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Multi-Layer Broadcasting over a Block Fading MIMO Channel

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***Abstract*—This paper introduces extensions for the broadcast approach for a multi-input multi-output (MIMO) block fading channel, with receiver only channel state information (CSI). Previous works have not been able to fully characterize the fundamental MIMO broadcasting upper bound. As it seems that analytical solution for this problem is quite difficult to achieve, we consider here sub-optimal schemes, for which achievable rates may be computed. In particular, finite level coding over a MIMO channel instead of continuous layering is analyzed, the expressions derived for decoding probability regions allow numerical computation of finite level coding upper bounds. Noticing that the gains of two level coding over a MIMO channel are rather small, we consider sub-optimal techniques, which are more straightforward to implement. Among these techniques is the multiple-access channel (MAC) approach with single level coded streams, which is similar in concept to V-BLAST. Closed form expressions for probabilities of decoding regions here are derived, allowing numerical evaluation. We further consider multi-access permutation codes (MAPC). A Hadamard transform is compared with a suggested diagonal permutation code, which are shown to have similar performance, while diagonal permutation has lower implementation complexity. For all approaches, we derive information theoretic upper bounds of achievable rates.**

***Index Terms*—MIMO, MISO, MAC, broadcasting, code layering, permutation codes, MAPC.**

I. Introduction

THEORETICAL limitations of transmission rates in point to point wireless communications have been a growing field of interest. In particular, for a block fading channel, when transmitter has no access to CSI, and only knows the fading statistics, the achievable rate for a transmission block is unknown. That is, even if the receiver can perform channel estimation, and has perfect CSI, the transmitter cannot predict the achievable rate of some arbitrary transmission block. The theoretical achievable rate for block k, characterized by a channel matrix H_k, is given by $I_k = \log\det(I + \frac{P}{M} H_k H_k^H)$, where P is the transmit power, and M is the number of transmit antennas. In traditional single-level coded communication, the transmitter sends coded data using one code at some fixed rate, say R_1. Then, in case the channel conditions are sufficiently good the data may be reliably decoded, which means $R_1 \leq I_k$. In case of bad channel conditions, that is $R_1 > I_k$, the receiver cannot decode its received signal, and an outage event is declared. Such a model gives rise to the notion of outage capacity [1], [2, see references therein]. The achievable rate of the described setting is $R_{1,avg} = \Pr(R_1 \leq I_k) \cdot R_1$.

However, the outage capacity is not the fundamental upper bound of achievable rates for the block fading channels (with receiver only CSI). Consider the following transmission scheme. Let the transmitter perform multi-layer coding. A practical example is super-position coding. Let the receiver decode as many layers as possible, and return a feedback to the transmitter indicating which layers were successfully decoded. Then, the transmitter reschedules only the undecoded layers for retransmission. This basically describes the essence of the broadcast approach [3],[4]. Clearly with only one layer the maximal achievable rate is the outage capacity, and with an unlimited number of layers the fundamental broadcasting upper bound can be obtained. Cover in his original paper [5] suggested broadcasting for the compound channel, where for every realization of the compound channel parameter a different set of users may reliably decode the transmission. The fading channels without transmitter CSI may be viewed as a compound channel with the instantaneous fading realization as the parameter of the compound channel.

In [4], the broadcast approach was fully analyzed for the single-input-single-output (SISO) channel. That is, optimal power distribution was derived, obtaining the fundamental broadcasting upper bound for continuous layering (unlimited number of layers). Then, the broadcast approach in [4] was considered for the MIMO channel as well, however only sub-optimal approaches have been analyzed, as the MIMO fading channel may be viewed as a non-degraded broadcast channel. A sub-optimal ranking using majorization theory was attempted, and other sub-optimal approaches - non showing any advantage over the outage approach for a 2×2 channel.

