) = \delta(f)$ into (6) and using tools from [18], equality (12) is easily shown.

Let us now consider the rate (8) achievable by inter-cell conferencing. We start with the observation that for $\tilde{P} \rightarrow 0$ and any finite K, we have $C/K > R_{upper}$ so that the first term in

(8) is dominant and rate (8) is given by:

$$R = \max_{P_p, P_c, \mathbf{h}_c} \int_0^1 \log\left(1 + P_p H(f)^2 + P_c H(f)^2 \left|H_c(f)\right|^2\right) df. \tag{13}$$

The optimization problem (13) (with constraints (9)) is generally not convex so that finding a global optimal solution is not an easy task [20]. For this reason, we focus on a sub-optimal feasible solution that is asymptotically (in the sense of Proposition 4) optimal and allows to gain insight into the performance of inter-cell conferencing. This solution is based on the observation that, from Remark 1 and from the discussion above, the asymptotically optimal power allocation is $P_c = \tilde{P}$ (and $P_p = 0$) and the optimal filter $\mathbf{h}_c$ satisfies $\left|H_c(f)\right|^2 = S(f) = \delta(f)$. Accordingly, with the stated power allocation, here we design for any finite (but large) K the filter $\left|H_c(f)\right|^2$ so as to approximate the (asymptotically) optimal $\left|H_c(f)\right|^2 = \delta(f)$ by an ideal low-pass filter with frequency response:

$$\left|H_c(f)\right|^2 = \begin{cases} \frac{1}{2W} & -W \leq f \leq W \\ 0 & \text{otherwise} \end{cases}, \tag{14}$$

where the bandwidth W satisfies $W \simeq 1/K \ll 1$. Clearly, frequency response (14) can only be approximated by a FIR filter, but the approximation is acceptable for large K. Hence, under the low-SNR condition and assuming large K, the rate (13) is given by

$$R \simeq 2\int_0^{1/K} \log\left(1 + \frac{1}{2}K\tilde{P}H(f)^2\right) df, \tag{15}$$

so that the minimum energy can be calculated following [18] and after some algebra[4], as

$$\left.\frac{E_b}{N_0}\right|_{\min} \simeq \frac{\ln 2}{(1+2\alpha)^2\left(1 - \frac{8\alpha\pi^2}{3(1+2\alpha)K^2}\right)}. \tag{16}$$

[4]We use the second-order approximation:

$$H(f)^2 \cong (1+2\alpha^2)\left(1 - \frac{2\alpha}{1+2\alpha^2}f^2\right) + o(f^4)\,.$$

From the previous equation, it is clear that the minimum energy per bit of inter-cell conferencing (16) is a decreasing function of the number of conferencing rounds K and, as expected from Proposition 4, tends to the optimal performance (12) for $K \to \infty$.

F. *Numerical results*

In this section, we present some numerical examples in order to assess the performance of the discussed inter-cell conferencing scheme. Similarly to the previous section, since the optimization problem (8) that yields the considered achievable rate R is generally non-convex, here we focus on a simple feasible solution that is asymptotically (in the sense of Proposition 4) optimal and allows to gain interesting insight into the system performance. As discussed in Remark 1, for $C \to \infty$, $K \to \infty$ and $C/K \geq R_{upper}$, the (global) optimal power allocation is $P_c = \tilde{P}$ (and $P_p = 0$) and the optimal frequency response $|H_c(f)|^2$ satisfies $|H_c(f)|^2 = S(f)$. Based on this result, for any choice of the parameters, first the $2K + 1$ taps of filter $\mathbf{h}_c$ are generated according to the frequency-sampling method with target amplitude of the frequency response given by the waterfilling solution $\sqrt{S(f)}$ [17] (the filter is scaled to satisfy the constraint (9b)). Then, for fixed filter $\mathbf{h}_c$, the optimization problem (8) is convex in the powers (P_c, P_p), and can be solved efficiently by using standard numerical methods [20]. Illustration of the performance of the frequency-sampling filter design for different values of K is shown in Fig. 4 for $\tilde{P} = 3dB$ and $\alpha = 0.2$. It can be seen that with K large enough the FIR filter $H_c(f)$ in (10) is able to approximate closely the (asymptotically) optimal waterfilling solution $S(f)$.

As discussed above, increasing K is always beneficial to obtain a better approximation of the waterfilling strategy (7). However, due to the finite conferencing capacity C, it is not necessarily advantageous in terms of the achievable rate (8). To show this, Figs. 5 and 6 present the achievable rate (8) versus the inter-cell gain α along with the lower bound (4) and upper bound (6) for $J = 1$, and $C = 1$ and $C = 10$, respectively. Fig. 5 shows that, with $C = 1$, while increasing the conferencing rounds from $K = 1$ to 2 increases the achievable rate, further increments of the conferencing capacity C are disadvantageous, according to the trade-off mentioned above. With a larger capacity $C = 10$, Fig. 6 shows that very relevant performance gains can be harnessed by increasing the number of conference rounds, especially from $K = 1$ to $K = 2$. Moreover, as expected from Proposition 4, having sufficient large conferencing capacity C and number of conference rounds K (with $C/K \geq R_{upper}$) enables to the upper bound (6) to be approached.

III. Intra-cell conferencing

In this section, we study a different extension of the linear Wyner model, where there exist conferencing channels that link MSs within the same cell so as to enable intra-cell conferencing. Notice that here we do not assume intra-cell TDMA, that is, same-cell MSs are allowed to transmit to the BSs at the same time.

