Broadcast Cooperation Strategies for Two Colocated Users

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***Abstract*—This work considers the problem of communication between a remote single transmitter and a destined user, with helping colocated users, over an independent block Rayleigh fading channel. The colocation nature of the users allows cooperation, which increases the overall achievable rate, from transmitter to destination. The transmitter is ignorant of the fading coefficients, while receivers have access to perfect channel state information (CSI). We propose, for this setting, a multi-layer broadcast transmission approach. The broadcast approach transmission enables enhanced cooperation between the colocated users. That is due to the nature of broadcasting, where the better the channel quality, the more layers that can reliably be decoded. The cooperation between the users is performed over additive white Gaussian noise channels (AWGN), with a relaying power constraint, and unlimited bandwidth. Three commonly used cooperation techniques are studied: amplify-forward (AF), compress-forward (CF), and decode-forward (DF). These techniques are extended by using the broadcast approach, for the case of relaxed decoding delay constraint. For this case a separate processing of the layers, which includes multi-session cooperation is shown to be beneficial. Further, closed form expressions for infinitely many AF sessions and recursive expressions for the more complex CF are given. Numerical results for the various cooperation strategies demonstrate how the multi-session cooperation outperforms conventional relaying techniques.**

***Index Terms*—Single-user broadcasting, code layering, ad-hoc networks, amplify-and-forward, decode-and-forward, compress-and-forward, multi-session cooperation.**

I. Introduction

In recent years, interest in communication networks has increased, and various applications of it, such as sensor networks [1],[2],[3] energy sensitive networks [4],[5] and Ad-hoc networks [6], have gained popularity. In this field, networks with colocated receivers and colocated transmitters constitute a substantial part, since they allow increased cooperation [7], thus improving the overall networks' throughput [8], [9]. Specifically, many contributions deal with the various aspects of such cooperation, such as transmitters cooperation in ad-hoc networks [10], transmitters cooperation in a multiple access channel (MAC) [11], receivers cooperation [12], [13], [14], [15], [16], and both transmitters and receivers cooperation [17],[18]. In source related networks, such as the sensors network, the cooperation is slightly different, since the objective is to convey a source with a distortion (e.g. the reach-back problem [19]), rather than ensuring reliable communication. The compress forward (CF) and amplify forward (AF) techniques make use of lossy source coding techniques, to ensure high communication rates, when the cooperative receiver does not decode the message. This is studied in [20], [21], [22], among many others. Here, we deal with one transmitter that sends the same information to two colocated cooperating users, where only one of them is the destination. Receiver cooperation in general appears to be less understood than transmitter cooperation; for example, when orthogonal links exist between two colocated users, where one of them is the transmitter, the capacity region is known [11]. This is not the case for the receiver side cooperation [23].

The transmitter in our work sends the same information to two colocated receivers (where only one of them is its destination), over independent block Rayleigh fading channels, as in [24]. Such channels have zero Shannon capacity, and usually one turns to rate versus outage probability [25], [26]. When considering the average throughput or delay as figures of merit, it is beneficial to use the broadcast approach [27]. The broadcast strategy for a single-user facilitates reliable transmission rates adapted to the actual channel conditions, without providing any feedback from the receiver to the transmitter [27], [28]. The single-user broadcasting approach hinges on the broadcast channel, which was first explored by Cover [29]. In a broadcast channel, a single transmission is directed to a number of receivers, each experiencing possibly different channel conditions, reflected in their received signal to noise ratios (SNR). Here, every fading gain is associated with another virtual user. The higher the fading gain, the higher is the achievable rate. This broadcasting scheme is also referred to as the continuous broadcast approach, where every fading gain is associated with a code layer. The continuous broadcasting facilitates an upper bound on the average achievable throughput in fading channels with no transmit CSI. Although not practical for implementation, it may well serve as a design goal for finite level coding, e.g. [30].

The broadcast approach has been studied in [31], [32] for a two hop relay channel, where the efficiency of ad-hoc cooperation in a two-hop relay setting was demonstrated, when a direct link from source to destination is not available. Several broadcasting strategies were investigated for relaying techniques such as decode forward (DF), AF, and CF. In our setting, a direct link from source to destination exists in addition to the cooperation link, which motivates multi-session cooperation, and different broadcasting approaches for maximizing average throughput. In [15], cooperation among densely packed K-colocated receivers is studied, where the

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users with better channel conditions decode the message faster, and join the senders to the destined user, thus allowing the destined user to decode the original message even if a severe fading occurs on the source destination link. Notice that transmission and cooperation in [15] take place within a single block, whereas in our work we consider multi-session cooperation where each session starts after the transmission of another block is complete. In [16], a similar network setting is considered, with a single source transmitting to two colocated users, where a Wyner-Ziv (WZ) CF single session cooperation is studied. The WZ-CF in [16] does not assume knowledge of the actual fading realization on the source-destination link. In our work, we assume that prior to the WZ compression, the destination sends the relay its actual fading gain, and thus we incorporate continuous broadcasting with optimal power allocation, as the transmitter views a single equivalent fading gain. In this case a cooperation scheme, such as AF or CF, can be efficiently analyzed by transforming the network setting into an equivalent point-to-point fading channel (usually non-Rayleigh) between the transmitter and the destination, and an adapted broadcast approach can be used. It is observed that CF offers the best performance which, given the colocation of the users, is in agreement with the result obtained for the multiple-relay channel in which CF approaches capacity when the relays are close to the destination [33].

We consider the case where the two receivers can cooperate between themselves, so that they can improve reception at the destination receiver, via DF or via source related techniques such as AF or CF. Since these users are colocated, the probability of a multi-path non-line-of-sight channel, such as the channel from the transmitter, is low, so the cooperation takes place over the additive white Gaussian noise (AWGN) channel, with a relaying power constraint, and unlimited bandwidth. In addition to single session cooperation, we study multi-session cooperation schemes, like was done by [34] for the binary erasure channel. By combining the broadcasting approach with multi session cooperation, the efficiency of each session is increased by reducing information layers that were decoded in previous sessions. This way, we can surpass the naive cooperation performance.

The rest of the paper is organized as follows. The main contributions of this work are described in section II, and the channel model is specified in III. Upper and lower bounds are stated in section IV. Section V deals with cooperation through the simpler amplify and forward, and section VI improves the achievable rates of the previous section, by using Wyner-Ziv (CF). Section VII describes the broadcast approach with DF cooperation. Then, section VIII gives numerical results, comparing the achievable rates of the various cooperation schemes. The paper ends with concluding remarks, and a discussion of a simple extension of our results to the case of receiving common information at two cooperating colocated end-users.

Throughout the paper $E(\cdot)$ stands for the expectation operator. Boldface variables are used for vectors. The log function is in natural logarithm basis. Without loss of generality we assume here that the destination is user $i = 1$. The terms second user ($i = 2$) and relay are synonymously used.

II. MAIN CONTRIBUTIONS

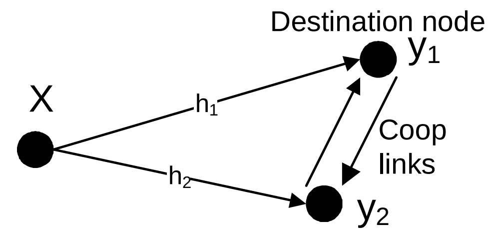

Fig. 1. A schematic diagram of a source transmitting to two colocated users, with multi-session cooperation.

We study a wireless network consisting of a single antenna transmitter, which sends information to a single destination. Near the destination there is a colocated user, allowed to cooperate with the destination receiver. In general, if the only cooperation link was from relay (cooperating user) to destination, the problem setting would completely match the classical relay channel [35] (with an additional block fading process). In our setting, the destination and relay are assumed to be symmetric in capabilities, and therefore exchange information via multi-session cooperation. This is depicted in Figure 1. All cooperation sessions are assumed to take place on channels orthogonal to the direct link (transmitter - destination). Furthermore, when multiple sessions are allowed, a corresponding bandwidth expansion is used, under a fixed power constraint. In addition, the cooperation links are mutually orthogonal and each receiver is full-duplex, i.e. it receives information on direct link and simultaneously transmits cooperation information from previous sessions to the other user. Another important aspect of the problem setting is that transmitter has no CSI. This gives rise to single-user broadcasting [27], [28], which allows maximizing the average throughput over fading channels, with transmitter uncertainties.

The main contributions of this paper are summarized in the following. For some specific cases, explicit maximal achievable average rates are obtained using the continuous broadcast approach, and different cooperation schemes. In other cases, where the broadcasting power distribution cannot be optimized, an equivalent fading gain is obtained, which turns the cooperation into a simple point to point SISO equivalent problem, where broadcasting achievable rates are obtained.

Three types of cooperation strategies are considered (AF, CF, and DF). The first is based on the low complexity amplify-and-forward (AF) relaying by a network user to the destination user, over the cooperation link:

1) *Naive AF* - A relaying user scales its input and forwards it to the destined user, who jointly decodes the signal from the direct link, and the relay. A closed form expression for maximal achievable rate in a broadcast approach is obtained.
2) *Separate preprocessing AF* - A more efficient form of single session AF is the separate preprocessing, where the colocated users exchange the values of the estimated fading gains, then individually decode the layers up to

the smallest fading gain. The relaying user subtracts this decoded common information from its received signal and performs AF to the destined user. An equivalent fading gain is derived, and achievable rates are computed for this case using sub-optimal power distribution for the broadcast approach.

3) *Multi-session AF* - Repeatedly separate preprocessing is followed by a transmission of cooperation information at both relay and destination sides (on orthogonal channels). The preprocessing basically includes individual decoding with the available received information from the direct link and previous cooperation sessions. During the cooperation sessions the transmission of the next block already takes place. This means that our multi-session cooperation introduces additional decoding delays, **without reducing the overall throughput**. Simultaneous transmission of the next block requires that overall, in each block time slot, processing of many blocks be performed, as well as cooperation channel uses. This requires multiple parallel cooperation channels between the cooperating users, as illustrated in Figure 3. In order to incorporate practical constraints on the multi-session approach, the total power of a multi-session cooperation is restricted to P_r, which is identical to a the power restriction in single session cooperation.

The capacity of a multi session cooperation channel is $C_{coop} = P_r$, see (4), in the limit of infinitely many sessions. The other cooperation schemes, namely naive AF, and separate preprocessing AF, cannot efficiently use an unlimited bandwidth. Single session wide-band AF means duplicating the AF signal while proportionally reducing its power. This results in no gain over narrow-band cooperation. Therefore a narrow-band cooperation channel is used for these two schemes, with $C_{coop} = \log(1 + P_r)$. Other broadcast cooperative strategies are based on the WZ [36] CF relaying:

1) *Naive CF* - According to this strategy the relaying user performs WZ-CF over the cooperation link. Prior to the WZ compression, the destination informs the relay of the actual fading gain it has estimated. The destination performs optimal decoding using its own copy of the signal from the direct link and the WZ compressed signal forwarded over the cooperation link. An equivalent fading gain is obtained between transmitter and destination receiver, of a virtual point-to-point equivalent SISO channel, and maximal achievable rate is derived in closed form.
2) *Separate preprocessing CF* - Here, the users first decode independently up to the highest common decodable layer. Then a WZ-CF cooperation takes place on the residual relay signal (like the naive CF). Here an equivalent gain is obtained, and achievable broadcasting rates are derived.
3) *Multi-session CF* - Multi session cooperation, in the same steps as described for AF, only adhering to successive refinement WZ [37]. Recursive expressions for the equivalent fading gain are obtained.

We consider also DF cooperation, where the cooperating users are colocated [38], and the source transmitter performs broadcasting. However, the DF cooperation is not suitable for multi-session cooperation. That is, after a first session of cooperation, where the relay has sent to the destination its decoded layers (on top of those decoded independently at the destination), the destination cannot send any information back to the relay in order to decode more layers. In a DF scheme, the achievable rate is limited by the maximal rate achievable independently by each user. This type of limitation does not exist in AF and CF cooperation, which are beneficial to both users, with the stronger and weaker channels [39]. Similarly to single-session CF, a wide-band DF cooperation can also be used here, and numerical results show that wide-band DF cooperation closely approximates the DF upper bound. The DF upper bound is obtained by taking $C_{coop} \to \infty$ in (51), which is also the selection diversity achievable rate, according to the distribution specified in (12).

All results in this paper can be straightforwardly extended to the case of a single transmitter sending common information to two cooperating users. See section IX for more details.

III. Channel Model

Consider the following single-input multiple-output (SIMO) channel (we use boldfaced letters for vectors) ,

$$\mathbf{y}_i = h_i \mathbf{x}_s + \mathbf{n}_i \quad , \; i = 1, 2 \tag{1}$$

where $\mathbf{y}_i$ is a received vector by user i, of length L, which is also the transmission block length. The length L is assumed to be large enough such that transmission rates close to the mutual information are reliably decoded. $\mathbf{x}_s$ is the original source transmitted vector. $\mathbf{n}_i$ is the additive noise vector, with elements that are complex Gaussian i.i.d with zero mean and unit variance, denoted $\mathcal{CN}(0, 1)$, and h_i is the (scalar) fading coefficient. The fading coefficient h_i is assumed to be perfectly known by receiver i. The fading h_i is distributed according to the Rayleigh distribution $h_i \sim \mathcal{CN}(0, 1)$, and remains constant for the duration of every transmission block (adhering to a block fading channel). This also means that the two users have equal average SNR, which is realistic due to their colocation. Nevertheless, the results may be straightforwardly extended to the case of unequal average SNRs. Receivers colocation may also suggest channel realization correlation (h_1 and h_2). In case of such correlation the cooperation gains are expected to be smaller, since even the joint decoding channel capacity decreases. We assume, for simplicity of analysis, fully independent fading channel realizations.