This paper extends the results of [4], and enhances the understanding of potential gains in MIMO broadcasting through finite code layering or other sub-optimal approaches. The main contributions of this paper are as follows:

- **Continuous broadcasting** - we review the continuous broadcast approach [4], which serves as the multi-layer coding fundamental upper bound. The SISO case is extended to the multi-input-single-output (MISO) channel, which unlike the MIMO case can be fully analyzed, and its maximal achievable rate may be analytically obtained.
- **Finite level coding** - motivated by the potential gains of the above continuous broadcasting relative to the conventional outage approach, we consider practical finite-level coding for the MIMO channel. Since in the MIMO case the continuous broadcasting upper bound was not obtained analytically [4], the gains of finite level coding in MIMO may indicate some of the potential gains in

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continuous MIMO broadcasting. In particular, two level coding is analyzed. As demonstrated in Section II, two level coding captures most of the broadcasting gain w.r.t. an outage approach for channels with one degree of freedom. This emphasizes the importance of understanding the achievable rates with finite level coding.
- **Multi-access permutation codes (MAPC)** - since the implementation of MIMO broadcasting is quite involved, we consider MAPC [7] (also known as multi-stratum codes). The main motivation here is to obtain fundamental achievable upper bounds for rather simple MAPC coding schemes. The first transmission scheme considered is a multi-access (MAC) outage. This approach is basically similar to V-BLAST, where different streams are transmitted using different antennas. Explicit expressions for the probability of decoding regions are derived, which enable direct computation of achievable rates. The second coding scheme considered is the space-time Hadamard permutation, also used in [7]. Another suggested scheme is a diagonal permutation. We show that the last two permutation codes achieve outage capacity under the optimal joint decoding rule, however a performance penalty is evident for sub-optimal successive decoding.

The structure of the paper is as follows. In section II the MISO broadcasting is considered. Then MIMO finite-level code layering in discussed in section III. In section IV, several MAPCs are considered for the MIMO channel, where achievable rates are characterized and evaluated. A summary of numerical results, and concluding remarks follow.

II. The Continuous Broadcast Approach

The SISO broadcasting strategy was discussed in detail in [4]. For completeness, we briefly review the SISO broadcast approach, and then extend it to the MISO channel. The results in this section provide a full characterization of the ultimate broadcast achievable upper bounds for channels with one degree of freedom. Consider the following SISO channel model,

$$y = hx + n\,, \tag{1}$$

where y is a scalar received signal, h is a channel fading coefficient, and n is the additive noise. Both h and n are complex Gaussian i.i.d $\mathcal{CN}(0,1)$ distributed. In this block fading model h remains fixed throughout every transmission block.

The concept of the broadcast approach is demonstrated in Fig. 1. As may be noticed, in the outage approach transmission is either reliably decoded or completely fails (outage). Whereas in the broadcast approach the number of decoded layers varies w.r.t. the channel fading gain. The undecoded layers induce an outage region. In the continuous broadcast approach, every layer is associated with a fading gain $s = |h|^2$, and is allocated a fractional power $\rho(s)ds$. Thus the incremental rate associated with layer s is [4]

$$dR(s) = \log\left(1 + \frac{\rho(s)sds}{1+I(s)s}\right) = \frac{\rho(s)sds}{1+I(s)s}, \tag{2}$$

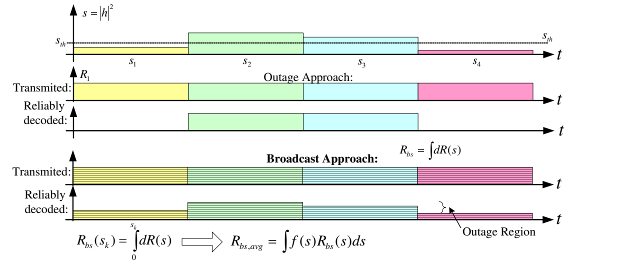

Fig. 1. A timing diagram illustrating the concept of the broadcast approach versus the conventional outage approach.

where $I(s)$ is the residual interference function, such that $I(0) = P$, and $\rho(s) = -\frac{d}{ds}I(s)$ is the power allocation density function. The maximal average rate is expressed as follows

$$R_{bs,avg} = \max_{I(s)} \int_0^\infty ds(1-F(s))\frac{\rho(s)s}{1+I(s)s} \tag{3}$$

where $F(s)$ is the cumulative distribution function (cdf) of the fading gain random variable. It can be shown [4] that the optimal power allocation for (2) is given by

$$I_{SISO}(s) = \begin{cases} P & s < s_0 \\ \frac{1-F(s)-s\cdot f(s)}{s^2 f(s)} & s_0 \le s \le s_1 \\ 0 & s > s_1 \end{cases} \tag{4}$$

where s_0, s_1 are specified by the boundary conditions $I_{SISO}(s_0) = P$, $I_{SISO}(s_1) = 0$, respectively.