A. System model

The basic linear Wyner model with multiple active users per cell, say $J \geq 1$, is defined as follows. Denoting as $X_{j,m}$ the input symbol of the jth MS ($j = 1, ..., J$) in the mth cell, the signal received by the mth BS is given by ($X_{j,m} = 0$ for $m > M$ and $m < 1$):

$$Y_m = \sum_{j=1}^{J} X_{j,m} + \alpha \left(\sum_{j=1}^{J} X_{j,m-1} + \sum_{j=1}^{J} X_{j,m+1} \right) + N_m,\ m = 1, ..., M, \tag{17}$$

As in the previous section, the per-user power constraint is $E[|X_{j,m}|^2] = P$ so that a total power constraint per cell of $\tilde{P} = JP$ is enforced.

The basic Wyner model is now extended to allow intra-cell conferencing. We consider M *intra-cell* multicast channels with capacity C', one per cell: each such channel connects a MS to all the other same-cell MSs, and is accessed by *only one MS at each time* in a TDMA fashion (see Fig. 7). Such channels are orthogonal for different cells and with respect to the main uplink channel. As in Sec. II-A, transmission of a given set of messages $W_{j,m} \in \mathcal{W} = \{1, 2, ..., 2^{NR/J}\}$ for the (j, m)th MS with $j = 1, ..., J$ and $m = 1, ..., M$, occurs in two phases that are arranged in a frame structure as shown in Fig. 2.

In the conferencing phase, we assume again JK rounds of conferencing[5]. Each (j,m)th MS at any kth round transmits a message $d_{k,j,m}$ to all the other MSs in the mth cell (see Fig. 7), which is a deterministic function of the previously received messages (recall (2)):

$$d_{k,j,m} = h'_{k,j,m}(\{d_{1:k-1,j,m}\}_{j=1}^{J}) \in \mathcal{D}_{k,j,m} \cup \{\varnothing\}, \tag{18}$$

for a given deterministic mapping $h'_{k,j,m}$, and alphabet $\mathcal{D}_{k,j,m}$. Notice that, in order to deal with the issue of multiple access to conferencing channels by the same-cell MSs, we have extended the alphabet of symbols used for conferencing with a symbol $\varnothing$, which represent no transmission. Moreover, similarly to (3), the finite capacity of the conferencing links imposes the condition

$$\frac{1}{N}\sum_{k=1}^{K} \log |\mathcal{D}_{k,j'(k,m),m}| \leq C', \tag{19}$$

where with a slight abuse of notation we have defined as $j'(k,m)$ the MS that uses the conferencing channel in the mth cell at round k. Finally, since only one (j,m)th MS in cell m can transmit in a given round k, we have that if $d_{k,j,m} \neq \varnothing$ then $d_{k,j',m} = \varnothing$ for all $j' \neq j$.

In the transmission phase, encoding at each mth MS takes place according to a deterministic mapping $f_{j,m}(\cdot)$ from the message set and the received conferencing messages to the codebook as $\mathbf{x}_{j,m} = f_{j,m}(w_{j,m}, \{\mathbf{d}_{j,m}\}_{j=1}^{J}) \in \mathbb{C}^N$ for $w_{j,m} \in \mathcal{W}$. Finally, decoding is based on the $N \times M$ signal $\mathbf{y}$ according to the deterministic mapping $g(\cdot)$: $\mathbb{C}^{N\times M} \rightarrow \mathcal{W}^{J\times M}$ as $\hat{\mathbf{w}} = g(\mathbf{y})$.

B. Reference results

In this section, we present relevant upper and lower bounds on the achievable rate of the linear Wyner model with intra-cell conferencing presented above. We first notice that a lower bound on the achievable rate is still set by (4), which corresponds to the case of no conferencing ($C' = 0$). We now discuss a useful upper bound.

[5]Notice that in the previous section, we considered intra-cell TDMA so that the total number of conferencing rounds was JK.

Proposition 4 (upper bound, perfect conferencing): An upper bound on the rate achievable with intra-cell conferencing on the Wyner model (with $M \to \infty$) is given by

$$R'_{upper} = \int_0^1 \log\left(1 + J\tilde{P} \cdot H(f)^2\right) df. \tag{20}$$

Similarly to Proposition 2, Proposition 4 follows by assuming a perfect conferencing phase where each (j,m)th MS is able to deliver the entire message $W_{j,m}$ to all the other in-cell MSs. In fact, under such assumption, we observe that all the J MSs in any mth cell can be seen as a "super-MS" with input symbol $\tilde{X}_m = \sum_{j=1}^{J} X_{j,m}$ (recall (17)) and power constraint $J\tilde{P}$ due to *coherent power combining*.

C. An achievable rate

Here we provide an achievable rate for the linear Wyner model with intra-cell conferencing and describe the transmission scheme that is able to attain it.

Proposition 5 (achievable rate): The following rate is achievable on the linear Wyner model with intra-cell processing and $M \to \infty$:

$$\begin{aligned} R' &= \max_{P_c,P_p} \min\left\{\int_0^1 \log\left(1 + (P_p + JP_c)H(f)^2\right) df,\right. \\ &\qquad \left.\int_0^1 \log\left(1 + P_pH(f)^2\right) df + C'\right\}, \end{aligned} \tag{21}$$

with constraint (9a) and definition (5).

A brief sketch of the proof of achievability is in order. The details are worked out in Appendix-C. Each (j,m)th MS first splits its message $W_{j,m}$ into two parts, say *private* ($W_{p,j,m}$) and *common* ($W_{c,j,m}$). The common part $W_{c,j,m}$ is then communicated to all the MSs belonging to the same cell in one conference round (a total number of $K = J$ conference rounds is thus employed). In the transmission phase, all the MSs in a cell cooperate to achieve coherent power combining on the common part of the message, which is transmitted by each user with power P_c/J and received with power JP_c. The private message is instead jointly encoded by each MS on top of the common message and carries power P_p/J.