The source transmitter has no CSI, and the power constraint at the source is given by $E\left(|x_s|^2\right) \leq P_s$, where x_s represents the random variable of the source transmitted signal every channel use.

The fading gains $|h_i|^2$ $(i = 1, 2)$ are both known at each receiver. This assumption requires a short transaction between the two cooperating users, in order to exchange their local fading gains. Since only one scalar has to be exchanged for every block, its overhead is neglected. Furthermore, in a realistic setting, a quantized version of $|h_i|^2$ is exchanged. It is expected that a few bits of feedback will be sufficient, as was observed in the work on the broadcast approach with quantized feedback [40].

The cooperation channels between the users are modeled by AWGN channels as follows

$$\begin{aligned} \mathbf{y}_{2,1}^{(k)} &= \mathbf{x}_1^{(k)} + \mathbf{w}_1^{(k)} \\ \mathbf{y}_{1,2}^{(k)} &= \mathbf{x}_2^{(k)} + \mathbf{w}_2^{(k)} \end{aligned} \tag{2}$$

where $\mathbf{y}_{2,1}^{(k)}$ is the second user's received cooperation vector (of length L) from the destination ($i = 1$), on the k^{th} cooperation link, and vise-versa for $\mathbf{y}_{1,2}^{(k)}$. $\mathbf{x}_i^{(k)}$ is the cooperation signal from user i, on the k^{th} cooperation link, and $\mathbf{w}_i$ is the noise vector with i.i.d elements distributed according to $\mathcal{CN}(0,1)$. For a single session cooperation $k = 1$, and the power of $x_i^{(1)}$ is limited by $E\left(|x_i^{(1)}|^2\right) \leq P_r$ (for $i = 1,2$). However, for a wide-band cooperation $k = 1,2,...,K$, which models $K-$parallel cooperation channels for each user. The power constraint here is specified by $E\left(\sum\limits_{k=1}^{K} |x_i^{(k)}|^2\right) \leq P_r$ (for $i = 1,2$). So K is the bandwidth expansion that results from the multi-session cooperation. It is assumed that the receiver is capable of full-duplex communications, and can receive next block while transmitting a cooperation message of previous blocks. This is also done without interference, as orthogonal channels are assumed for this purpose.

Throughout this work, the power constraint P_r is a short term power constraint, applied per cooperation session. For a long term average power constraint, the cooperation efficiency may increase, by using a different cooperation power allocation scheme. See also comment following proposition 5.1.

Naturally, the link capacity of a single session narrow-band cooperation over the AWGN channel defined in (2) is given by

$$C_{coop,NB} = \log(1 + P_r). \tag{3}$$

In the limit of $K \to \infty$ with a power constraint for multi-session cooperation, the cooperation link capacity is given by

$$C_{coop,WB} = \int\limits_0^\infty dR(s) = \int\limits_0^\infty \rho(s)ds = P_r, \tag{4}$$

where $dR(s)$ is the fractional rate of a session associated with parameter s, and $dR(s) = \log(1 + \rho(s)ds)$. The fractional power at the s^{th} session is $\rho(s)$. The multi-session power constraint implies $\int\limits_0^\infty \rho(s)ds = P_r$, which justifies the last equality in (4).

In view of a single-session cooperation, only CF and DF approaches may utilize a cooperation channel bandwidth expansion of the form $C_{coop,WB}$, in (4), for improving the cooperation efficiency. This is also considered in sections VI and VII.

IV. Upper and Lower bounds

In order to evaluate the benefit of cooperation among receivers in a fading channel following the model described in (1)-(2), we bring here some upper and lower bounds.

There are basically three types of bounds relevant for our channel model. The first is the outage capacity, which is the ultimate average rate achievable using a single level code (without multi-layer coding). The broadcasting achievable rates, refer to a scheme using continuous broadcast approach. And finally, the ergodic capacity gives the ultimate upper bound on average rates, by averaging maximal rates obtained with full transmitter CSI.

The lower bounds are obtained by considering no-cooperation. That is a single transmitter receiver pair, with no cooperating user. Hence all lower bounds correspond to SISO fading channel capacities. The upper bounds refer to the case where a colocated cooperating user exists, and the two users can share all resources and information, in order to perform optimal joint decoding, of the two received signals. Such upper bound is essentially a SIMO fading channel model with two receiving antennas, for which capacities are specified in the following. In all cases the bounds relate to a Gaussian block fading channel, adhering to (1)-(2).

A. Lower Bounds

One immediate lower bound is the single receiver lower bound. That is, the outage and broadcasting average rates [28] are computed for a single user, assuming there are no available users for cooperation. The distribution of the fading gain of a single user over a Rayleigh channel is given by $F(u) = 1 - e^{-u}$.

1) Outage lower bound: The achievable single-level coding average rate is given by

$$R_{outage,LB} = \max_{u_{th}>0} \{(1 - F(u_{th})) \log(1 + u_{th} P_s)\} \tag{5}$$

where the optimal threshold u_{th} which maximizes (5) is given by $u_{th,opt} = \frac{P_s - W(P_s)}{W(P_s)P_s}$. The function $W(x)$ is the Lambert-W function [41], also known as the Omega function.

2) Broadcasting lower bound: The broadcasting lower bound is based on a SISO block fading channel, where only receiver has perfect CSI. The maximal achievable broadcasting rate was generally obtained in [28], for any fading gain distribution. The bound here is specified for a Rayleigh fading channel. The average achievable broadcasting rate is given by [28],

$$R_{bs,LB} = e^{-1} - e^{-s_0} + 2E_1(s_0) - 2E_1(1) \tag{6}$$

where $s_0 = 2/(1 + \sqrt{1 + 4P_s})$, and $E_1(x)$ is the exponential integral function $E_1(x) = \int_x^\infty dt \frac{e^{-t}}{t}$ for $x \geq 0$. Further insight and a short overview of the broadcast approach can be found in Section V.

3) Ergodic lower bound: The ergodic capacity for a general SIMO channel with m receive antennas is [42]

$$C_{erg}(m) = \int\limits_0^\infty u^{m-1} e^{-u} \log(1 + P_s u) du, \quad m = 1, 2, ... \tag{7}$$

and for a SISO channel we get the ergodic lower bound

$$C_{erg,LB} = C_{erg}(1) = e^{1/P_s} E_1(1/P_s). \tag{8}$$

B. Upper Bounds

A natural upper bound here is the joint decoding upper bound. In this case a single receiver with two antennas and optimal processing is assumed. The distribution of an equivalent fading gain of a channel with two fully-cooperating agents is $F_{UB}(u) = 1 - e^{-u} - ue^{-u}$.

1) Outage upper bound: An outage bound for fully cooperating users is derived similarly to (5), with $F_{UB}(u)$ as the fading gain distribution function.

2) Broadcasting upper bound: The broadcasting upper bound is based on a SIMO block fading channel with two receiver antennas, which is also a special case of [28]. The bound here is specified for a Rayleigh fading channel. The corresponding average broadcasting rate is

$$\begin{aligned} R_{bs,UB} = \quad & s_1 e^{-s_1} - e^{-s_1} - 3E_1(s_1) \\ & -(s_0 e^{-s_0} - e^{-s_0} - 3E_1(s_0)) \end{aligned} \tag{9}$$

where s_0 and s_1 are determined by the boundary conditions $I_{UB}(s_0) = P_s$ and $I_{UB}(s_1) = 0$, respectively. The residual interference $I_{UB}(x)$ is given by $I_{UB}(x) = (1 + x - x^2)/x^3$.

3) Ergodic upper bound: The ergodic upper bound for two fully cooperative users is given by $C_{erg}(2)$,

$$\begin{aligned} C_{erg,UB} \quad = \quad & C_{erg}(2) = 1 + e^{1/P_s} E_1(1/P_s) \\ & -1/P_s e^{1/P_s} E_1(1/P_s). \end{aligned} \tag{10}$$

Figure 2 illustrates the lower and upper bounds of two cooperating users.

4) Single session Cut-set upper bound: Another upper bound considered is the classical cut-set bound of the relay channel [35]. This bound may be useful for single session cooperation, where the capacity of the cooperation link is rather small.

Using the relay channel definitions in (1)-(2), and assuming a single cooperation session $K = 1$, the cut-set bound for a Rayleigh fading channel is given by:

$$\begin{aligned} C_{cut-set} \quad = \quad & \sup_{p(x_s),p(x_2)} \min\{I(x_s; y_1|h_1) + I(x_2; y_{1,2}), \\ & \qquad I(x_s; y_1, y_2|h_1, h_2)\} \\ & = \min\{C_{erg}(1) + C_{coop},\ C_{erg}(2)\} \end{aligned} \tag{11}$$

where the ergodic capacity $C_{erg}(m)$ is given by (7), and $C_{erg}(1)$, $C_{erg}(2)$ are specified in (8), (10), respectively. Specifically, the cut-set bound is tight only when $C_{erg}(1) + C_{coop} \leq C_{erg}(2)$, since otherwise the cut-set bound coincides with the ergodic upper bound $C_{erg,UB}$ in (10).

5) DF upper bounds: The above upper bounds serve well as DF upper bounds, however tighter bounds can be obtained by noticing that the maximal achievable DF rate is the maximal separately achievable rate by the two users. Therefore the DF upper bound distribution is the distribution of $s = \max\left(|h_1|^2, |h_2|^2\right)$. Hence the cdf of s is given by

$$F_{DF,UB}(s) = 1 + e^{-2s} - 2e^{-s}. \tag{12}$$

The above outage and broadcasting capacities may be computed for the DF, as upper bounds following the same steps with the cdf in (12).

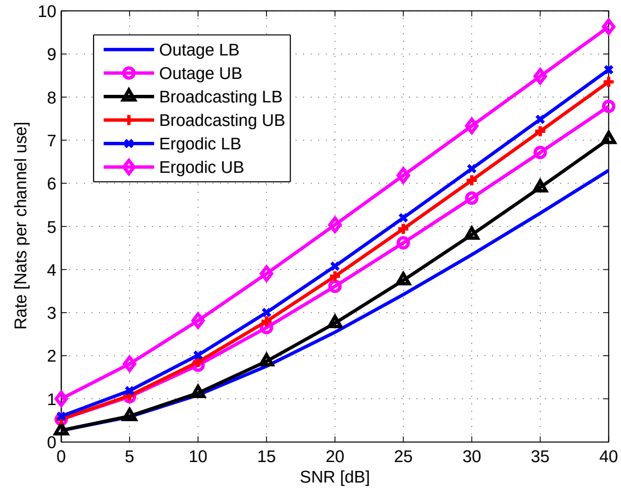

Fig. 2. Ranges of the average rates for both outage and broadcast approaches, over the cooperation channel, which were calculated using these approaches for either single antenna user (LB) or two antennas user (UB). The corresponding rate-range for an ergodic channel from (8) and (10) is also given for comparison.

V. AMPLIFY FORWARD COOPERATION

In what follows, we consider three types of AF cooperation schemes:

1) *Naive* - In this cooperation scheme the relaying user directly scales its input to the available transmit power P_r, and forwards the scaled channel output to the destination user using a single session $K = 1$. The destination then decodes the data based on its direct link channel output y_1 and the output of the cooperation link $y_{1,2}^{(1)}$.
2) *Separate preprocessing* - Relay first removes separately decodable common layers, then transmits the residual signal to destination. That is, each receiver attempts decoding on its own. Then both users exchange the index of the highest reliably decoded layer. The relay removes the commonly decoded layers from its input signal. Finally, the relaying user scales the residual signal to P_r, and forwards it to destination. This forms a single cooperation session ($K = 1$). The destination then optimally combines its own copy of the residual signal and the relayed version, and decodes as many layers as possible.
3) *Multi-session* - This scheme consists of multiple cooperation blocks ($K \to \infty$) with separate preprocessing per cooperation session, and a total power constraint P_r for all the cooperation sessions. In order to maintain maximal average throughput, a wide-band cooperation link, as specified in (4), is required. In this setting, common layers are removed before every AF session by both users, and after every AF transmission each user tries to decode more layers based on all received AF signals and its original received input signal. A closed form expression for the achievable rate is derived for unlimited number of sessions, assuming an overall

power constraint P_r for all sessions.