Consider now a MISO channel, with M transmit antennas,

$$y = (\mathbf{x_S} + \mathbf{x_I})\mathbf{h} + n\,, \tag{5}$$

where y is a scalar received signal, $\mathbf{h}$ is a $[M \times 1]$ channel fading vector with complex Gaussian $\mathcal{CN}(0,1)$ i.i.d elements. The decodable data vector is denoted $\mathbf{x_S}$, and the undecodable data to be treated as additive interference is denoted $\mathbf{x_I}$. The mutual information $I(y;\mathbf{x_S})$ is

$$I(y;\mathbf{x_S}) = \log\left(1 + \frac{\frac{P_S}{M}\sum\limits_{m=1}^{M}|h_m|^2}{1+\frac{P_I}{M}\sum\limits_{m=1}^{M}|h_m|^2}\right). \tag{6}$$

The distribution of $I(y;\mathbf{x_S})$ depends on the distribution of $s \triangleq \sum\limits_{m=1}^{M}|h_m|^2$ which defines an equivalent fading parameter s. This allows specification of the incremental transmission rate $dR(s)$, following similar steps to the derivation of (2),

$$dR(s) = \log\left(1 + \frac{s\rho(s)ds}{M+sI(s)}\right) = \frac{s\rho(s)ds}{M+sI(s)}. \tag{7}$$

Like demonstrated in Fig. 1, the average achievable rate over all possible channel realizations is $R_{bs,MISO} = \int\limits_0^\infty(1 - F(s))dR(s)$. Optimization of $R_{bs,MISO}$ with respect to the power distribution $I(s)$, under the power constraint P, may be done by solving the associated constrained Eüler equation

[8]. The MISO broadcasting optimal residual interference distribution is thus

$$I(s) = \frac{M(1 - F(s) - sf(s))}{s^2 f(s)}, \quad s_0 \leq s \leq s_1 \tag{8}$$

where $I(s_0) = P$ and $I(s_1) = 0$.

Numerical results for the MISO and SIMO Rayleigh fading channels are given in Figs. 2-3. The ergodic capacity upper bound serves as a performance reference. Two-level coding achievable rate R_{2L} is compared to the broadcasting achievable rate R_{bs}. In the SISO case, the largest performance gain of broadcasting over finite layer coding is observed. In a MISO setting, with $M = 8$, the broadcasting gain over an outage approach is marginal. This is due to the less randomness (hardening) associated with the MISO/SIMO relatively to the SISO channel.

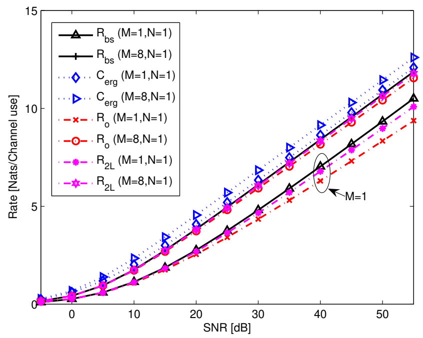

Fig. 2. MISO broadcasting maximal achievable rates, compared with single, two-level coding and ergodic capacity, for $M = 1, 8$ and $N = 1$.

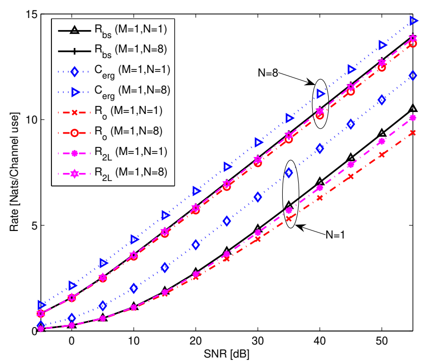

Fig. 3. SIMO broadcasting maximal achievable rates, compared with single, two-level coding and ergodic capacity, for $N = 1, 8$ and $M = 1$.

III. Finite Level Code Layering

Motivated by the potential gains of the continuous broadcasting relative to the conventional outage approach, as demonstrated in Section II, we consider practical finite-level coding for the MIMO channel. Since, for the MIMO case, a continuous broadcasting upper bound was not obtained in [4], and seems to be hard to determine analytically, the gains of finite level coding in MIMO may indicate some of the potential gains in continuous MIMO broadcasting. In particular, this section focuses on two level coding. As observed in Section II, two level coding captures most of the broadcasting gain w.r.t. an outage approach for channels with one degree of freedom. This emphasizes the importance of understanding the achievable rates with finite level coding.