Remark 2: It should be noticed that rate (21) is achieved with multiple MSs simultaneously active in each cell. By comparison with rate (8), which is achievable with intra-cell TDMA, it can be seen that, in case intra-cell conferencing is allowed, intra-cell TDMA is not optimal. In fact, as explained above, simultaneous transmission of multiple MSs after intra-cell conferencing allows coherent power combining to be achieved. This lack of optimality of intra-cell TDMA in the presence of intra-cell conferencing clearly contrasts with the results in [3] for the case of no conferencing (see Proposition 1).

D. Conditional optimality of the considered scheme

Similarly to the case of inter-cell processing, the considered scheme based on rate splitting is optimal if the conferencing capacity is large enough. However, in contrast with the previously considered scenario (see Proposition 3), here optimality is obtained for finite conferencing capacity C'.

Proposition 6 (conditional optimality): The transmission scheme achieving the rate (8) is optimal if $C' \geq R'_{upper}$.

Proof: We need to prove that the rate (21) equals the upper bound (20) under the conditions in the proposition above. This follows easily by setting $P_c = \tilde{P}$ (and $P_p = 0$).

Remark 3: The argument in the proof above shows that for $C' \geq R'_{upper}$ it is optimal to allocate all the available power to the common messages ($P_c = \tilde{P}$ and $P_p = 0$).

E. Discussion: the low-SNR regime

For the sake of completeness, similarly to Sec. II-E, here we assess the performance of intra-cell conferencing in the low-SNR regime by calculating the minimum energy per bit $E_b/N_0|_{\min}$ required for reliable communications. This task is pretty straightforward since the advantages of intra-cell conferencing are related to the power gain achievable through coherent power combining, which, differently from the waterfilling advantage of inter-cell conferencing, is immediate to account for in the low-SNR regime. In particular, the energy $E_b/N_0|_{\min,upper}$ obtained by the upper bound (20) is given by

$$\left.\frac{E_b}{N_0}\right|_{\min,upper} = \frac{\ln 2}{J(1+2\alpha^2)},$$

which, when compared to the lower bound (11), clearly shows the coherent power gain by J due to cooperation. As proved in Proposition 6, under the assumption $C' \geq R'_{upper}$, the achievable rate (21) attains the upper bound, so that we clearly have $E_b/N_0|_{\min} = E_b/N_0|_{\min,upper}$ for $C' \geq R'_{upper}$.

F. Numerical results

Fig. 8 shows the achievable rate (21) versus the transmitted power per cell $\tilde{P}$ along with the lower bound R_{lower} (4) and the upper bounds R_{upper} (6) and R'_{upper} (20) for $\alpha = 0.6$, $C' = 1$, 2 and $J = 2$ MSs per cell. Notice that (21) is a convex problem so that global optimality can be attained by using standard numerical methods [20]. From the figure, it is seen that increasing the intra-cell conferencing capacity C' allows the upper bound R'_{upper} to be approached and eventually reached (as stated in Proposition 6). Moreover, it is interesting to observe that the best performance achievable with intra-cell conferencing (R'_{upper}) is preferable to the best rate attainable with inter-cell conferencing (R_{upper}), lending evidence to the effectiveness of coherent power combining.

IV. Conclusions

Most of the current proposals for the enhancement of cellular-based wireless networks, such as the IEEE 802.16j standard, are based on cooperative technologies. Among such solutions, multi-cell processing, where cooperation is at the BS level, is receiving an increasing attention for its significant potential enabled by the high-capacity backbone connecting the BSs. In this paper, we have looked at an extension to this technology, where, beside multi-cell processing, partial cooperation is allowed at the MS level as well. In particular, additional system resources are assumed to be available to provide conferencing channels of finite capacity between nearby MSs. Two limiting scenarios have been considered: one in which conferencing is allowed between MSs belonging to adjacent cells (as is reasonable in the presence of small cells) and another where conferencing is possible only among MSs belonging to the same cell. In both cases, a transmission scheme based on rate splitting and cooperative transmission has been proven to be optimal when the conferencing capacity is large enough.

A relevant extension of this work, that is currently under study, is to consider achievable rates for a two-dimensional cellular systems in the spirit of the hexagonal-cell models presented in [3]. The main problem in such scenarios is the propagation of the conferencing messages, which, given the geometry at hand, could possibly benefit from network coding.

Another interesting issue left open by this work is the establishment of capacity-achieving schemes for any value of the conferencing capacity and finite number of cell-sites. The main challenge in this regard appears to be the extension of the converse result in [9] to the scenario at hand. In particular, it remains to be determined whether, unlike the simpler model in [9], interactive communications among the MSs during the conferencing phase is necessary to achieve capacity. The results of this paper have shown that this is not the case in the regime of high conferencing capacity.

V. APPENDIX

A. Appendix-A: Closed-form expression of the upper bound (6)-(7) for the low-α large-power regime

In this section, we reconsider the upper bound given in Proposition 1 based on waterfilling power allocation and present a closed-form analytical expression of (6)-(7) that hold in a specific regime of low inter-cell gain α and high-power. We remark that in other regimes (large α and/ or small power) we were not able to obtain such compact expressions.