A. Naive AF Cooperation

In the naive AF strategy, the relaying user ($i = 2$) scales its input to the available transmit power P_r, and forwards the signal to the destination user ($i = 1$). The received signal at the destination, after AF, is

$$\mathbf{y}_b = \begin{bmatrix} \mathbf{y}_{1,2}^{(1)} \\ \mathbf{y}_1 \end{bmatrix} = \begin{bmatrix} \alpha h_2 \mathbf{x}_s + \alpha \mathbf{n}_2 + \mathbf{w}_2 \\ h_1 \mathbf{x}_s + \mathbf{n}_1 \end{bmatrix} = \begin{bmatrix} (\sqrt{\beta}\mathbf{x}_s + \widetilde{\mathbf{w}}_2) \cdot \sqrt{1+\alpha^2} \\ h_1 \mathbf{x}_s + \mathbf{n}_1 \end{bmatrix} \quad (13)$$

where $\mathbf{y}_b$ is the signal to be decoded at the destination, and the scaling factor α scales the transmit power to P_r, thus $\alpha = \sqrt{\frac{P_r}{1+P_s s_2}}$, where $s_i = |h_i|^2$. The normalized noise vector $\widetilde{\mathbf{w}}_2$ has i.i.d elements distributed $\mathcal{CN}(0,1)$, hence the normalized signal gain after the scaling of user $i = 2$ is

$$\beta = \frac{P_r s_2}{1 + P_s s_2 + P_r}. \quad (14)$$

The achievable rate as a function of the channel fading gains is given by the following mutual information

$$I(x_s; \mathbf{y}_b | h_1, h_2) = \log(1 + P_s s_b) = \log\left(1 + P_s\left(s_1 + \frac{P_r s_2}{1 + P_s s_2 + P_r}\right)\right), \quad (15)$$

where $s_b = s_1 + \beta$, and therefore the continuous broadcasting equivalent fading parameter is s_b. This requires the derivation of the cdf of s_b, [28]

$$F_{s_b}(x) = Prob(s_b \le x) = \int_0^\infty du f_{s_1}(u) \int_0^{\max\left(0, x - \frac{P_r u}{1+P_s u + P_r}\right)} dv f_{s_2}(v), \quad (16)$$

where $f_{s_i}(u)$ is the pdf of s_i. For a Rayleigh fading channel with $f_{s_i}(u) = e^{-u}$ the cdf of s_b is explicitly given by

$$F_{s_b}(x) = \begin{cases} 0 & \cdots\cdots x \le 0 \\ 1 - e^{-\frac{(1+P_r)x}{P_r - P_s x}} - \int_0^{\frac{(1+P_r)x}{P_r - P_s x}} du \cdot e^{-u - x + \frac{P_r u}{1+P_s u + P_r}} & \cdots\cdots 0 \le x < \frac{P_r}{P_s} \\ 1 - \int_0^\infty du \cdot e^{-u - x + \frac{P_r u}{1+P_s u + P_r}} & \cdots\cdots x \ge \frac{P_r}{P_s} \end{cases} \quad (17)$$

The corresponding pdf $f_{s_b}(x)$ is given by

$$f_{s_b}(x) = \begin{cases} 0 x \le 0 & \\ \int_0^{\frac{(1+P_r)x}{P_r - P_s x}} du \cdot e^{-u - x + \frac{P_r u}{1+P_s u + P_r}} & 0 \le x < \frac{P_r}{P_s} \\ \int_0^\infty du \cdot e^{-u - x + \frac{P_r u}{1+P_s u + P_r}} & x \ge \frac{P_r}{P_s} \end{cases} \quad (18)$$

We can now state the outage and broadcasting achievable rates for the naive AF.

1) Outage approach: Using the result of the fading power distribution in (17)-(18), one can optimize for maximum average rate using a single level code. Since it has a SISO equivalent representation with fading cdf specified in (17), the maximal average outage approach rate is

$$R_{out} = \max_{x>0} (1 - F_{s_b}(x)) \log(1 + x P_s), \quad (19)$$

where the transmitter uses code rate which is given by $\log(1 + x P_s)$. The rate R_{out} can be numerically evaluated.

2) Broadcast approach: In this approach the transmitter performs continuous code layering, matched to the equivalent fading random variable s_b from equation (16). For completeness of presentation, we quickly review the principles of the broadcast approach, the incremental rate as function of power allocation is [28]

$$dR(u) = \log\left(1 + \frac{\rho(u) u du}{1 + I(u) u}\right) = \frac{\rho(u) u du}{1 + I(u) u} \quad (20)$$

where $I(u)$ is the residual interference function, such that $I(0) = P_s$, and $\rho(u) = -\frac{d}{du} I(u)$ is the power allocation density function. The maximal average rate is expressed as follows

$$R_{bs,avg} = \max_{I(u)} \int_0^\infty du (1 - F_{s_b}(u)) \frac{\rho(u) u}{1 + I(u) u} \quad (21)$$

where $F_\nu(u)$ is the cumulative distribution function (cdf) of the fading gain random variable. It can be shown [28] that the optimal power allocation is given by

$$I_{NAF}(u) = \begin{cases} P_s & u < u_0 \\ \frac{1 - F_{s_b}(u) - u \cdot f_{s_b}(u)}{u^2 f_{s_b}(u)} & u_0 \le u \le u_1 \\ 0 & u > u_1 \end{cases} \quad (22)$$

where u_0 and u_1 are obtained from the boundary conditions $I_{opt}(u_0) = P_s$, and $I_{opt}(u_1) = 0$, respectively. The broadcasting gain is always compared to the single level coding under the same fading gain distribution.

Using the equivalent SISO channel model which is governed by s_b , specified in (15), with its cdf $F_{s_b}(u)$ in (17), the optimal power allocation for naive-AF can be specified, following (22). Notice that $I_{NAF}(u)$ is non-increasing, starting from P_s at $u = 0$. The average rate is explicitly given by

$$R_{NAF} = \int_0^\infty dx \left[\frac{2(1 - F_{s_b}(x))}{x} + \frac{(1 - F_{s_b}(x)) f'_{s_b}(x)}{f_{s_b}(x)}\right]. \quad (23)$$

The first derivative of the pdf of s_b is denoted by $f'_{s_b}(x)$.

B. Amplify Forward with Separate Preprocessing

In this approach, we assume that every user attempts decoding as many layers as possible independently, before cooperation. Then both users exchange the index of the highest layer successfully decoded. Every user re-encodes the decoded data, up to the lower index (reconstructing only common information) and subtracts it from the original received signal.

The relaying user scales the result into power P_r and transmits over the cooperation link to the destination $i=1$. This is better than the naive AF, since the cooperation is more efficient, resulting in higher equivalent gains. Like the naive AF, it requires only single session $K=1$, but unlike the naive AF, it requires the knowledge of the destination fading gain at the relay. The received signal at the second user side can be expressed as follows,

$$\boldsymbol{y}_2 = h_2(\boldsymbol{x}_{s,D} + \boldsymbol{x}_{s,I}) + \boldsymbol{n}_2, \tag{24}$$

where $\boldsymbol{x}_{s,D}$ is the part of the source data successfully independently decoded by user $i=2$. The residual interference signal is then denoted $\boldsymbol{x}_{s,I}$, which includes coded layers not decoded independently.

Assuming that $s_1 \geq s_2$, then the decoded data in $\boldsymbol{x}_{s,D}$ will include layers up to the parameter s_2. Let the residual interference power be denoted by $I(s)$, where s is the fading gain equivalent. Thus after removing layers up to s_2 the residual interference power is given by $I(s_2)$. The residual signals at both sides (before a cooperation session) are then given by

$$\boldsymbol{y}_{1,I} = h_1 \boldsymbol{x}_{s,I(s_2)} + \boldsymbol{n}_1. \tag{25}$$
$$\boldsymbol{y}_{2,I} = h_2 \boldsymbol{x}_{s,I(s_2)} + \boldsymbol{n}_2. \tag{26}$$

It can be shown, following the same lines of AF derivation, that the equivalent fading gain, after amplifying and forwarding $y_{2,I}$, is (27). In general, the cooperating user removes only common information from its input signal and forwards the residual signal to the destination. That is, each user tries decoding separately as many layers as possible. The destination user receives a forwarded residual signal, containing only its undecoded layers, when the helping user has better channel conditions. If the helping user has worse channel conditions, it transmits its scaled residual interference, including layers which could be independently decoded by the destination. The equivalent fading gain observed by the destination, and its distribution are stated in the following proposition.

Proposition 5.1: In an AF with separate preprocessing cooperation strategy, with a single cooperation session $K=1$ (power P_r), the highest decodable layer is associated with an equivalent fading gain determined by

$$s_a = s_1 + \frac{P_r s_2}{1 + s_2 \cdot \max(I(s_1), I(s_2)) + P_r}, \tag{27}$$

with the following cdf for a Rayleigh fading channel,

$$F_{s_a}(x) = \int_0^{\phi_1^{-1}(x)} [\exp(-2u) - \exp(-u - \phi_2(u)) - \exp(-u - \phi_3(u))] du. \tag{28}$$

where

$$\begin{aligned} \phi_1(u) &= u + \tfrac{uP_r}{1+uI(u)+P_r} \\ \phi_2(u) &= \max\left(u, x - \tfrac{uP_r}{1+uI(u)+P_r}\right) \\ \phi_3(u) &= \max(u, \phi_4(x-u)) \end{aligned} \tag{29}$$

where

$$\phi_4(x-u) = \begin{cases} \frac{(1+P_r)(x-u)}{P_r - I(u)(x-u)} & P_r - I(u)(x-u) > 0 \\ \infty & P_r - I(u)(x-u) <= 0 \end{cases} \tag{30}$$

Proof: See Appendix A.

Note that in the AF with separate preprocessing strategy, we have implicitly assumed that if both users can decode all layers independently, then no forwarding is done. This saves a fraction of the relaying power P_r, and under long-term power constraint on the relay, AF transmission power may be increased by $\frac{1}{1-P_{bs}}$, where P_{bs} is the probability that both users will successfully decode all layers. The probability P_{bs} clearly depends on the available power for cooperation $\frac{1}{1-P_{bs}} P_r$, which gives a recursive definition for P_{bs}. Thus the extension of the results for the case of long-term average cooperation power constraint is non-trivial.

The expressions for the broadcasting average rate include the function $I(s)$ as part of the equivalent fading gain cdf $F_{s_a}(x)$, and in an integral form. This turns the optimization problem of the average rate to be a difficult one. And it seems that no closed analytical solution for optimal $I(s)$ can be found. We suggest a few sub-optimal approaches to maximize the achievable average broadcasting rate:

1) *One step sub-optimal* $I_{sub-opt}(s)$. Use a sub-optimal power allocation, $I_{NAF}(s)$, which is the optimal naive AF power allocation, specified in (22), to compute the corresponding cdf of the equivalent fading gain is (28). Then use these distributions to compute $R_{bs,sub-opt}$.
2) *Iterative solution of* $I(s)$. Assume at the first iteration that $I_0(s)$ is given by the naive AF function specified in (22). Calculate $F_{s_a,1}(x)$ using $I_0(s)$, and compute the corresponding average rate $R_{1,bs}$. In the second iteration, calculate $I_1(s)$ using $F_{s_a,1}(x)$ and equation (22). Go back to (27) and solve for $F_{s_a,2}(x)$ using $I_1(s)$, and compute $R_{2,bs}$. Repeat the same procedure till the difference $|R_{k,bs} - R_{k-1,bs}|$ is sufficiently small.
3) *Finite level coding*. The derivation of $F_{s_a}(x)$ in this case is doable, and the maximal average rates may be numerically computed. Although with two level coding, the efficiency of separate preprocessing may be very limited, since there are only two thresholds involved. So that separate preprocessing may help only when both users successfully decode the first layer, and could not decode the second layer.

C. Multi-Session Amplify and Forward with Separate Preprocessing

We consider here the multi-session AF with separate preprocessing per session. The total power allocation available for all sessions is P_r, where unlike previous schemes, here $K=\infty$. In this setting, common layers are subtracted before every AF session by both users, and after every AF transmission each user tries to decode more layers based on all received AF signals and its original received input signal. We find the average rate for unlimited number of sessions, assuming only an overall power constraint for all sessions. It should be emphasized that the multi-session is performed

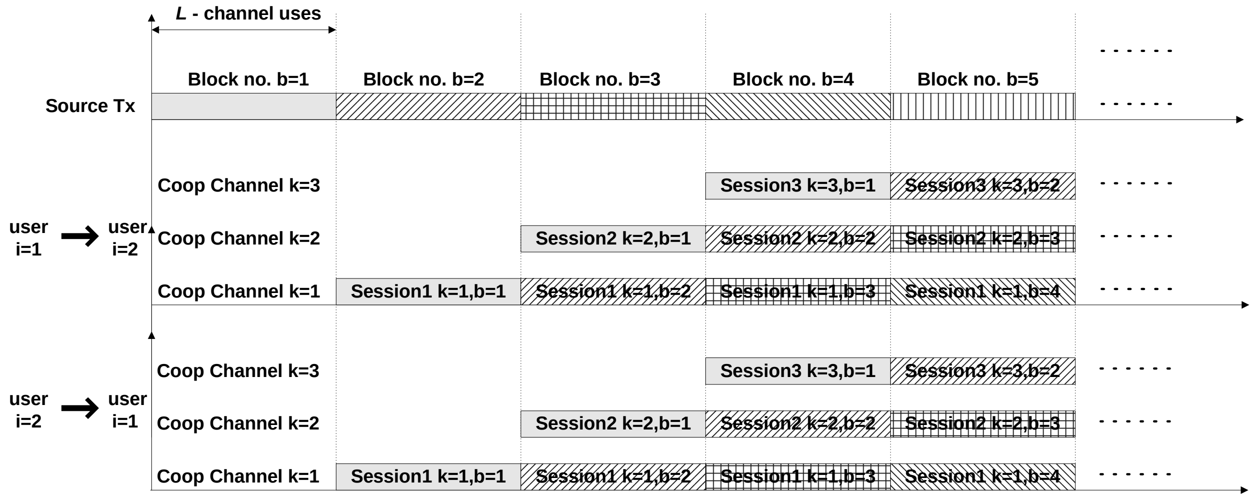

Fig. 3. Illustration of multi-session AF cooperation with $K = 3$ cooperation sessions per block. Block b refers to the b^{th} transmission for which there is a fixed fading level. The source transmits continuously information blocks, and simultaneous cooperation sessions take place on parallel channels.

over parallel channels (for example, OFDM), as illustrated in Figure 3, in such way that the source transmission is block-wise continuous. For example, during the k^{th} cooperation session of the 1^{st} transmitted block (from the source), the 1^{st} cooperation session for the $k-1$ transmitted block takes place. As the overall multi-session power is limited to P_r, at every block epoch a total power of P_r is used.