Consider the following flat fading MIMO channel,

$$\mathbf{y} = H\mathbf{x} + \mathbf{n}\,, \tag{9}$$

where $\mathbf{x}$ is the $[M \times 1]$ input vector, $\mathbf{n}$ is the $[N \times 1]$ noise vector with complex Gaussian i.i.d $\mathcal{CN}(0, 1)$ distributed elements. The propagation matrix H $[N \times M]$ consists of i.i.d $\mathcal{CN}(0, 1)$ distributed elements. The received $[N \times 1]$ vector is denoted by $\mathbf{y}$. We adhere to the non-ergodic case, where H is fixed throughout the codeword transmission, and perfectly known at the receiver.

We adhere now to two level code layering for $M = N = 2$, and derive single integral expressions for the probability regions of joint/successive decoding. This allows computation of the achievable rates for this coding scheme, which is later optimized numerically.

Consider a two layer code, with rate $2\alpha R$ and power βP for the first layer, and $2(1 - \alpha)R$, $(1 - \beta)P$ pair for the second, where $0 \leq \alpha \leq 0.5$, and $0 \leq \beta \leq 0.5$ without loss of generality. Single layer decoding regions are denoted $\mathcal{A}$, $\mathcal{B}$, and the joint decoding region is denoted $\mathcal{C}$,

$$\begin{aligned} &\mathcal{A} : 2\alpha R \leq I(\mathbf{y}; s_1) \\ &\mathcal{B} : 2(1 - \alpha)R \leq I(\mathbf{y}; s_2) \\ &\mathcal{C} : \begin{cases} \mathcal{C}_1 : \;\; 2\alpha R \leq I(\mathbf{y}; s_1 | s_2) \\ \mathcal{C}_2 : \;\; 2(1 - \alpha)R \leq I(\mathbf{y}; s_2 | s_1) \\ \mathcal{C}_3 : \;\; 2R \leq I(\mathbf{y}; \mathbf{s}) = I(\mathbf{y}; \mathbf{x}) \end{cases} \end{aligned} \tag{10}$$

A. Optimal decoding

The optimal decoding approach considers first a joint decoding strategy for the two layers. When not successful, single layer decoding is attempted. The probability of joint decoding is denoted by $P_C \triangleq \Pr\{\mathcal{C}\}$ (10). A rate $2\alpha R$ is achieved when layer s_1 is decoded while joint decoding fails, with probability $P_{AC^c} \triangleq \Pr\{\mathcal{C}^c \cap \mathcal{A}\}$, and similarly $P_{BC^c} \triangleq \Pr\{\mathcal{C}^c \cap \mathcal{B}\}$, where $\mathcal{C}^c$ denotes the complementary of $\mathcal{C}$. The achievable average rate is then

$$R_{2L,opt} = 2R \cdot (P_C \; + \; \alpha P_{AC^c} \; + \; (1 - \alpha)P_{BC^c})\,. \tag{11}$$

The probability density for all mutual information expressions depends only on the eigenvalues distribution, since there is no restriction on the utilization of the transmit antennas. Our main result here is the single integral expressions for computation of P_C, P_{AC^c}, and P_{BC^c}. These allow numerical evaluation of $R_{2L,opt}$, which otherwise cannot be evaluated [4]. The following proposition specifies P_C.

Proposition 3.1: The probability of successful joint decoding of two layers, in any two level coding scheme over a 2×2

MIMO channel (9), is given by

$$\begin{aligned} P_C &= P_{pdf}(0,\infty,x_C,\infty), \quad \text{s.t.} \\ x_C &= \max\Bigg\{0, \frac{1}{P}\left(\frac{e^{2R}}{1+P\lambda_2}-1\right), \\ &\quad \frac{1}{\beta P}\left(\frac{e^{2\alpha R}}{1+\beta P\lambda_2}-1\right), \\ &\quad \frac{1}{(1-\beta)P}\left(\frac{e^{2(1-\alpha)R}}{1+(1-\beta)P\lambda_2}-1\right)\Bigg\}. \end{aligned} \quad (12)$$

where

$$P_{pdf}(a_0,a_1,b_0,b_1) = \int_{a_0}^{a_1} d\lambda_2 \; F_1(\lambda_2,b_0,b_1), \quad (13)$$

and

$$\begin{aligned} F_1(\lambda_2,b_0,b_1) &\triangleq 4e^{-2\lambda_2}\Bigg[\left((b_0-\lambda_2)^2+(b_0-\lambda_2)+\tfrac{1}{2}\right) \\ & e^{-2b_0} - \left((b_1-\lambda_2)^2+(b_1-\lambda_2)+\tfrac{1}{2}\right)e^{-2b_1}\Bigg]. \end{aligned}$$

Proof: See Appendix A.