Proposition 7: Assume $0 \le \alpha < 1/2$ and

$$\tilde{P} \ge \frac{1}{(1-2\alpha)^2} - \frac{1}{(1-4\alpha^2)^{3/2}}, \tag{22}$$

then the upper bound (6)-(7) becomes

$$R_{upper} = \log\left(\tilde{P} + \frac{1}{(1-4\alpha^2)^{\frac{3}{2}}}\right) + 2\log\left(\frac{1+\sqrt{1-4\alpha^2}}{2}\right). \tag{23}$$

Proof: Under the assumption that the power $\tilde{P}$ is sufficiently large so that

$$\mu \ge \max_{0 \le f \le 1/2} \frac{1}{\tilde{P}H(f)^2} = \frac{1}{\tilde{P}(1-2\alpha^2)} \tag{24}$$

(i.e., the high-power regime), the constraint (7) can be written as

$$\begin{aligned} 1 &= \int_0^1 \left(\mu - \frac{1}{\tilde{P}H(f)^2}\right) df = \mu - 2\int_0^{1/2} \frac{1}{\tilde{P}(1+2\alpha\cos(2\pi f))^2} df = \\ &= \mu - \frac{1}{\tilde{P}(1-4\alpha^2)^{\frac{3}{2}}}, \end{aligned} \tag{25}$$

where the last equality follows from formula 3.661.4 in [24] and some algebra. Hence, from (24) and (25) the high-power regime is defined by condition (22), and the waterfilling constant μ is given by

$$\mu = 1 + \frac{1}{\tilde{P}(1-4\alpha^2)^{\frac{3}{2}}}. \tag{26}$$

Finally, the rate expression is given by

$$\begin{aligned} R_{upper} &= \int_0^1 \log\left(1 + \tilde{P}\cdot H(f)^2\left(\mu - \frac{1}{\tilde{P}H(f)^2}\right)\right) df = \\ &= \log\mu + 2\int_0^1 \log(\tilde{P}(1+2\alpha\cos\theta))df \\ &= \log\left(\tilde{P} + \frac{1}{(1-4\alpha^2)^{\frac{3}{2}}}\right) + 2\log\left(\frac{1+\sqrt{1-4\alpha^2}}{2}\right), \end{aligned} \tag{27}$$

where the last equality is achieved by applying formula 4.224.12 of [24].

B. Appendix-B: Proof of Proposition 3

In this section, the proof of achievability of rate (8) stated in Proposition 3 is provided. For simplicity of notation, we consider $J = 1$ since the extension to $J > 1$ requires only straightforward modifications given the intra-cell TDMA assumption. We consider conference and transmission phases separately.

1) Conference phase: As discussed in Sec. II, the first step is to split the message of each MS into private and common parts. More precisely, as in [9], each mth MS partitions the message set $\mathcal{W}$ into R_c bins, each containing 2^{NR_p} elements with $R_p = R - R_c$. Index $W_{c,m} \in \mathcal{W}_c = \{1, 2, ..., 2^{NR_c}\}$ is used to identify the bins and index $W_{p,m} \in \mathcal{W}_p = \{1, 2, ..., 2^{NR_p}\}$ to identify the given message within the bin. The index $W_{c,m}$ is communicated via conferencing to $2K$ neighboring MSs in K rounds: in the first round each mth MS communicates a given $w_{c,m}$ to the two neighbors ($c_{1,m\rightarrow m+i} = w_{c,m}$ for $i = -1, 1$). In any kth round with $k > 1$, the MSs propagate what they received in the previous round as $c_{k,m\rightarrow m+i} = c_{k-1,m-i\rightarrow m}$ ($i = -1, 1$). At the end of the conference, message $W_{c,m}$ is known at terminals $m, m \pm 1, ..., m \pm K$. The procedure explained above entails the following constraint on the common rate:

$$R_c \leq \frac{C}{K}. \tag{28}$$

2) Transmission phase (after conference): After the conference, each mth MS has two kinds of information, a private message $W_{p,m}$ with rate $R_p = R - R_c$ and $2K + 1$ common messages $\{W_{c,m+k}\}_{k=-K}^{K}$ each with rate R_c.

Codebook generation: The codebooks are generated as follows. For each m, we generate a codebook of 2^{NR_c} independent codewords $\mathbf{z}_m = [z_{1,m} \cdots z_{N,m}]^T$ according to a distribution $\mathbf{Z}_m \sim \prod_{n=1}^{N} p(z_{n,m})$. We label these sequences as $\mathbf{z}_m(W_{c,m})$. Now, for each m and common messages sets $\{W_{c,m+k}\}_{k=-K}^{K}$, we generate 2^{NR_p} $(N \times 1)$ independent codewords $\mathbf{x}_m$ according to a distribution $\mathbf{X}_m \sim \prod_{n=1}^{N} p(x_{n,m}|\{\mathbf{z}_m(W_{c,m+k})\}_{k=-K}^{K})$, and label them as $\mathbf{x}_m(W_{p,m}|\{W_{c,m+k}\}_{k=-K}^{K})$.

In order to achieve rate (8), we further specialize the distribution $p(x_{n,m}|\{\mathbf{z}_m(W_{c,m+k})\}_{k=-K}^{K})$ as (dropping the dependence on the time index with a slight abuse of notation)

$$X_m = V_m + \sum\nolimits_{k=-K}^{K} h_{c,k} \cdot Z_{m+k} = \\ = V_m + h_{c,m} * Z_m, \quad (29)$$

where $\|\mathbf{h}_c\|_2^2 = 1$, $V_i \sim \mathcal{CN}(0, P_p)$ independent of all $\mathbf{Z}_m$ and "$*$" denotes convolution. From (29) it is clear that the overall impact of conferencing here is to enable linear precoding via the FIR filter $\mathbf{h}_c$ of the signal encoding common messages (Z_m) in the spatial domain (recall Fig. 3).

Encoding: Each mth MS encodes messages $W_{p,m}$ and $\{W_{c,m+k}\}_{k=-K}^{K}$ as $\mathbf{x}_m(W_{p,m}|\{W_{c,m+k}\}_{k=-K}^{K})$.