Since the cooperation is performed over parallel channels, with infinitesimal power $\rho(s)$ allocated per channel, the capacity of this wide-band cooperation link is the capacity of the corresponding parallel channel. The power allocation constraint enforces $\int_0^\infty \rho(s)ds = P_r$. The fractional rate per sub-band is then $dR(s) = \log(1+\rho(s)ds) = \rho(s)ds$, [43]. Therefore, the average capacity of this wide-band cooperation link, regardless of the actual power allocation density is $C_{coop} = P_r$, see (4). Notice that we use AF, which can not effectively use such capacity increase in a single session cooperation ($P_r > \log(1+P_r)$). Note that this capacity is available in two directions: relay-destination and destination-relay. It is required that information is exchanged in both directions, otherwise multi-session cooperation becomes inefficient, and unidirectional transmission (only relay to destination) will not benefit from multi-session relaying.

In the case of unlimited sessions, the scalar equivalent fading gain can be derived for a given broadcasting power allocation $I(s)$. From the equivalent fading gain a cdf can be computed, from which the average achievable rate can be obtained.

Proposition 5.2: In a multi-session AF ($K \to \infty$, cooperation power constraint P_r) with separate preprocessing cooperation strategy, the highest decodable layer is associated with an equivalent fading gain determined by

$$s_{ms} = \begin{cases} s_a^* & s_1 \geq s_2 \\ s_b^* & s_1 < s_2 \end{cases} \tag{31}$$

where s_b^* is the solution of the following equation,

$$\int_{s_2}^{s_b^*} \frac{s_1}{(s_1+s_2-\sigma)^2}[1+s_1 I(\sigma)]d\sigma = P_r, \tag{32}$$

and by using s_b^*,

$$s_a^* = s_1 + s_2 \frac{Z(s_b^*)}{1+Z(s_b^*)}. \tag{33}$$

where

$$Z(s) = \int_{s_2}^{s} \frac{1+s_1 I(\sigma)}{(1+s_2 I(\sigma))} \frac{s_1}{(s_1+s_2-\sigma)} d\sigma \tag{34}$$

Proof: See Appendix B.

For a given power allocation $I(s)$, computation of the cdf of s_{ms} is quite involved, as it requires solving equation (32) for every pair (s_1, s_2) subject to $s_1 \geq s_2$. Hence the optimization of $I(s)$ to maximize the achievable rate does not seem doable. For the numerical results we use $I(s)$ corresponding to optimal broadcasting in presence of optimal joint decoding. This selection is demonstrated (see Section VIII) to be a good one, particularly for high P_s and P_r, as such conditions allow approximation of optimal performance with multi-session AF cooperation.

We have used a continuous power allocation function for the multi-session cooperation link power $\delta(s)$ (see appendix B), such that for every session, different power may be used (s here serves as a continuous session index). Identical $\delta(s)$ for both cooperation directions are used to simplify derivation, although such restriction is suboptimal, since $\delta(s)$ is chosen to maximize s_b^*, which is not equal to averaged rate (which includes also s_a^*). In addition, the scheme is suboptimal by letting user $i = 1$ forward layers from $s_b^{(k)}$, rather than $s_a^{(k)}$, so it forwards a layered transmission instead of a direct transmission (which is more efficient, since the cooperative channel is non-fading).

Notice that both s_a^* and s_b^* reach $s_1 + s_2$ when $P_r \to \infty$, which is the same case with the other AF approaches. The difference, however, is in the convergence rate to s_1+s_2 of the various cooperation schemes. This is demonstrated in the numerical results section VIII.

VI. Compress Forward Cooperation

In this section we consider compress forward (CF) cooperation. Both users are capable of quantizing and compressing their received signals and forwarding the result to one another. The compression here relies on the well known Wyner-Ziv [36] compression using side information at the decoder. Similar to the AF, here too, we consider three ways of implementing the basic cooperation.

A. Naive CF Cooperation

Consider the channel model in (1)-(2). The signal to be sent to the destination user $\hat{\boldsymbol{y}}_1$, is compressed in the Wyner-Ziv spirit, and is given by

$$\hat{\boldsymbol{y}}_2 = \boldsymbol{y}_2 + \boldsymbol{n}_c = h_2 \boldsymbol{x}_s + \boldsymbol{n}_2 + \boldsymbol{n}_c, \tag{35}$$

where $n_c \sim \mathcal{CN}(0, \sigma^2)$ is the compression noise, which is independent of y_2. Then the achievable rate, for the destination user ($i=1$), is given by

$$\begin{aligned} &R_{WZ,2}(h_1,h_2) = I(x_s; y_1, \hat{y}_2 | h_1, h_2) \\ &\text{s.t.} \quad I(y_2;\hat{y}_2|h_2) - I(y_1;\hat{y}_2|h_1,h_2) \le C_{coop} \end{aligned} \tag{36}$$

where $R_{WZ,2}(h_1,h_2)$ is maximized when the constraint is met with equality. The constraint C_{coop} represents cooperation link capacity. We consider two cases for the naive CF:

1) *Narrow-band naive CF* - In this case the cooperation bandwidth is equal to the source-relay link bandwidth ($K=1$), and therefore the cooperation capacity is $C_{coop} = \log(1+P_r)$.
2) *Wide-band naive CF* - In this case the cooperation bandwidth is unlimited ($K=\infty$), and according to (4) the cooperation capacity is $C_{coop}=P_r$, when fractional power is allocated per sub-band.

When requiring that the constraint in (36) will be met by equality, with a narrow-band cooperation link, the resulting quantization noise variance σ^2 is (C.7),

$$\sigma^2_{NB} = \frac{1+s_1P_s+s_2P_s}{P_r(1+s_1P_s)} \tag{37}$$

Proposition 6.1: In a Narrow-band Naive Wyner-Ziv compression cooperation strategy, the highest decodable layer is associated with an equivalent fading gain determined by

$$s_{NWZ} = s_1 + \frac{s_2(1+s_1P_s)P_r}{(1+P_r)(1+s_1P_s)+s_2P_s}. \tag{38}$$

The distribution $F_{s_{NWZ}}(u)$ of s_{NWZ} over a Rayleigh fading channel with $F_{s_i}(u) = 1-e^{-u}$, for $i=1,2$ is, when $u \ge \frac{P_r}{P_s}$:

$$F_{s_{NWZ}}(u) = 1 - e^{-u}\left(1+\frac{P_r(uP_s+1)}{P_s(P_r+1)}\right) + \frac{P_r(uP_s+1)^2}{P_s^2(P_r+1)^2} e^{-\frac{uP_rP_s-1}{P_s(P_r+1)}}\,\mathrm{Ei}\left(1,\frac{uP_s+1}{P_s(P_r+1)}\right) \tag{39}$$

and when $u < \frac{P_r}{P_s}$:

$$F_{s_{NWZ}}(u) = 1 - e^{-u}\left(1+\frac{P_r(uP_s+1)}{P_s(P_r+1)}\right) - \frac{P_r-uP_s}{(1+P_r)P_s} e^{-\frac{u}{P_r-uP_s}} + \frac{P_r(uP_s+1)^2}{P_s^2(P_r+1)^2} e^{-\frac{uP_rP_s-1}{P_s(P_r+1)}} \left(\mathrm{Ei}\left(1,\frac{uP_s+1}{P_s(P_r+1)}\right) - \mathrm{Ei}\left(1,\frac{P_r(uP_s+1)^2}{P_s(P_r+1)(P_r-uP_s)}\right)\right). \tag{40}$$

Proof: See Appendix C.

In the same lines of derivation for the narrow-band cooperation, when requiring that the constraint in (36) will be met by equality, for a wide-band cooperation link ($C_{coop}=P_r$), the resulting $\mathrm{E}|n_c|^2=\sigma^2$ is (C.8),

$$\sigma^2_{WB} = \frac{1+s_1P_s+s_2P_s}{(e^{P_r}-1)(1+s_1P_s)} \tag{41}$$

where it may be noticed that in a wide-band cooperation regime the noise variance of the compressed signal decays exponentially fast with P_r.

B. Wyner-Ziv Compress and Forward with Separate Preprocessing

Let us repeat what was done for the Amplify and Forward with separate preprocessing in subsection V-B, for Wyner-Ziv compression. For consistency, assume that $s_1 > s_2$, and then replace P_s by $I(s_2)$ in (37), by introducing the preprocessing, and letting the receivers subtract the decoded message before compressing and forwarding. We get that (37) is now

$$\sigma^2 = \frac{1+s_2I(s_2)+s_1I(s_2)}{P_r[1+s_1I(s_2)]} \tag{42}$$

and the equivalent signal to noise ratio at $i=1$, after the first iteration is now written by (38) and (42) as

$$s_a = s_1 + \frac{s_2P_r[1+s_1I(s_2)]}{(1+P_r)[1+s_1I(s_2)]+s_2I(s_2)}. \tag{43}$$

When further iterations are involved, we will denote s_a by $s_a^{(i)}$. Specifically, $s_a^{(1)}$ is already given in (43).

C. Multiple Sessions with Wyner-Ziv Compression and Separate Preprocessing

A similar approach to multi-session AF is considered for the Wyner-Ziv compress and forward. For this to be performed, several definitions are in order. Notice that each step of Wyner-Ziv compression can use all information collected in the previous sessions, in the form of side information.

Define $\hat{\boldsymbol{y}}_1^{(k)} = \boldsymbol{y}_1 + \boldsymbol{n}_{c,1}^{(k)}$, where $n_{c,1}^{(k)}$ is independent of y_1, as the compressed signal that is transmitted from $i=1$ to the colocated user, $i=2$. We refer the reader to [37], for successive Wyner-Ziv overview. Here, we deal with the case where the message that is transmitted in each session has better side information than the previous session, since more layers are decoded. Further, the second session can use the information sent by all the previous sessions, in order to improve performance. Since the power that is used by each session is a control parameter, rather than a fixed parameter, the use of an auxiliary variable that is transmitted during a session, but decoded only at the next session is superfluous (due to the better side information, declared as V in [37]). Next, using [37], the following Markov chain is defined, where unlike [37], we are interested in independent averaged distortion, rather than plain averaged distortion. The main feature here is that the compression noise $\boldsymbol{n}_{c,i}^{(k)}$ should decrease from iteration to iteration, ending up with a sequence of degraded channels $\hat{y}_i^{(k)}$, following the Markov chain:

$$y_2 - x_s - y_1 - \hat{y}_1^{(k)} - \hat{y}_1^{(k-1)} - \cdots - \hat{y}_1^{(1)} \quad (44)$$

$$y_1 - x_s - y_2 - \hat{y}_2^{(k)} - \hat{y}_2^{(k-1)} - \cdots - \hat{y}_2^{(1)} \quad (45)$$

The equivalent fading gains after every iteration of the multi-session cooperation are stated in the following proposition.

Proposition 6.2: The achievable rate in the multi-session with separate preprocessing and successive refinement WZ is given in a recursive form for the k^{th} session,

$$R_{WZ}^{(k)} = E_{s_{ms}^{(k)}} \log(1 + s_{ms}^{(k)} P_s) \quad (46)$$

where

$$s_{ms}^{(k)} = \begin{cases} s_a^{(k)} & s_1 \geq s_2 \\ s_b^{(k)} & s_1 < s_2 \end{cases} \quad (47)$$

and

$$s_a^{(k)} = s_1 + \frac{s_2}{1 + (\sigma_2^{(k)})^2} \quad (48)$$

$$s_b^{(k)} = s_2 + \frac{s_1}{1 + (\sigma_1^{(k)})^2}, \quad (49)$$

where $\sigma_j^{(k)}$ is specified in (50) for $j = 1, 2$, and where $\delta_j^{(k)}$ is the fractional power assigned to user j for the k^{th} cooperation session.

Proof: See Appendix D.

VII. Decode Forward Cooperation

We consider here the well known form of cooperation, namely Decode and Forward (DF). We present here bounds for the DF strategy, where the clear upper bound is the strongest user achievable rate (similar to selection diversity). From the nature of this approach, there is no place for considering multi-session, as after one session there is nothing the destination can send back to the relay for improving upon its independent decoding. For a fair comparison of DF cooperation to other multi-session techniques we consider both wide-band cooperation, where $C_{coop} = P_r$, see (4), and narrow-band cooperation (corresponding to the single session relaying techniques), where the cooperation link capacity is only $C_{coop} = \log(1 + P_r)$, see (3).