The probability of single layer decoding, when joint decoding fails (P_{AC^c}) is computed similarly,

$$\begin{aligned} P_{AC^c} &= \Pr\Bigg\{e^{2R} > \min\Big[(1+P\lambda_1)(1+P\lambda_2), \\ &\quad e^{2(1-\alpha)R}(1+\beta P\lambda_1)(1+\beta P\lambda_2), \\ &\quad e^{2\alpha R}(1+(1-\beta)P\lambda_1)(1+(1-\beta)P\lambda_2)\Big] \text{ and} \\ &\quad e^{2\alpha R} \le G_{ys_1}(\lambda_1,\lambda_2)\Bigg\}, \end{aligned} \quad (14)$$

where $G_{ys_1}(\lambda_1,\lambda_2) \triangleq \left(1+\frac{\beta P\lambda_1}{1+(1-\beta)P\lambda_1}\right)\left(1+\frac{\beta P\lambda_2}{1+(1-\beta)P\lambda_2}\right)$. And in a similar manner the decoding of the second layer only probability P_{BC^c} is derived. Further details may be found in [16].

B. Successive decoding

The probability of successively decoding both layers P_{succ} is the union of two regions: $AC_2 \triangleq \mathcal{A}\cap\mathcal{C}_2$ (10) is the region of reliable decoding when starting from layer s_1, and similarly $BC_1 \triangleq \mathcal{B}\cap\mathcal{C}_1$ corresponds to initially decoding s_2. The intersection of these regions is the single-user decoding region, denoted $AB \triangleq \mathcal{A}\cap\mathcal{B}$. The probability of two layers successive decoding is

$$P_{succ} = P_{AC_2} + P_{BC_1} - P_{AB}, \quad (15)$$

where P_{AC_2} is given by

$$\begin{aligned} P_{AC_2} = \Pr\Bigg\{ & e^{2\alpha R} \le G_{ys_1}(\lambda_1,\lambda_2) \text{ and } e^{2(1-\alpha)R} \\ & \le (1+(1-\beta)P\lambda_1)(1+(1-\beta)P\lambda_2)\Bigg\}. \end{aligned} \quad (16)$$

Other probabilities P_{BC_2}, and P_{AB} are similarly derived, and a single integral expression is obtained like in Proposition 3.1. The average achievable rate with successive decoding is

$$R_{2L,bs} = 2R\cdot\left(P_{succ} + \alpha P_{AC_2^c} + (1-\alpha)P_{BC_1^c}\right), \quad (17)$$

where $P_{AC_2^c}$ and $P_{BC_1^c}$ are the probabilities of decoding layer s_1 only and s_2 only, respectively. Again all the above probability expressions can be evaluated following Proposition 3.1, and thus numerical results may be obtained for two level coding. Further details may be found in [16].

Interestingly, the case of equal rate and power assignment to both layers suggests that separate single user decoding will perform like successive decoding since it implies that $I(\mathbf{y};s_1) = I(\mathbf{y};s_2)$. In this case, successive decoding is not necessary, a single layer decoding achieves the same performance. Therefore, it is expected that unequal rate and power assignment to both layers would allow higher average rates, and when optimizing jointly α,β for each SNR, the performance of this approach is expected to be lower bounded by the outage capacity, and upper bounded by the optimal two layer scheme introduced in the previous subsection.

IV. Permutation Codes

We consider now coding schemes, which are more straightforward for implementation than the finite level coding schemes discussed in Section III. We adhere to some space-time coding approaches, namely multi-access permutation codes (MAPC) [15], which utilize the diversity inherent in the MIMO channel, and obtain achievable rates for the different MAPC schemes. The achievable rates shed light on the efficiency of the MAPC schemes, and the expected degradation due to their inherent sub-optimality. In the first example, a MAC-outage approach is considered, where every antenna transmits its separately coded block. In other schemes, the two coded blocks are permuted such that each coded block is transmitted using both antennas, by altering the antennas in time according to some permutation.