Decoding: Decoding is based on the received signal $\mathbf{y}$ and joint typicality: the decoder decides for $\{W_{p,m}, W_{c,m}\}_{m=1}^{M}$ if and only if sequences $(\mathbf{y}, \{\mathbf{z}_m(W_{c,m})\}_{m=1}^{M}, \{\mathbf{x}_m(W_{p,m}|\{W_{c,m+k}\}_{k=-K}^{K})\}_{m=1}^{M})$ are jointly typical and no other triplet of sequences is.

Analysis of probability of error: From [22] (see Sec. VII therein), we conclude that the following $2(2^M - 1)$ conditions guarantee vanishing error probability for block-length $N \to \infty$:

$$|\mathcal{P}|R_p \leq I(\mathbf{X}_\mathcal{P}; \mathbf{Y}|\mathbf{X}_{\mathcal{P}^c}, \mathbf{Z}) \quad (30)$$

$$MR_p + |\mathcal{C}|R_c \leq I(\mathbf{X}; \mathbf{Y}|\mathbf{Z}_{\mathcal{C}^c}) \quad (31)$$

for any two subsets $\mathcal{P}, \mathcal{C} \subseteq \mathcal{M} = \{1, ..., M\}$. Notationwise, we have defined $\mathbf{X}_\mathcal{P} = \{\mathbf{X}_m\}_{m\in\mathcal{P}}$ and $\mathbf{X} = \mathbf{X}_\mathcal{M}$ (and similarly for $\mathbf{Z}$), while $|\cdot|$ denotes the cardinality of the argument set. In order to facilitate calculation of the required mutual informations, let us substitute (29) into (1) so as to obtain the received signal as a function of "private" and "common" symbols V_m and Z_m, respectively:

$$Y_m = V_m * h_m + Z_m * h_m * h_{c,m} + N_m, \quad (32)$$

where

$$h_m = \delta_m + \alpha\delta_{m-1} + \alpha\delta_{m+1}. \tag{33}$$

Consider at first the $2^M - 1$ constraints (30). From (32), it is easy to see that

$$I(\mathbf{X}_{\mathcal{P}}; \mathbf{Y}|\mathbf{X}_{\mathcal{P}^c}, \mathbf{Z}) = I(\mathbf{V}_{\mathcal{P}}; \tilde{\mathbf{Y}}|\mathbf{V}_{\mathcal{P}^c}) \tag{34}$$

with the mth element of the $M \times 1$ vector $\tilde{\mathbf{Y}}$ being

$$\tilde{Y}_m = V_m + \alpha(V_{m-1} + V_{m+1}) + N_m. \tag{35}$$

Since (35) is a regular Wyner model as in [3], and from Theorem 2.1 therein, we can conclude the dominating condition among the first $2^M - 1$ (30) is obtained for $\mathcal{P} = \mathcal{M}$, i.e.,[6]

$$R_p \leq \min_{\mathcal{P}\subseteq\mathcal{M}} \frac{1}{|\mathcal{P}|} I(\mathbf{X}_{\mathcal{P}}; \mathbf{Y}|\mathbf{X}_{\mathcal{P}^c}, \mathbf{Z}) = \frac{1}{M} I(\mathbf{X}; \mathbf{Y}|\mathbf{Z}). \tag{36}$$

Now consider the remaining $2^M - 1$ inequalities (31). In the following we would like to show that, coupled with (36), these conditions identify the achievable (R_p, R_c) region sketched in Fig. 9, which is characterized by the conditions

$$R_p \leq \frac{1}{M} I(\mathbf{X}; \mathbf{Y}|\mathbf{Z}) \tag{37a}$$

$$R_p + R_c \leq \frac{1}{M} I(\mathbf{X}; \mathbf{Y}). \tag{37b}$$

Since the right-hand side of (37b) corresponds to the condition (31) with $\mathcal{C} = \mathcal{M}$, proving the previous statement amounts to: (*i*) pointing out that the right hand side of (31) $I(\mathbf{X}; \mathbf{Y}|\mathbf{Z}_{\mathcal{C}^c}) = H(\mathbf{Y}|\mathbf{Z}_{\mathcal{C}^c}) - H(\mathbf{N})$ is a non-decreasing function of $|\mathcal{C}|$ and (*ii*) showing that for private rate equal to $R_p = 1/M \cdot I(\mathbf{X}; \mathbf{Y}|\mathbf{Z})$ the maximum common rate R_c according to (31) is a non-increasing function of $|\mathcal{C}|$. This latter conclusion can be obtained as follows. Substituting $R_p =$

[6] The reader is referred to [3] for a thorough discussion of the border effects in this argument.

$1/M \cdot I(\mathbf{X};\mathbf{Y}|\mathbf{Z})$ into (31) (taken with equality), we obtain the following chain of equalities:

$$\begin{aligned} R_c &= \frac{1}{|\mathcal{C}|}(I(\mathbf{X};\mathbf{Y}|\mathbf{Z}_{\mathcal{C}^c}) - I(\mathbf{X};\mathbf{Y}|\mathbf{Z})) = \\ &= \frac{1}{|\mathcal{C}|}(H(\mathbf{Y}|\mathbf{Z}_{\mathcal{C}^c}) - H(\mathbf{Y}|\mathbf{X},\mathbf{Z}_{\mathcal{C}^c}) - H(\mathbf{Y}|\mathbf{Z}) + H(\mathbf{Y}|\mathbf{X},\mathbf{Z})) \\ &\overset{(a)}{=} \frac{1}{|\mathcal{C}|}I(\mathbf{Z}_{\mathcal{C}};\mathbf{Y}|\mathbf{Z}_{\mathcal{C}^C}), \end{aligned} \tag{38}$$

where (a) is a consequence of the Markov condition $\mathbf{Z} \rightarrow \mathbf{X} \rightarrow \mathbf{Y}$. Now, since the channel seen by $\mathbf{Z}$ is an ISI channel with channel response $h_m * h_{c,m}$ and additive noise, we can again apply the same approach as in [3] (see proof of Theorem 2.1 therein) to show that $1/|\mathcal{C}| \cdot I(\mathbf{Z}_{\mathcal{C}};\mathbf{Y}|\mathbf{Z}_{\mathcal{C}^C})$ is non-increasing with $|\mathcal{C}|$.