The DF strategy may be described as follows. The source performs continuous broadcasting, and two copies of the transmitted signal are received at destination and relaying side, as described by the channel model in (1)-(2). Recalling that the destination is denoted by user $i = 1$, then for $s_1 \geq s_2$ the destination user can decode at least as many layers as the relaying user. Hence there is place for DF cooperation **only** when $s_1 < s_2$, as in this case the relaying user can decode more layers than the destination. The **additional** layers decoded by the relay (for $s \in (s_1, s_2]$) are encoded by the relay and forwarded, constrained by the cooperation link capacity C_{coop}. For $P_r >> P_s$, a practically unlimited cooperation channel is available all additional information may be sent to destination and the DF upper bound is obtained.

Denote the decodable rate associated with a fading gain s by $R(s)$, where $R(s) = \int_0^s du \frac{\rho(u)u}{1+I(u)u}$. Say that before cooperation starts, user i decodes $R(s_i)$. As mentioned, cooperation is required for $s_1 < s_2$, and is limited by the relay link capacity C_{coop}. Hence for the pair (s_1, s_2), the achievable broadcasting rate is given by

$$R_{DF}(s_1, s_2) = \begin{cases} \min\{R(s_1) + C_{coop}, R(s_2)\} & s_2 > s_1 \\ R(s_1) & \text{otherwise} \end{cases}. \quad (51)$$

The optimal broadcasting power distribution maximizes the average rate, and the optimization problem is stated as follows,

$$\begin{aligned} \overline{R}_{DF} &= \max_{\rho(s) \geq 0, \text{s.t.} \int_0^\infty ds \rho(s) \leq P_s} E_{s_1,s_2} R_{DF}(s_1, s_2) \\ &= \max_{\rho(s) \geq 0, \text{s.t.} \int_0^\infty ds \rho(s) \leq P_s} \int_0^\infty ds_2 \int_0^{s_2} ds_1 f(s_1) f(s_2) \\ &\quad \min\{R(s_1) + C_{coop}, R(s_2)\} \\ &\quad + \int_0^\infty ds_2 \int_{s_2}^\infty ds_1 f(s_1) f(s_2) R(s_1) \end{aligned} \quad (52)$$

where $\rho(s) = -\frac{d}{ds} I(s)$ is the power density function. Finding the optimal power allocation seems intractable analytically, however $\overline{R}_{DF}$ could be computed for sub-optimal power distributions, such as the strongest user optimal $I_{sel,opt}(s)$, or for the no cooperation lower bound $I_{SU,opt}(s)$, and for $I_{Joint,opt}(s)$. These are defined and demonstrated in section VIII.

VIII. Numerical Results

In this section, we compare the broadcasting and outage achievable rates of the various cooperation methods, with narrow band cooperation links for all schemes, besides the multi-session with infinite bandwidth. Figures 4-5 demonstrate by numerical results the broadcasting AF and CF cooperation gains. Average achievable rates are computed for AF cooperation with a single session, which we have referred to as naive AF cooperation, and separate preprocessing. For the separate preprocessing we have used a sub-optimal power allocation which admits the optimal power allocation of naive AF broadcasting. Thus, in both cases we have the same power allocation, only in the latter, common information is removed prior to relaying. It may be noticed that when the SNR on the cooperation link satisfies $P_r \geq P_s$, the achievable rates are close to the joint processing upper bounds, where separate preprocessing is slightly better compared to the naive AF. However, when $P_r < P_s$ the separate preprocessing can introduce substantial gains over the naive AF. Note also that the computed separate preprocessing rate is yet a lower bound, since the optimal power allocation was not obtained. Separate preprocessing AF surpasses the outage upper bound

$$\left(\sigma_j^{(k)}\right)^2 = \left(\sigma_j^{(k-1)}\right)^2 \frac{1 + s_j I(s^{(k-1)}) + s_{3-j} I(s^{(k-1)})}{(1 + s_{3-j} I(s^{(k-1)}))\left[1 + \delta_j^{(k)}\left(1 + \left(\sigma_j^{(k-1)}\right)^2\right)\right] + s_j I(s^{(k-1)})(1 + \delta_j^{(k)})} \tag{50}$$

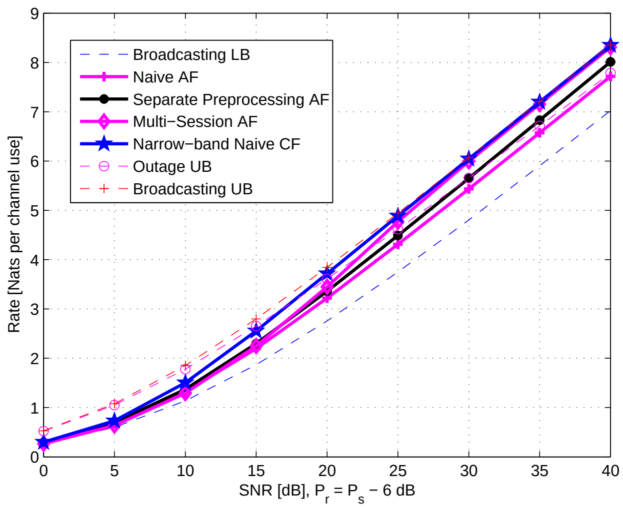

Fig. 4. Broadcast approach: average rates of Naive AF, AF with separate preprocessing, multi sessions AF and narrow-band (NB) naive CF compared to upper and lower bounds ($P_r = P_s - 6$ dB).

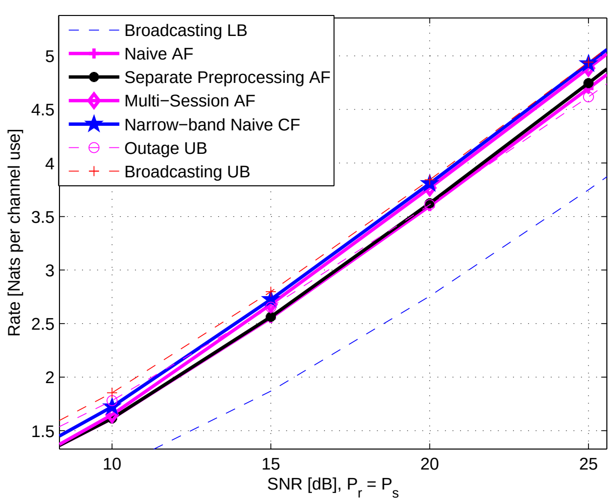

Fig. 5. Broadcast approach: average rates of Naive AF, AF with separate preprocessing, multi sessions AF and narrow-band (NB) naive CF compared to upper and lower bounds ($P_r = P_s$).

(joint processing with an outage approach) for high SNRs. For example, in Figure 4, where $P_r = P_s - 6$ dB, the separate preprocessing AF achieves a ~ 1 dB gain over the outage upper bound. The multi-session achievable rates are computed using proposition 5.2, for the broadband cooperation channel ($K \rightarrow \infty$). The sub-optimal power distribution function $I(s)$ used for the rate computation is the one corresponding to the broadcasting upper bound, specified in (9), which is

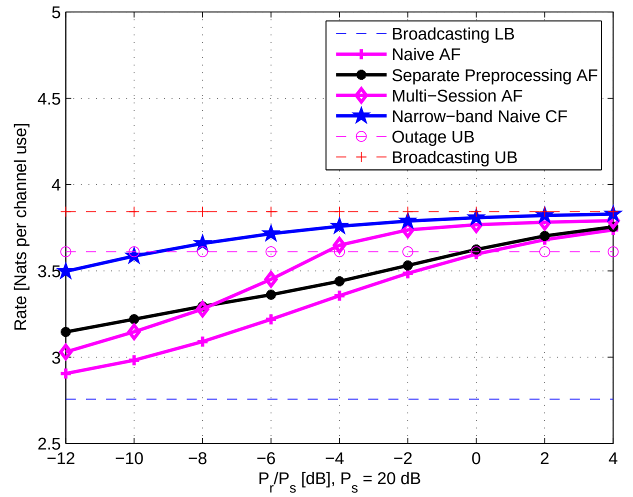

Fig. 6. Broadcast approach: average rates of Naive AF, AF with separate preprocessing, multi sessions AF and narrow-band (NB) naive CF compared to upper and lower bounds, as function of the channels quality ratio $\frac{P_r}{P_s}$. ($P_s = 20$ dB).

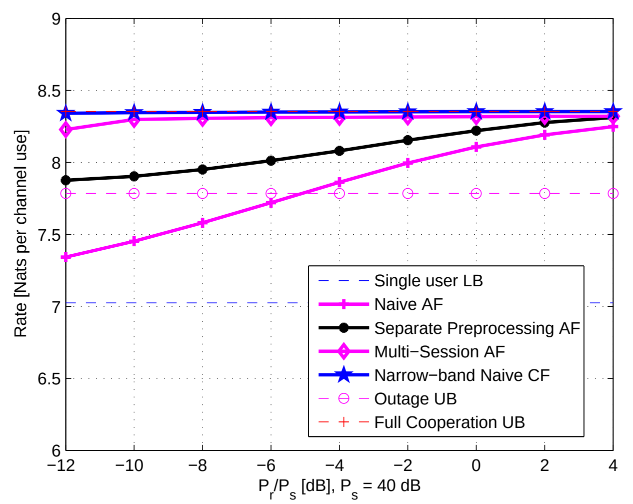

Fig. 7. Broadcast approach: average rates of Naive AF, AF with separate preprocessing, multi sessions AF and narrow-band (NB) naive CF compared to upper and lower bounds, as function of the channels quality ratio $\frac{P_r}{P_s}$. ($P_s = 40$ dB).

$I_{Joint,opt}(s) = \frac{1}{s^3} + \frac{1}{s^2} - \frac{1}{s}$. Interestingly, the average achievable rates with multi-session, with a sub–optimal power allocation approximate the broadcasting upper bound, for moderate and high SNRs, and for both $P_r/P_s = -6$, and 0 dB ratios in Figures 4 and 5, respectively. Another efficient approach is the narrow-band naive CF which uses the Wyner-Ziv (WZ) compression based cooperation. This approach seems to be the

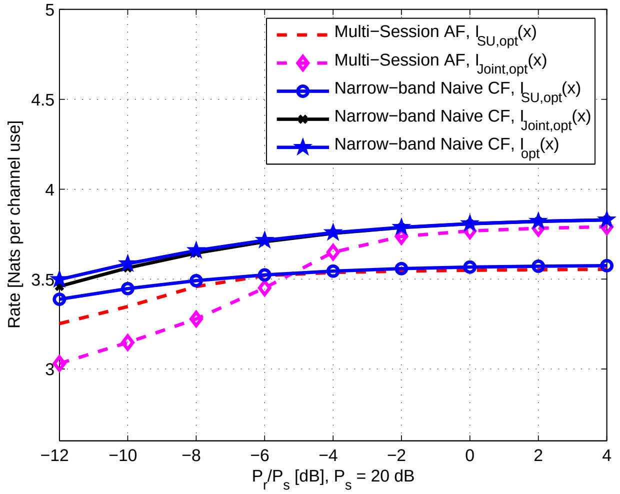

Fig. 8. Broadcast approach: average rates of multi sessions AF and narrow-band (NB) naive CF, as function of the channels quality ratio $\frac{P_r}{P_s}$. The approaches are compared as function of the broadcasting power allocation function, where $I_{SU,opt}(x)$ refers to the single user optimal power allocation, and $I_{Joint,opt}(x)$ denotes the function corresponding to full-cooperation joint decoding bound, and $I_{opt}(x)$ is the naive WZ optimal power allocation ($P_s = 20$ dB).

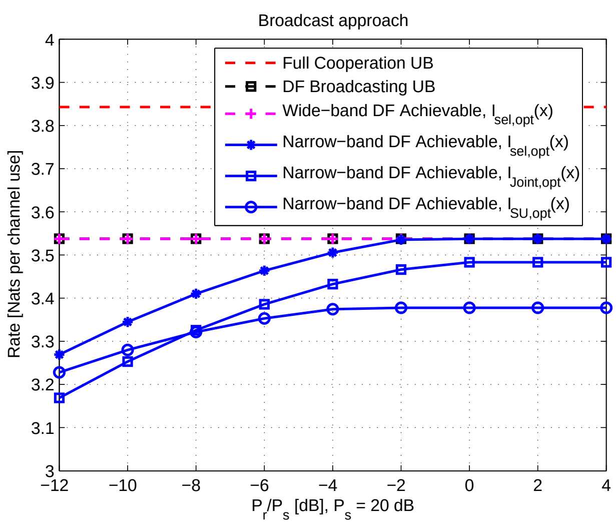

Fig. 9. Broadcast approach: average rates of DF achievable approaches, as function of the channels quality ratio $\frac{P_r}{P_s}$. The approaches are compared as function of the broadcasting power allocation function, where $I_{SU,opt}(x)$ refers to the single user optimal power allocation, and $I_{Joint,opt}(x)$ denotes the function corresponding to full-cooperation joint decoding bound, and $I_{sel,opt}(x)$ is the DF upper bound optimal power distribution. The DF achievable rates are computed using (52). Note that for single session $C_{coop} = \log(1 + P_r)$, and for multi-session $C_{coop} = P_R$. ($P_s = 20$ dB).