A. MAC Outage

Consider the $M = N = 2$ MIMO channel model (9). In the MAC outage approach, like in V-BLAST, every single level coded stream is associated with a transmit antenna. Thus for the case of 2 transmit antenna the first stream denoted s_1 in (10) corresponds to x_1. The specification of the decoding regions relies on the definitions in (10). Furthermore, since transmitter has no CSI information, equal rate R and equal power $P/2$ are used for each stream. Hence, using notations of previous section, $\alpha = 0.5$, and $\beta = 0.5$ in (10).

1) Optimal Decoding: The decoder initially attempts decoding both streams adhering to the optimal joint detection. If that fails it tries to decode either of the streams, treating the other stream as additive interference, while optimally preprocessing for the channel response H, using an MMSE decoder [12], [13]. We use the term joint decoding to characterize this detection procedure, which resembles the strongest user detector concept, first introduced in [11].

The joint decoding region is specified in (10) and its probability corresponds to $P_C \triangleq \Pr\{\mathcal{C}\}$. Probability of decoding of stream s_1 only is given by $P_{AC^c} = \Pr\{\mathcal{A}\cap\mathcal{B}^c\cap\mathcal{C}^c\}$. The next lemma will help simplifying P_{AC^c}.

Lemma 1: The probabilities $\Pr\{\mathcal{A}\cap\mathcal{B}\} = \Pr\{\mathcal{A}\cap\mathcal{B}\cap\mathcal{C}\}$.

Proof: The left hand side probability is the probability of successfully decoding both users in a single user decoding strategy. The right hand side probability is the probability of successfully decoding both users in a single user strategy and in an optimal (joint) decoding strategy. Clearly, if the single user decoding strategy is successful, optimal decoding will surely succeed. ■

Using Lemma 1, it is immediate to see that

$$P_{AC^c} = \Pr\{\mathcal{A}\} - \Pr\{\mathcal{A}\cap\mathcal{C}\}. \tag{18}$$

A similar result holds for P_{BC^c}. The average rate $R_{mac1,opt}$ is thus specified in (11) with $\alpha = 0.5$. However the probabilities of the decoding regions are definitely different from those of two level coding, due to the MAC-outage setting constraints. Due to symmetry of power and rate allocation the single stream decoding probabilities match, that is $P_{AC^c} = P_{BC^c}$. The following proposition explicitly states these decoding probabilities.

Proposition 4.1: Probabilities of joint decoding of two streams P_C, and single stream decoding P_{AC^c}, in a two stream MAC-outage scheme over a 2×2 MIMO channel (9), are given by

$$\begin{aligned} P_C &= \frac{2}{P}\left(2x-2-x_2\right)e^{-x_2} \\ &+ \frac{2}{P}\left(2x-x_2+2\right)e^{-2x} + (x+1)^2 e^{-2x}. \end{aligned} \tag{19}$$

where $x \triangleq \frac{2}{P}(e^R-1)$, and $x_2 \triangleq \frac{2}{P}(e^{2R}-1)$. Probability of decoding first stream only (18) is

$$P_{AC^c} = P_{A1} - P_{A2} \tag{20}$$

where

$$P_{A1} = \frac{(8+8x+12Px+2P^2x^3+P^3x^3+6P^2x^2+8Px^2)}{(2+Px)^3}e^{-x}, \tag{21}$$

and

$$\begin{aligned} P_{A2} = &\frac{(8P+32Px+12P^2x+6P^3x^2+P^4x^3+24P^2x^2+8P^3x^3+P^4x^4+16)}{P(2+Px)^3} \\ &e^{-2x} - \frac{(16Px+4P^2x^2+16)}{P(2+Px)^3}e^{-2x-\frac{Px^2}{2}} \end{aligned} \tag{22}$$

Proof: See Appendix B.

2) Successive Decoding: We adhere now to a sub-optimal decoding strategy, known as successive decoding. Successive decoding is also implied in the MAC broadcast strategy [4]. The decoding regions are again derived from (10). The probability of successive decoding P_{succ} based on (15), where due to symmetry in rate and power allocation $P_{AC_2} = P_{BC_1}$, thus

$$P_{succ} = 2P_{AC_2} - P_{AB}. \tag{23}$$

In order to use previous results to compute P_{succ}, we prove the following Lemma.