Having established that the region of achievable rates (R_p, R_c) is (37), we now need to calculate the two right-hand sides in the limit $M \rightarrow \infty$. Following [3], we have

$$R_p \leq \int_0^1 \log\left(1 + P_p H(f)^2\right) df \tag{39a}$$

$$R_p + R_c \leq \int_0^1 \log\left(1 + P_p H(f)^2 + P_c H(f)^2 |H_c(f)|^2\right) df. \tag{39b}$$

Finally, using (39a) and (39b) with (28), we obtain (8).

C. Appendix-C: Proof of Proposition 5

As for the proof of Proposition 3, we consider conference and transmission phases separately. The treatment follows closely Appendix-B, so that here we emphasize only the major differences.

1) Conference phase: Each (j,m)th MS splits its message into private and common parts by partitioning the message set $\mathcal{W}$ in R_c bins, each containing 2^{NR_p} elements with $R_p = R'/J - R_c$. Index $W_{c,j,m} \in \mathcal{W}_c = \{1, 2, ..., 2^{NR_c}\}$ is used to identify the bins and index $W_{p,j,m} \in \mathcal{W}_p = \{1, 2, ..., 2^{NR_p}\}$ to identify the given message within the bin. From the discussion in Sec. III, the conferencing messages (18) are then selected as $d_{j,j,m} = w_{c,j,m}$ for $j = 1, ..., J$, so that $K = J$. Furthermore, the finite capacity constraints (19) impose the condition

$$R_c \leq \frac{C'}{J}. \tag{40}$$

2) Transmission phase (after conference): After the conference, each (j,m)th MS has two kinds of information, a private message $W_{p,j,m}$ with rate $R_p = R'/J - R_c$ and J same-cell common messages $\tilde{W}_{c,m} = \{W_{c,j,m}\}_{j=1}^{J}$ each with rate R_c so that the overall common message $\tilde{W}_{c,m}$ for the mth cell has rate JR_c.

Codebook generation: The codebooks are generated as follows. For each m, we generate 2^{NJR_c} $N \times 1$ independent codewords $\mathbf{u}_m$ according to a distribution $\mathbf{U}_m \sim \prod_{n=1}^{N} p(u_{n,m})$ and label these sequences as $\mathbf{u}_m(\tilde{W}_{c,m})$. Now, for each (j,m) and for each common message $\tilde{W}_{c,m}$, we generate 2^{NR_p} $(N \times 1)$ independent codewords $\mathbf{x}_{j,m}$ according to a distribution $\mathbf{X}_{j,m} \sim \prod_{n=1}^{N} p(x_{n,j,m}|\mathbf{u}_m(\tilde{W}_{c,m}))$, and label them as $\mathbf{x}_{j,m}(W_{p,j,m}|\tilde{W}_{c,m})$.

In order to achieve rate (21), we consider the specific distributions (dropping the dependence on the time index with a slight abuse of notation) $U_m \sim \mathcal{CN}(0, P_c/J)$ and $X_{j,m} = V_{j,m} + U_m$, with $V_{j,m} \sim \mathcal{CN}(0, P_p/J)$ independent of U_m.

Encoding: Each mth MS encodes messages $W_{p,j,m}$ and $\tilde{W}_{c,m}$ as $\mathbf{x}_m(W_{p,j,m}|\tilde{W}_{c,m})$.

Decoding: Decoding is based on the received signal $\mathbf{y}$ and joint typicality: the decoder decides for $\{\{W_{p,j,m}\}_{j=1}^{J}, \tilde{W}_m\}_{m=1}^{M}$ if and only if sequences $(\mathbf{y}, \{\mathbf{u}_m(\tilde{W}_{c,m})\}_{m=1}^{M}, \{\{\mathbf{x}_{j,m}(W_{p,j,m}|\tilde{W}_{c,m})\}_{j=1}^{J}\}_{m=1}^{M})$ are jointly typical and no other triplet of sequences is.

Analysis of probability of error: Following [22] (see Sec. VII therein), we conclude that the following $(2^{MJ}-1)+(2^{M}-1)$ conditions guarantee vanishing error probability for block-length $N \to \infty$:

$$|\mathcal{P}|R_p \leq I(\mathbf{X}_{\mathcal{P}}; \mathbf{Y}|\mathbf{X}_{\mathcal{P}^c}, \mathbf{U}) \tag{41a}$$

$$MJ \cdot R_p + |\mathcal{C}|JR_c \leq I(\mathbf{X}; \mathbf{Y}|\mathbf{U}_{\mathcal{C}^c}), \tag{41b}$$

for any two subsets $\mathcal{P} \subseteq \{1, ..., M\} \times \{1, ..., J\}$, $\mathcal{C} \subseteq \{1, ..., M\}$. We can now follow similar steps as in Sec. V-B.2 to show that for $M \rightarrow \infty$ the inequalities (41) reduce to

$$R_p \leq \frac{1}{J}\int_0^1 \log\left(1 + P_p H(f)^2\right) df \tag{42a}$$

$$R_p + R_c \leq \frac{1}{J}\int_0^1 \log\left(1 + (P_p + JP_c)H(f)^2\right) df. \tag{42b}$$

Finally, recalling that the rate per cell is given by $R' = J(R_c + R_p)$ and using (42a) and (42b) with (40), we obtain (21).