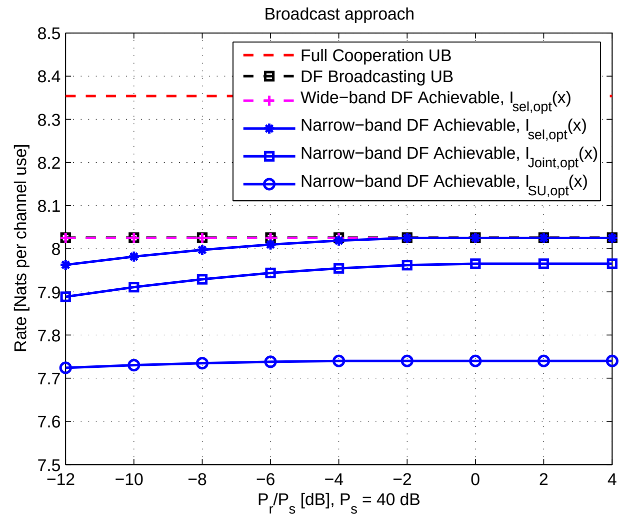

Fig. 10. Broadcast approach: average rates of DF achievable approaches, as function of the channels quality ratio $\frac{P_r}{P_s}$. The approaches are compared as function of the broadcasting power allocation function, where $I_{SU,opt}(x)$ refers to the single user optimal power allocation, and $I_{Joint,opt}(x)$ denotes the function corresponding to full-cooperation joint decoding bound, and $I_{sel,opt}(x)$ is the DF upper bound optimal power distribution. The DF achievable rates are computed using (52). Note that for single session $C_{coop} = \log(1 + P_r)$, and for multi-session $C_{coop} = P_R$. ($P_s = 40$ dB).

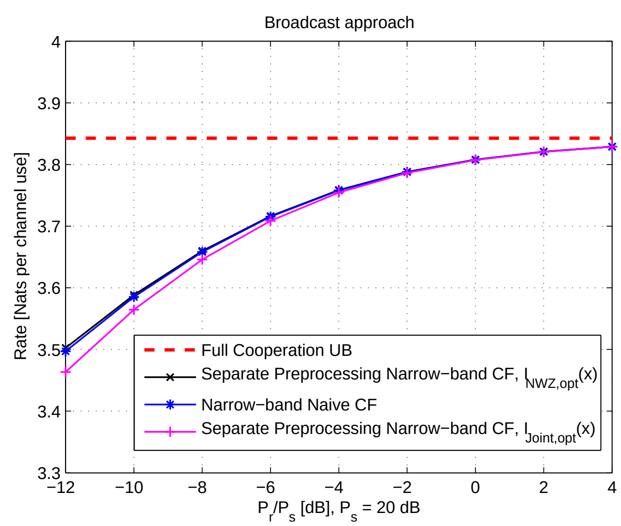

Fig. 11. Broadcast approach: average rates of CF achievable approaches, as function of the channels quality ratio $\frac{P_r}{P_s}$. The approaches are compared as function of the broadcasting power allocation function, where $I_{Joint,opt}(x)$ denotes the power distribution corresponding to full-cooperation joint decoding bound, and $I_{NWZ,opt}(x)$ is the optimal power allocation for naive WZ processing. The CF achievable rates are computed using (42) ($P_s = 20$ dB).

best approach out of all numerically evaluated settings. The naive WZ cooperation even closely approximates the separate processing WZ cooperation, as will be demonstrated in the following.

Figures 6-7 show a comparison between the naive AF, separate preprocessing AF, multi-session AF, and narrow-band naive CF, as function of the cooperation link quality (P_r/P_s). As may be noticed from these figures, the lower P_r/P_s is, the higher the gains of separate preprocessing AF, over the naive approach. For $P_s = 20$ dB, both approaches achieve gains over the outage upper bound for $P_r/P_s \geq 0$ dB. However, for $P_s = 40$ dB, the separate preprocessing AF substantially outperforms the outage upper bound for any $P_r/P_s \geq -12$ dB. In view of the multi-session AF in Figures 6-7, it seems

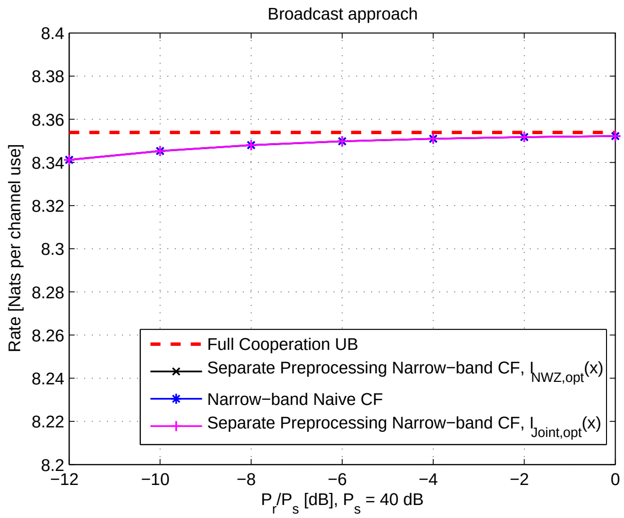

Fig. 12. Broadcast approach: average rates of CF achievable approaches, as function of the channels quality ratio $\frac{P_r}{P_s}$. The approaches are compared as function of the broadcasting power allocation function, where $I_{Joint,opt}(x)$ denotes the power distribution corresponding to full-cooperation joint decoding bound, and $I_{NWZ,opt}(x)$ is the optimal power allocation for naive WZ processing. The CF achievable rates are computed using (42) ($P_s = 40$ dB).

that for moderate to high P_s and P_r, the multi-session AF approximates the broadcasting upper bound. The naive CF, again, outperforms all other approaches, and approximates the broadcasting upper bound even on a wider range of P_r values.

Figure 8 demonstrates the implications of using sub-optimal power allocation for broadcasting in the AF multi-session and the narrow-band naive CF approaches. It may be noticed that for $P_r/P_s > -5$ dB it is more efficient to use the full cooperation optimal $I(s)$, however for lower relaying power values it is already preferable to use the single user optimal broadcasting power allocation. In the narrow-band naive CF approach the full cooperation optimal power distribution $I_{Joint,opt}(s)$ is highly efficient and approximates well the throughput with an optimal power allocation derived from the WZ approach. However, in the low P_r/P_s values both power allocations show close performance to that with the single user optimal power allocation.

Figures 9-10 demonstrate achievable rates of the DF approach with single session only (as there is no place for multi-session cooperation with a DF strategy only). Achievable rates are computed using (52) for narrow-band and wide-band cooperation channel link. The difference between the achievable rates is the power allocation strategy, which is sub-optimal for all three. This is since the optimal power distribution $I(x)$ for (52) is a difficult problem to solve analytically. The power allocations considered include $I_{SU,opt}(x)$ - single user optimal distribution; $I_{Joint,opt}(x)$ - full-cooperation joint decoding optimal distribution; and $I_{sel,opt}(x)$ is the optimal power distribution for strongest user (DF) upper bound. As may be noticed from the figures the best achievable rate of narrow-band cooperation uses $I_{sel,opt}(x)$, which also closely approximates the DF upper bound for $P_r \geq P_s$. For low SNRs and $P_r << P_s$ the achievable rate with $I_{SU,opt}(x)$ is slowest decaying, and will naturally be preferable in the extreme case of low P_r (which in the limit is the case of no effective cooperation). Additionally, over a wide-band cooperation link the DF cooperation closely approximates the DF upper bound in all considered P_r/P_s ratios and SNRs.

Figures 11-12, demonstrate achievable rates of the separate processing WZ cooperation scheme. As the optimal broadcasting power distribution does not lend itself to an analytical solution, sub-optimal power distributions are used. The power distributions used are $I_{Joint,opt}(x)$ and $I_{NWZ,opt}(x)$. The function $I_{Joint,opt}(x)$ is the optimal power allocation for a broadcasting with optimal joint decoding. This function is expected to closely approximate the optimal power allocation of separate processing WZ for $P_r \geq P_s$, and high SNRs, which is also the case where the naive WZ cooperation closely approximates the broadcasting upper bound (see also Figure 8). The function $I_{NWZ,opt}(x)$ is the optimal power distribution for naive WZ cooperation. As may be noticed from Figures 11-12, the separate processing WZ with these sub-optimal power distributions gains only marginally compared to the naive WZ cooperation. The largest evident gain is for $P_s = 20$ dB, and for $P_r << P_s$ (Figure 11) with $I_{NWZ,opt}(x)$. Using $I_{Joint,opt}(x)$ in these cases may turn out to be even less efficient than naive WZ cooperation. For $P_s = 40$ dB it may be noticed that the separate processing WZ gain is negligible. These results indicate that naive WZ is already highly efficient, and that separate processing might provide significant gains, however we are unable to fully justify this, since the optimal power distribution for separate processing WZ is unknown.

IX. Discussion

We have considered two relaying techniques - AF and CF, for a multi-session cooperation, when the transmitter employs a broadcast transmission approach. Our cooperation strategies are designed to enhance the overall throughput of a destination user, while a colocated user receives another copy of the original transmitted signal over an independent fading channel. Essentially, the results here are also valid for the case when a single source sends common information for two users, and they cooperate following the described schemes as to maximize their individual throughput. One may consider the following communication schemes, among others:

1) **Information enhancement** - in this case two users are receiving common information such as digital TV broadcasting, and the cooperation allows for image quality enhancement. Progressive transmission of images is also a useful application here [44],[45], where refinement of image quality is achieved through decoding of more coded layers. In this transmission scheme, no acknowledge (ACK) signal to the transmitter is required.
2) **Reliable throughput enhancement (RTE)** - common information streaming of data packets. In this case it is required that both users receive exactly the same information reliably. Hence after the end of the last cooperation session, an ACK signal is returned to the transmitter indicating what the highest common decoded layer was.

The *information enhancement* setting for transmitting common information for two users essentially achieves the **same average rates** as those derived for a single user above.

Our results can be adapted to the *reliable throughput enhancement* setting in the following way. As the maximal decoded layer depends on the actual fading gain, the reliable broadcasting rate is controlled by the smaller equivalent fading after cooperation. This is specified for all cooperation schemes:

- *Naive AF* - the equivalent fading gain $s_b = s_1 + \frac{P_r s_2}{1+P_s s_2+P_r}$ is replaced by $s_b^{RTE} = \min\left\{s_1 + \frac{P_r s_2}{1+P_s s_2+P_r}, s_2 + \frac{P_r s_1}{1+P_s s_1+P_r}\right\}$. Then the corresponding cdf allows computation of optimal power allocation and average rates for RTE setting, as in (16).
- *Separate Preprocessing AF* - the equivalent fading gain $s_a = s_1 + \frac{P_r s_2}{1+s_2\cdot\max(I(s_1),I(s_2))+P_r}$, is replaced by $s_a^{RTE} = \min\left\{s_1 + \frac{P_r s_2}{1+s_2\cdot\max(I(s_1),I(s_2))+P_r}, s_2 + \frac{P_r s_1}{1+s_1\cdot\max(I(s_1),I(s_2))+P_r}\right\}$. Then the corresponding cdf allows computation of optimal power allocation and average rates for RTE setting, as in (27).
- *Multi-session AF with broadband cooperation* - the equivalent fading gain, specified in (31), is replaced by s_b^*, which is defined there, below (31).
- *Naive CF* - the fading gain $s_{NWZ} = s_2 + \frac{s_1(1+s_2P_s)P_r}{(1+P_r)(1+s_2P_s)+s_1P_s}$ is replaced by $s_{NWZ}^{RTE} = \min\left\{s_1 + \frac{s_2(1+s_1P_s)P_r}{(1+P_r)(1+s_1P_s)+s_2P_s}, s_2 + \frac{s_1(1+s_2P_s)P_r}{(1+P_r)(1+s_2P_s)+s_1P_s}\right\}$, where the cdf of s_{NWZ}^{RTE} allows computation of maximal achievable RTE rates, as in (39)-(40).
- *CF with Separate Preprocessing* - similarly, the equivalent fading gain $s_a^{(1)} = s_1 + \frac{s_2P_r(1+s_1I(s_2))}{(1+P_r)(1+s_1I(s_2))+s_2I(s_2)}$ has to be replaced by $s_a^{(1)} = \min\left\{s_1 + \frac{s_2P_r(1+s_1I(s_2))}{(1+P_r)(1+s_1I(s_2))+s_2I(s_2)}, s_2 + \frac{s_1P_r(1+s_2I(s_1))}{(1+P_r)(1+s_2I(s_1))+s_1I(s_1)}\right\}$.
- *Multi-session CF with separate preprocessing* - the equivalent fading gain s_{ms}, as in (47), is replaced by $s_b^{(k)}$, which is defined in (49).

These direct permutations of the equivalent fading gains allow analysis of the RTE setting. This turns the extension of our results for the RTE scheme to be straightforward.