Lemma 2: $\Pr\{\mathcal{A}\cap\mathcal{C}_2\} = \Pr\{\mathcal{A}\cap\mathcal{C}\}$.

Proof: Existence of $\mathcal{A}\cap\mathcal{C}_2$ suggests that $I(\mathbf{y};x_1) > R$ and $I(\mathbf{y};x_2|x_1) > R$. Clearly, from the first inequality $I(\mathbf{y};x_1|x_2) \geq I(\mathbf{y};x_1) > R$, therefore $I(\mathbf{y};x_1|x_2) > R$. And since $I(\mathbf{y};x_1,x_2) = I(\mathbf{y};x_2|x_1) + I(\mathbf{y};x_1)$, where each element in the right hand side is greater than R, we have $I(\mathbf{y};x_1,x_2) > 2R$. ■

This means that $P_{AC_2} = P_{AC}$, which is specified in closed form in (22). The rest of the probabilities P_{AB}, and $P_{AC_2^c}$, and $P_{BC_1^c}$ can be computed in closed form, using the same steps as in Proposition 4.1. More details regarding these derivations may be found in [16].

B. Diagonal Permutation

Consider the channel model defined in Eq. (9). Let the input coded layers be denoted by $s_1(n)$ and $s_2(n)$, where n is a discrete time index (within a single transmission block). The diagonal permutation switches between antennas for each channel use, yielding transmitted signals

$$\begin{bmatrix} x_1(2n) \\ x_2(2n) \\ x_1(2n+1) \\ x_2(2n+1) \end{bmatrix} = \tag{24}$$

$$M_{perm}\cdot\begin{bmatrix} s_1(2n) \\ s_2(2n) \\ s_1(2n+1) \\ s_2(2n+1) \end{bmatrix}, \; M_{perm} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The next proposition summarizes main result for optimal decoding using diagonal permutation.

Proposition 4.2: The maximal achievable rate using a diagonal permutation for a 2×2 MIMO channel (9) is the outage capacity, that is

$$R_{diag,opt} = \Pr\left(\log\det(I + \frac{P}{M}\Lambda_H) > 2R\right)\cdot 2R \tag{25}$$

Proof: Using the same tools as for the proof of Proposition 4.1, $I(\mathbf{y};s_1|s_2)$ is given by

$$\begin{aligned} I(\mathbf{y};s_1|s_2) &= \frac{1}{2}\log\det\left(1+\frac{P}{2}w_1\right) \\ &+ \frac{1}{2}\log\det\left(1+\frac{P}{2}w_2\right), \end{aligned} \tag{26}$$

where w_i are defined in (B.1). Due to the symmetry in the diagonal permutation we have $I(\mathbf{y};s_1|s_2) = I(\mathbf{y};s_2|s_1)$. In addition, the optimal decoding region is solely governed by a single mutual information threshold $I(\mathbf{y};\mathbf{s}) \geq 2R$. This can be verified by using

$$I(\mathbf{y};\mathbf{s}) = \log\left[\left(1+\frac{P}{2}w_1\right)\left(1+\frac{P}{2}w_2\right) - \frac{P^2}{4}w_3\right] \tag{27}$$

which results from the mutual information definition. From (26) and (27) it is clear that $2I(\mathbf{y};s_1|s_2) \geq I(\mathbf{y};\mathbf{s})$. Now since the two streams are allocated equal rates it is clear that optimal decoding may be successfully performed when $I(\mathbf{y};\mathbf{s}) = \log\det\Lambda_{\mathbf{y}} \geq 2R$. ■

Consider now successive decoding with the diagonal permutation specified by M_{perm} in (24). The next proposition summarizes the result of the achievable rate.

Proposition 4.3: The maximal achievable rate using a diagonal permutation for a 2×2 MIMO channel (9) is the outage capacity, that is

$$R_{diag,bs} = 2RP_A \tag{28}$$

where

$$\begin{aligned} P_A = &\int_0^\infty dw_1 \int_0^\infty dw_2 \\ &\max\left\{0,\ \min\left[w_1 w_2,\ \tfrac{4}{P^2}(W_4 - \sqrt{W_4}e^R)\right]\right\} e^{-w_1-w_2} \end{aligned} \tag{29}$$

where $W_4 \triangleq \left(1+\frac{P}{2}w_1\right)\left(1+\frac{P}{2}w_2\right)$.