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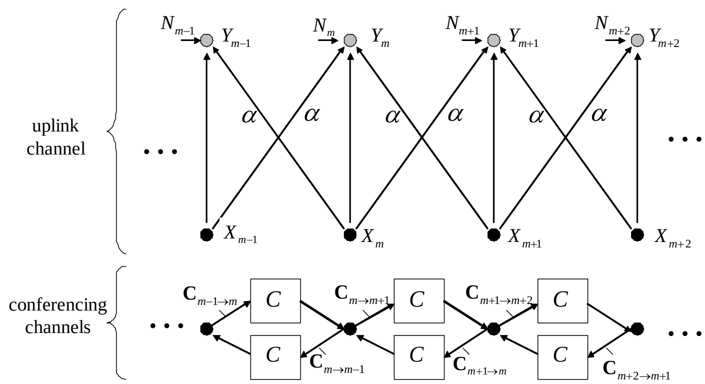

Fig. 1. Linear Wyner model with inter-cell conferencing and intra-cell TDMA studied in Sec. II.

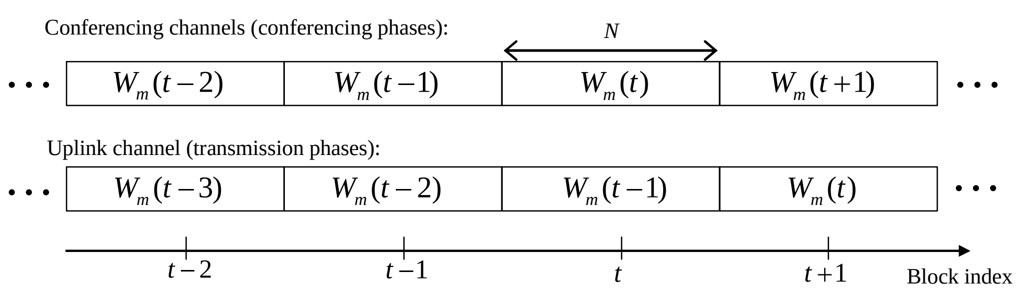

Fig. 2. Frame structure for transmission on the conferencing and uplink channels. The transmission phase of messages $\{W_m(t)\}_{m=1}^{M}$ occurs at slot $t+1$ after the corresponding conferencing phase.

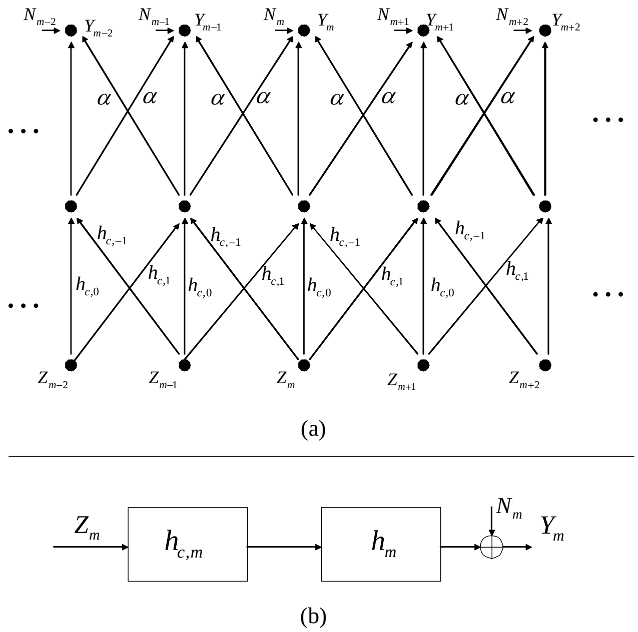

Fig. 3. (a) Equivalent channel seen by the common messages, encoded by symbols Z_m, after K rounds of the conference phase ($K = 1$); (b) Corresponding block diagram.

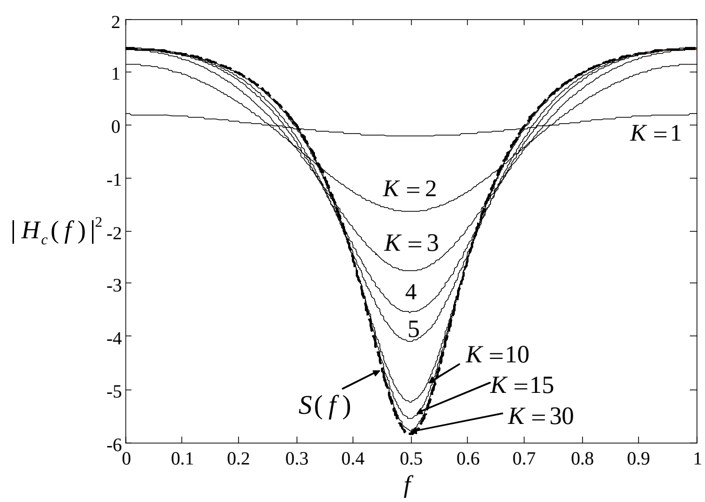

Fig. 4. Optimal waterfilling solution (7) and approximation obtained by the FIR pre-equalizer (10) for $\alpha = 0.2$ and $\tilde{P} = 3dB$.