X. Conclusion

We have considered several cooperation strategies for transmission to colocated users. The original data is intended to one of the users, and in the network setting examined, a colocated user receives another copy of the original signal and cooperates with the destined user to improve decoding at the destination. As the transmitter has no access to CSI, the broadcast approach is used along with the various cooperation strategies. We have examined the naive AF, and its improved version, namely separate preprocessing AF. In the latter, the users decode individually as many layers as they can, subtract the common information and forward a scaled version of the residual signal. In a multi-session AF approach, with a total cooperation power limitation P_r, each user tries to decode as many layers as possible using the inputs of the previous cooperation session. Then the users remove common information and scale the residual signal for the next session of cooperation. We give an explicit formulation for large number of sessions, with a fractional power allocation for every session, and an overall power constraint P_r for the whole cooperation duration. This approach may be used in applications where rapidly changing channel does not allow CSI acquisition at the transmitter, and the decoding processing delay constraints are relatively relaxed, as multi-session imposes additional delays on the processing per packet. We have considered the average throughput as subject for optimization, however when considering average delay as the figure of merit, other broadcasting strategies can be optimal [46], and the multi-session cooperation may possibly introduce a non-negligible additional delay overhead.

Another cooperation approach considered is CF. In a naive approach the colocated user performs Wyner-Ziv (WZ) compression, and forwards it to the destination user. For the naive CF we derive explicit expressions for the equivalent fading gains, which allows computation of maximal achievable rates. An improved version of this approach for multi-session cooperation, is presented, where for each session, the WZ compression uses all information collected in previous sessions as side information for decoding. This brings notions such as successive refinable WZ coding. Implicit expressions are derived for the equivalent fading gain in a multi-session WZ cooperation.

We consider also DF cooperation with a single session only. That is since DF is unsuitable for multi-session cooperation, as after the first session there is nothing the destination can send back to the other user to improve its decoding.

Numerical results show that narrow-band separate processing CF outperforms all other considered approaches (for which average rates were computed). The narrow-band naive CF already closely approximates the joint decoding broadcasting upper bound. The multi-session AF with a sub-optimal broadcasting power allocation also approximates the broadcasting upper bound in a wide range of SNRs and P_r/P_s ratios. In light of the multi-session gains in the AF technique, and the good performance of the CF, we expect the multi-session CF, to present very good results, even for small SNRs. The DF numerical results show that the DF broadcasting upper bound is achieved by wide-band cooperation, and only marginally degrades for a narrow-band cooperation link.

Even though broadcasting optimal power allocation was obtained only for naive AF and CF, we conjecture that the maximal average rate of separate CF broadcasting is superior to that of separate AF broadcasting, and similarly with multi-session rates. Our conjecture is based on the numerical results in the SNR ranges where both AF and CF approximate the broadcasting upper bound. The continuous broadcasting achievable rates obtained are especially useful in understanding where multi-session may be beneficial when the optimal naive cooperation is far from the broadcasting upper bound.

Possible future directions include assuming multiple (k) colocated user cooperation strategies. In addition, colocation suggests that some type of correlation is imposed on fading channel realizations between users. Obtaining the achievable throughput with the cooperation strategies studied above, for correlated fading channels, is another possible extension of this work.

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APPENDIX A
AF WITH SEPARATE PREPROCESSING EQUIVALENT FADING

The cdf of s_a can be derived separately for $s_1 > s_2$ and for $s_2 \geq s_1$. This results in the following expression

$$F_{s_a}(x) = \int_0^{\phi_1^{-1}(x)} du f_{s_1}(u) \int_u^{\phi_2(u)} dv f_{s_2}(v) + \int_0^{\phi_1^{-1}(x)} du f_{s_2}(u) \int_u^{\phi_3(u)} dv f_{s_1}(v) \quad \text{(A.1)}$$

where

$$\begin{aligned} \phi_1(u) &= u + \tfrac{uP_r}{1+uI(u)+P_r} \\ \phi_2(u) &= \max\left(u, x - \tfrac{uP_r}{1+uI(u)+P_r}\right) \\ \phi_3(u) &= \max\left(u, \phi_4(x-u)\right) \end{aligned} \quad \text{(A.2)}$$

where

$$\phi_4(x-u) = \begin{cases} \frac{(1+P_r)(x-u)}{P_r - I(u)(x-u)} & P_r - I(u)(x-u) > 0 \\ \infty & P_r - I(u)(x-u) <= 0 \end{cases} \quad \text{(A.3)}$$

For a Rayleigh fading channel, the cdf in (A.1) reduces to a single integral expression, as specified in (28).

APPENDIX B
MULTI-SESSION AF WITH SEPARATE PREPROCESSING EQUIVALENT FADING

We assume without loss of generality that $s_1 > s_2$, and that both users are aware of the index s_2. Thus they decode independently all layers up to s_2 and remove the decoded signal from the received channel output. Then, they exchange the residual signal, amplified to power δ_1. User $i = 1$ can now decode up to the layer associated with the equivalent fading, similar to (27), but with different AF power, namely

$$s_a^{(1)} = s_1 + \frac{\delta_1 s_2}{1 + s_2 I(s_2) + \delta_1} \quad \text{(B.1)}$$

where the superscript of s_a indicates the cooperation session index. The received signal of the first session, at $i = 1$, is given by

$$\boldsymbol{y}_{1,2}^{(1)} = \sqrt{\beta_b^{(1)}}(h_2 \boldsymbol{x}_I^{(1)} + \boldsymbol{n}_2) + \boldsymbol{n}_c^{(1)}, \quad \text{(B.2)}$$

where $n_c^{(1)}$ is the noise on the cooperation link, and $\beta_b^{(1)} = \frac{\delta_1}{1+I(s_2)s_2}$. Where for user $i = 2$ similar relations exist. In the second session there is a higher common decoded layer $s_b^{(2)} \geq s_2$, and since both users forwarded with the same power, $s_a^{(2)} \geq s_b^{(2)}$. Thus both users remove decoded layers up to $s_b^{(2)}$ and amplify the residual signal over to the other user. The received signal on the k^{th} session, at user $i = 1$ is given by

$$\boldsymbol{y}_{1,2}^{(k)} = \sqrt{\beta_b^{(k)}}(h_2 \boldsymbol{x}_I^{(k)} + \boldsymbol{n}_2) + \boldsymbol{n}_c^{(k)}, \quad \text{(B.3)}$$

where similar expression exists for user $i = 2$. In order to perform optimal decoding, the following is done. All decoded layers are cancelled out from the cooperation inputs, $\{\boldsymbol{y}_{1,2}^{(i)}\}_{i=1}^{k-1}$, for user $i = 1$, and a maximal ratio combining of all inputs is performed. Thus the equivalent SNR for decoding at the k^{th} session in user $i = 1$ is

$$s_a^{(k)} \triangleq s_1 + \frac{s_2 \sum_{i=1}^{k} \beta_b^{(i)}}{1 + \sum_{i=1}^{k} \beta_b^{(i)}}, \quad \text{(B.4)}$$

where

$$\beta_b^{(i)} = \frac{\delta_i}{1 + I\left(s_b^{(i)}\right) s_2}. \quad \text{(B.5)}$$

The common layer, decoded at the k^{th} session, is associated with $s_b^{(k)}$, which is equal to:

$$s_b^{(k)} \triangleq s_2 + \frac{s_1 \sum_{i=1}^{k} \beta_a^{(i)}}{1 + \sum_{i=1}^{k} \beta_a^{(i)}}. \quad \text{(B.6)}$$

where

$$\beta_a^{(i)} = \frac{\delta_i}{1 + I\left(s_b^{(i)}\right) s_1}. \quad \text{(B.7)}$$

Note that also $s_b^{(k)} \leq s_a^{(k)}$, so that $i = 1$ will decode all that is decoded by $i = 2$, which is true since they both forward through $y_{1,2}$ and $y_{2,1}$, with the same powers: $\{\delta_i\}_1^k$, and since $s_1 \geq s_2$. The equivalent fading after the k^{th} session at user $i = 1$ and $i = 2$ is explicitly given by using (B.4) and (B.5),

$$s_a^{(k)} = s_1 + s_2 \frac{\sum_{i=1}^{k} \frac{\delta_i}{1+I\left(s_b^{(i)}\right)s_2}}{1 + \sum_{i=1}^{k} \frac{\delta_i}{1+I\left(s_b^{(i)}\right)s_2}} \quad \text{(B.8)}$$

and respectively

$$s_b^{(k)} = s_2 + s_1 \frac{\sum_{i=1}^{k} \frac{\delta_i}{1+I\left(s_b^{(i)}\right)s_1}}{1 + \sum_{i=1}^{k} \frac{\delta_i}{1+I\left(s_b^{(i)}\right)s_1}}. \quad \text{(B.9)}$$

Since $I(s)$ is a decreasing function of s, $\{s_a^{(k)}\}$ is monotonically increasing, upper bounded by $s_1 + s_2$, and thus it also converges to a limit s_a^* which is upper bounded by s_u, given implicitly as

$$s_u = s_1 + \frac{P_r s_2}{1 + s_2 I(s_u) + P_r}. \quad \text{(B.10)}$$

Let us focus on the case of infinitely many sessions, each with some infinitely small power $\{\delta_i\}_{i=1}^{\infty}$. First, from equation (B.8) we can write

$$\begin{aligned}\Delta_b^k &\triangleq s_b^{(k)} - s_b^{(k-1)}\\ &= \frac{s_1\delta_k}{(1+X_{k-1})\left[\delta_k + \left(1+s_1 I\left(s_b^{(k)}\right)\right)(1+X_{k-1})\right]}\end{aligned} \tag{B.11}$$

where

$$X_{k-1} \triangleq \sum_{i=1}^{k-1} \frac{\delta_i}{1+s_1 I\left(s_b^{(i)}\right)}. \tag{B.12}$$

For infinitely many sessions $k \to \infty$, $\delta_k \to 0$, while $\sum_1^k \delta_i = P_r$.

$$\begin{aligned}\frac{\delta_k}{\Delta_b^k} &= \frac{1}{s_1}(1+X_{k-1})\\ &\quad \left[\delta_k + \left(1+s_1 I\left(s_b^{(k)}\right)\right)(1+X_{k-1})\right]\\ &= \frac{1}{s_1}(1+X_{k-1})\\ &\quad \left[\delta_k + \left(1+s_1 I\left(s_b^{(k-1)}+\Delta_b^k\right)\right)(1+X_{k-1})\right]\end{aligned} \tag{B.13}$$

Taking the limit of (B.13) results with

$$\lim_{\Delta_b^k \to 0} \frac{\delta_k}{\Delta_b^k} = \delta'(s_b) = \frac{1}{s_1}(1+s_1 I(s_b))(1+X(s_b))^2 \triangleq \rho(s_b), \tag{B.14}$$

where

$$\begin{aligned}X(s) &= \lim_{\Delta_b \to 0, k\to\infty} X_{k-1}\\ &= \lim_{\Delta_b \to 0} \sum_{j=1}^{k-1} \frac{\delta_j}{\Delta_b}\frac{\Delta_b}{1+s_1 I(s_2+j\Delta_b)} = \int_{s_2}^{s} \frac{\rho(\sigma)d\sigma}{1+s_1 I(\sigma)}.\end{aligned} \tag{B.15}$$

where we have assumed that $\Delta_b = \Delta_b^k \ \forall k$, which means that δ_k is chosen every session according to $\frac{d\delta(s)}{ds}$. Rewriting (B.14) gives

$$\frac{\rho(s)}{1+s_1 I(s)} = \frac{1}{s_1}(1+X(s))^2, \tag{B.16}$$

where the left hand side is the integrand in (B.15). Hence the following equality holds

$$X'(s) = \frac{1}{s_1}(1+X(s))^2, \tag{B.17}$$

which can be solved, using the initial condition $X(s_2)=0$,

$$X(s) = \frac{s-s_2}{s_1+s_2-s}. \tag{B.18}$$

This means that

$$\rho(s_b) = (1+s_1 I(s_b))\frac{s_1}{(s_1+s_2-s_b)^2}, \tag{B.19}$$

where using $\int_{s_2}^{s_b^*} \rho(s)ds = P_r$, we get the following implicit equation from which we can get the resulting s_b^*, which corresponds to the channel available at user $i=2$ after infinitely many conference sessions, with a total power of P_r,

$$\int_{s_2}^{s_b^*} \frac{s_1}{(s_1+s_2-\sigma)^2}[1+s_1 I(\sigma)]d\sigma = P_r. \tag{B.20}$$

The equivalent SNR s_a^* of user $i=1$ is more interesting, since it can decode more layers. From the above definition of $\beta_a^{(i)}$, in (B.7), in the limit of $\Delta_b \to 0$,

$$Z(s) = \lim_{k\to\infty}\sum_{i=1}^{k} \beta_a^{(i)} = \int_{s_2}^{s} \frac{\rho(\sigma)}{1+s_2 I(\sigma)}d\sigma \tag{B.21}$$

and using the result of (B.19), we get an implicit expression for $Z(s)$,

$$Z(s) = \int_{s_2}^{s} \frac{1+s_1 I(\sigma)}{(1+s_2 I(\sigma))}\frac{s_1}{(s_1+s_2-\sigma)}d\sigma \tag{B.22}$$

Once we have solved s_b^* from (B.20), we can find s_a^* by

$$s_a^* = s_1 + s_2\frac{Z(s_b^*)}{1+Z(s_b^*)}. \tag{B.23}$$

Note that due to the assumption that $s_1 \geq s_2$, the destination user can decode up to s_a^*. Clearly, for $s_1 < s_2$, the destination user will be able to decode only up to s_b^*. ∎