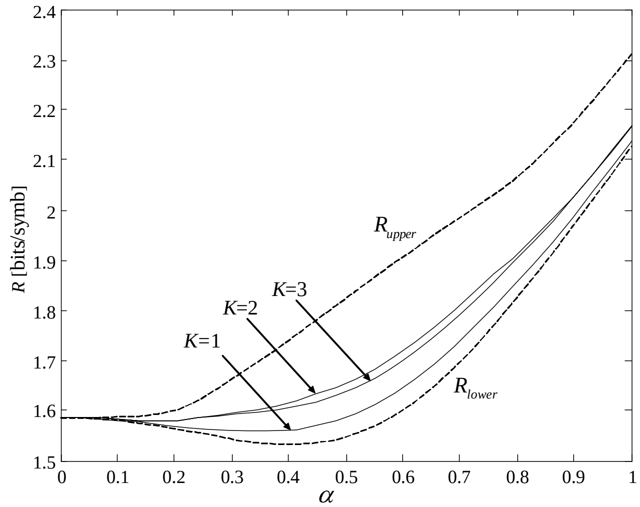

Fig. 5. Achievable rate (8) with inter-cell conferencing and intra-cell TDMA versus the inter-cell gain α. The lower bound (4) and upper bound (6) are also shown for reference ($\tilde{P} = 3dB, C = 1, J = 1$).

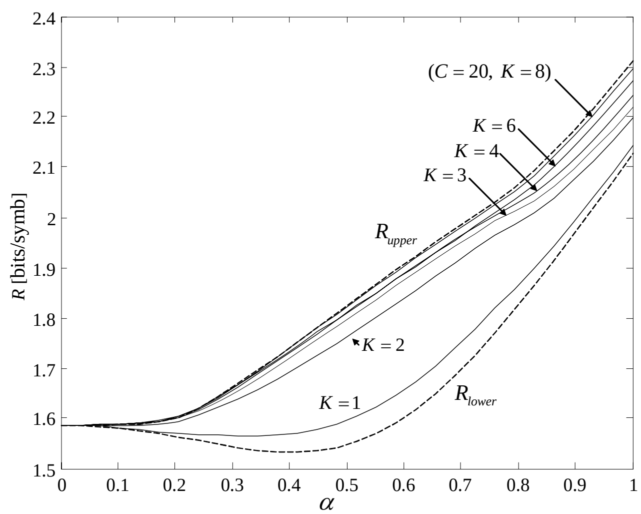

Fig. 6. Achievable rate (8) with inter-cell conferencing and intra-cell TDMA versus the inter-cell gain α. The lower bound (4) and upper bound (6) are also shown for reference ($\tilde{P} = 3dB, C = 10, J = 1$).

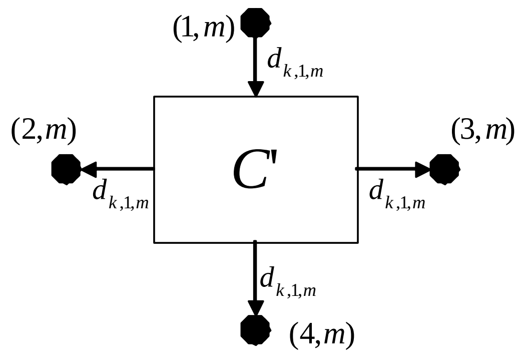

Fig. 7. Intra-cell conferencing channel in the mth cell with $J = 4$ users per cell: in the illustrated example, during the kth conferencing round, the $(1, m)$th MS is communicating message $d_{k,1,m}$ to the other same-cell MSs (multicast).

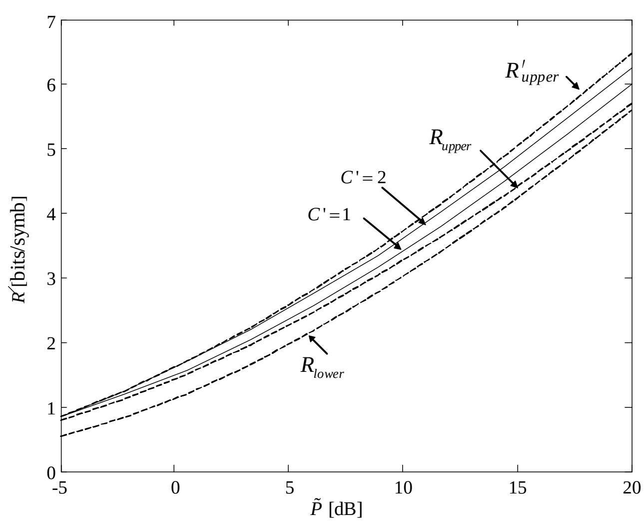

Fig. 8. Achievable rate (21) with intra-cell conferencing versus the transmitted power per cell $\tilde{P}$ along with the lower bound R_{lower} (4) and the upper bounds R_{upper} (6) (corresponding to inter-cell conferencing) and R'_{upper} (20) (intra-cell conferencing). Note that $R' = R'_{upper}$ if $C' \geq R'_{upper}$ ($\alpha = 0.6$, $C' = 1$, 2 and $J = 2$ MSs per cell).

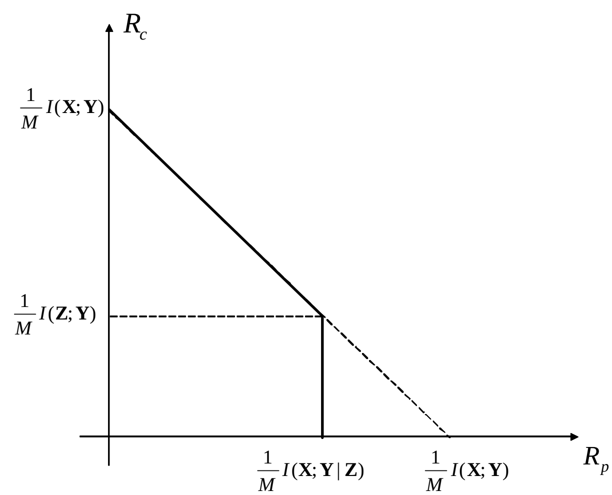

Fig. 9. Region of achievable rates (R_p, R_c) in the proof of Proposition 3.