APPENDIX C
Naive Compress and Forward Equivalent Fading

The mutual information expressions are directly derived from (35)-(37)

$$\begin{aligned}R_{WZ,2}(h_1,h_2) &= I(x_s;y_1,\hat{y}_2|h_1,h_2)\\ &= \log\left(1+s_1P_s+\frac{s_2P_s}{1+\sigma^2}\right),\end{aligned} \tag{C.1}$$

where $s_i = |h_i|^2$.
Let us evaluate σ from (C.1):

$$\begin{aligned}&I(y_2;\hat{y}_2|h_2) = \log(1+\frac{s_2P_s+1}{\sigma^2})\\ &I(y_1;\hat{y}_2|h_1,h_2) =\\ &\log\left(1+\frac{s_1s_2P_s^2}{(1+s_1P_s)(1+\sigma^2+s_2P_s)-s_1s_2P_s^2}\right).\end{aligned} \tag{C.2}$$

The derivation of $I(y_1;\hat{y}_2|h_1,h_2)$ follows from its definition, $I(y_1;\hat{y}_2|h_1,h_2) = h(\hat{y}_2|h_2) + h(y_1|h_1) - h(\hat{y}_2,y_1|h_1,h_2)$. It follows immediately that

$$\begin{aligned}h(\hat{y}_2|h_2) &= \log \pi e(1+\sigma^2+s_2P_s)\\ h(y_1|h_1) &= \log \pi e(1+s_1P_s)\end{aligned} \tag{C.3}$$

The covariance matrix of $(\hat{y}_2, y_1)$ is given by

$$\Lambda_{\hat{y}_2,y_1} = E\left[\begin{array}{c} y_1\\ \hat{y}_2\end{array}\right][y_1^* \ \hat{y}_2^*] = \left[\begin{array}{cc} 1+s_1P_s & h_2^*h_1P_s\\ h_2h_1^*P_s & 1+\sigma^2+s_2P_s\end{array}\right] \tag{C.4}$$

From the covariance matrix, the entropy of $(\hat{y}_2, y_1)$ can be computed

$$\begin{aligned}&h(\hat{y}_2,y_1|h_1,h_2) = \log\det(\pi e\Lambda_{\hat{y}_2,y_1}) =\\ &\quad \log \pi^2e^2\left((1+s_1P_s)(1+\sigma^2+s_2P_s)-s_1s_2P_s^2\right)\end{aligned} \tag{C.5}$$

Combining equations (C.3), and (C.5) gives $I(y_1;\hat{y}_2)$ in (C.2). Next we note that the Wyner-Ziv compression rate is

$$\begin{aligned}&I(y_2;\hat{y}_2|h_2) - I(y_1;\hat{y}_2|h_1,h_2) =\\ &\quad \log\frac{(1+\sigma^2+s_2P_s)(s_2P_s+(1+\sigma^2)(1+s_1P_s))}{\sigma^2\left[(1+\sigma^2)(1+s_1P_s)+s_2P_s+s_1s_2P_s^2\right]}\end{aligned} \tag{C.6}$$

The capacity of the cooperation channel C_{coop} restricts the compression rate, according to the condition in (36). When performing compression as function of (s_1, s_2), such that for every such pair there is a different codebook the best compression is achieved when condition (36) is satisfied with equality. This means that for the narrow-band link with $C_{coop} = \log(1 + P_r)$,

$$\sigma^2_{NB} = \frac{1 + s_1 P_s + s_2 P_s}{P_r(1 + s_1 P_s)}, \tag{C.7}$$

and for the wide-band transmission with $C_{coop} = P_r$,

$$\sigma^2_{WB} = \frac{1 + s_1 P_s + s_2 P_s}{(e^{P_r} - 1)(1 + s_1 P_s))} \tag{C.8}$$

To summarize the results, the achievable rate, governed by the cooperation channel capacity and fading gains for $C_{coop} = \log(1 + P_r)$, is given by

$$I(x_s; y_1, \hat{y}_2)_{NB} = \log\left(1 + s_1 P_s + \frac{s_2 P_s(1 + s_1 P_s)P_r}{(1 + P_r)(1 + s_1 P_s) + s_2 P_s}\right). \tag{C.9}$$

We continue with analysis of narrow-band cooperation only, as the same results can be directly obtained for the wide-band cooperation link. It may be noticed that the higher P_r is the closer the performance can get to coherent combining (SIMO processing).
Let us calculate the average rate which is decoded at each user, using the Wyner-Ziv compression. First we need to calculate $F_s(u)$ which is

$$F_s(u) = \text{Prob}(s \leq u) = \text{Prob}\left(s_1 + \frac{s_2(1 + s_1 P_s)P_r}{(1 + P_r)(1 + s_1 P_s) + s_2 P_s} \leq u\right) = \int f_{s_1}(v)\text{Prob}(s \leq u | s_1 = v)dv. \tag{C.10}$$

This integral can be written as:

$$\begin{aligned} F_s(u) = &\int_{\max\{\frac{P_s u - P_r}{P_s(1+P_r)}, 0\}}^{u} dv f_{s_2}(v) \int_0^{\frac{(u-v)(1+P_r)(1+vP_s)}{(1+vP_s)P_r - (u-v)P_s}} du f_{s_1}(u) \\ &+ \int_0^{\max\{\frac{P_s u - P_r}{P_s(1+P_r)}, 0\}} dv f_{s_2}(v) \\ &= 1 - e^{-u} - \int_{\max\{\frac{P_s u - P_r}{P_s(1+P_r)}, 0\}}^{u} dv \\ &\exp\left(-v - \frac{(u - v)(1 + P_r)(1 + vP_s)}{(1 + vP_s)P_r - (u - v)P_s}\right). \end{aligned} \tag{C.11}$$

Using the equality[1]:

$$\begin{aligned} \int dv e^{-v - \frac{(u-v)(1+P_r)(1+vP_s)}{(1+vP_s)P_r - (u-v)P_s}} = \\ e^{-\frac{P_r(1+uP_s)^2}{(1+P_r)P_s(v(1+P_r)P_s + P_r - uP_s)} - \frac{uP_rP_s - 1}{(P_r+1)P_s}} \left(v + \frac{P_r - uP_s}{(P_r + 1)P_s}\right) \\ - P_r(1 + uP_s)^2((P_r + 1)P_s)^{-2} e^{-\frac{uP_rP_s - 1}{(P_r+1)P_s}} \\ E_1\left(\frac{P_r(1 + uP_s)^2}{(1 + P_r)P_s(v(1 + P_r)P_s + P_r - uP_s)}\right) \end{aligned} \tag{C.12}$$

we find $F_s(u)$, when $u \geq \frac{P_r}{P_s}$:

$$\begin{aligned} F_s(u) = 1 - e^{-u}\left(1 + \frac{P_r(uP_s + 1)}{P_s(P_r + 1)}\right) + \frac{P_r(uP_s + 1)^2}{P_s^2(P_r + 1)^2} \\ e^{-\frac{uP_rP_s - 1}{P_s(P_r+1)}} E_1\left(\frac{uP_s + 1}{P_s(P_r + 1)}\right) \end{aligned} \tag{C.13}$$

and when $u < \frac{P_r}{P_s}$:

$$\begin{aligned} F_s(u) = 1 - e^{-u}\left(1 + \frac{P_r(uP_s + 1)}{P_s(P_r + 1)}\right) - \frac{P_r - uP_s}{(1 + P_r)P_s} \\ e^{-\frac{u}{P_r - uP_s}} + \frac{P_r(uP_s + 1)^2}{P_s^2(P_r + 1)^2} e^{-\frac{uP_rP_s - 1}{P_s(P_r+1)}} \\ \left(E_1\left(\frac{uP_s + 1}{P_s(P_r + 1)}\right) - E_1\left(\frac{P_r(uP_s + 1)^2}{P_s(P_r + 1)(P_r - uP_s)}\right)\right). \end{aligned} \tag{C.14}$$

APPENDIX D
MULTI-SESSION COMPRESS AND FORWARD EQUIVALENT FADING

Since $I(y_1; \hat{y}_1^{(k)} | \hat{y}_1^{(k-1)}, \ldots, \hat{y}_1^{(1)}, y_2, x_{s,D}^{(k-1)}, h_1, h_2) = I(y_1; \hat{y}_1^{(k)} | \hat{y}_1^{(k-1)}, y_2, x_{s,D}^{(k-1)}, h_1, h_2)$, the required colocation bandwidth for the k-th session is

$$\begin{aligned} \log(1 + \delta_1^{(k)}) &= I(y_1; \hat{y}_1^{(k)} | \hat{y}_1^{(k-1)}, y_2, x_{s,D}^{(k-1)}, h_1, h_2) \\ &= I(y_1; \hat{y}_1^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2) + I(y_2; \hat{y}_1^{(k-1)} | x_{s,D}^{(k-1)}, h_1, h_2) \\ &- I(y_1; \hat{y}_1^{(k-1)} | x_{s,D}^{(k-1)}, h_1, h_2) - I(y_2; \hat{y}_1^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2) \\ &= R_{WZ,C,1}^{(k)} - R_{WZ,C,1}^{(k-1)} \end{aligned} \tag{D.1}$$

(where $R_{WZ,C,i} \triangleq I(y_i; \hat{y}_i^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2) - I(y_{3-i}; \hat{y}_i^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2)$, $i = 1, 2$) and

$$\begin{aligned} \log(1 + \delta_2^{(k)}) &= I(y_2; \hat{y}_2^{(k)} | \hat{y}_2^{(k-1)}, y_1, x_{s,D}^{(k-1)}, h_1, h_2) \\ &= I(y_2; \hat{y}_2^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2) + I(y_1; \hat{y}_2^{(k-1)} | x_{s,D}^{(k-1)}, h_1, h_2) \\ &- I(y_2; \hat{y}_2^{(k-1)} | x_{s,D}^{(k-1)}, h_1, h_2) - I(y_1; \hat{y}_2^{(k)} | x_{s,D}^{(k-1)}, h_1, h_2) \\ &= R_{WZ,C,2}^{(k)} - R_{WZ,C,2}^{(k-1)} \end{aligned} \tag{D.2}$$

where R_{WZ} was defined in (36), and $x_{s,D}^{(k)}$ is the decoded signal in the k^{th} session, by both users. We notice that this can be further improved by using the extra side information at the stronger user, however, for the sake of brevity, we consider only the commonly decoded layers $x_{s,D}^{(k)}$. Considering

[1] $E_1(x)$ is the exponential integral, $E_1(x) = \int_x^\infty dt \frac{e^{-t}}{t}$

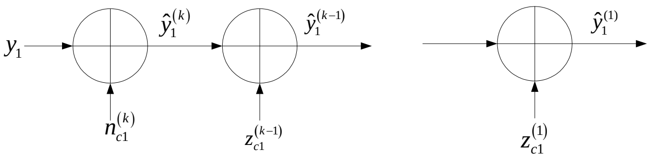

Fig. 13. The Markov relation between the compressed signals

that we deal with Gaussian channel, if $\{z_{c,j}^i\}_{j=1,i=1}^{2,k-1}$ are independent Gaussian variables with zero mean and variances of $\left\{\left(\sigma_j^{(i-1)}\right)^2 - \left(\sigma_j^{(i)}\right)^2\right\}$, where $\mathrm{E}\left|n_{c,j}^{(i)}\right|^2 = \left(\sigma_j^{(i)}\right)^2$, then

$$j = 1, 2: \; n_{c,j}^{(i)} = y_j + n_{c,j}^{(k)} + \sum_{l=i}^{k-1} z_{c,j}^{(l)}. \tag{D.3}$$

This is seen in figure 13. The compression quality in each session can be recursively calculated by ($\sigma_j^{(0)} = \infty$):

$$\frac{F(\sigma_j^{(k)}, k, s_j, s_{3-j}, s^{(k-1)})}{F(\sigma_j^{(k-1)}, k, s_j, s_{3-j}, s^{(k-1)})} = 1 + \delta_j^{(k)} \tag{D.4}$$

where $s^{(k-1)} = \min\{s_b^{(k-1)}, s_a^{(k-1)}\}$ ($s_b^{(k-1)}, s_a^{(k-1)}$ are defined according to (D.6) and (D.7)), and

$$F(\varsigma, k, s_j, s_{3-j}, s^{(k-1)}) \triangleq \frac{s_j I(s^{(k-1)}) + (1+\varsigma^2)(1 + s_{3-j} I(s^{(k-1)}))}{\varsigma^2(1 + s_{3-j} I(s^{(k-1)}))}. \tag{D.5}$$

Notice that when $k = 1$ in (D.4), the expression is indeed identical to the case of single session cooperation, given in equation (C.6). Solving equation (D.4) for $\sigma_j^{(k)}$ results with (50).

The achievable rate remains as in (C.1), which are calculated now by

$$s_a^{(k)} = s_1 + \frac{s_2}{1 + (\sigma_2^{(k)})^2} \tag{D.6}$$

$$s_b^{(k)} = s_2 + \frac{s_1}{1 + (\sigma_1^{(k)})^2}, \tag{D.7}$$

and

$$R_{WZ,1}^{(k)} = \log(1 + s_a^{(k)} P_s) \tag{D.8}$$

$$R_{WZ,2}^{(k)} = \log(1 + s_b^{(k)} P_s). \tag{D.9}$$

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