Proof: From the symmetry in rate and power allocations for the users, it is clear that if one of the streams is decoded successfully (when considering the other as interference), then the other user will also be successfully decoded. That is, the condition for successful decoding of both streams is $I(\mathbf{y};s_1) \geq R$, with probability $P_A = \Pr\{I(\mathbf{y};s_1) \geq R\}$, and thus achievable average rate is (28). Using the definitions of the mutual information and Λ_H (B.1), P_A can be computed from its condition $I(\mathbf{y};s_1) \geq R$,

$$\begin{aligned} &\log\left[\left(1+\frac{P}{2}w_1\right)\left(1+\frac{P}{2}w_2\right) - w_3\frac{P^2}{4}\right] \\ &- \frac{1}{2}\log\left(1+\frac{P}{2}w_1\right)\left(1+\frac{P}{2}w_2\right) \geq R \end{aligned} \tag{30}$$

and from the definition of the joint distribution of (w_1, w_2, w_3) (B.2), we get (29). ■

The results of Propositions 4.2, and 4.3 allow to numerically compute the average achievable rate with diagonal permutation, in a rather low complexity.

C. Hadamard Permutation

Consider the channel model defined in Eq. (9). Let the input coded layers be denoted by $s_1(n)$ and $s_2(n)$, where n is a discrete time index (within a single transmission block). The Hadamard permutation is used as the interlayer orthogonal transformation, which is rotated for increasing transmit diversity [15]. The transmitted streams from the two antennas are defined as in (24), where the permutation matrix M_{perm} is given by

$$M_{perm} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{31}$$

Consider first the problem of optimal decoding. That requires derivation of mutual information for the input stream and the channel output. As each input symbol is transmitted at two sequential epochs, the achievable rate is half the mutual information

$$I(\mathbf{s};\mathbf{y}) = \frac{1}{2}\log\det\left(I_4 + \frac{P}{4}H_4 H_4^\dagger\right), \tag{32}$$

where H_4 is the channel matrix given two consecutive channel uses, and input vector $\mathbf{s}$. After some algebra on (32), it turns out that the mutual information (MI) reduces to the standard 2×2 MI, achieved without any permutation, $I(\mathbf{s};\mathbf{y}) = \log\det(I + \frac{P}{M}\Lambda_H)$.

The mutual information of a single stream, after cancellation of the other is denoted $I(\mathbf{y};s_1|s_2)$. With s_2 known at the receiver, it can be removed from the received signal,

$$\begin{aligned} &\begin{bmatrix} y_1(2n) \\ y_2(2n) \\ y_1(2n+1) \\ y_2(2n+1) \end{bmatrix}_{s_2=0} \\ &= \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12} & -h_{11} \\ h_{22} & -h_{21} \end{bmatrix} \begin{bmatrix} s_1(2n) \\ s_1(2n+1) \end{bmatrix} + \mathbf{w}. \end{aligned} \tag{33}$$

The mutual information $I(\mathbf{y};s_1|s_2)$, is then given by

$$I(\mathbf{y};s_1|s_2) = \frac{1}{2}\log\det\left(I_2 + \frac{P}{4}H^\dagger H\right), \tag{34}$$

where H is the channel matrix in (33), and I_2 denotes a 2×2 identity matrix. An interesting observation is made in the following proposition.

Proposition 4.4: For the 2×2 Hadamard permutation codes $I(\mathbf{y};s_1|s_2) \geq \frac{1}{2}I(\mathbf{y};\mathbf{s})$.

Proof: See Appendix C.

We note here that due to the symmetry of the permutation $I(\mathbf{y};s_1|s_2) = I(\mathbf{y};s_2|s_1)$, which directly implies on $I(\mathbf{y};s_1) = I(\mathbf{y};s_2)$. From proposition 4.4, and by recalling that both layers have equal rates R, the joint detection region is specified solely by $I(\mathbf{y};\mathbf{s}) \geq 2R$, which is the outage capacity. Hence both permutations considered achieve outage capacity with optimal decoding, specified in Proposition 4.2.

Considering successive decoding, it is clear from above that the region of successive decoding, is given by the region of a single user decoding, specified by $I(\mathbf{y};s_1) \geq R$. The probability of a single user decoding is $P_A = \Pr\{I(\mathbf{y};\mathbf{s}) - I(\mathbf{y};s_1|s_2) \geq R\}$, and the achievable average rate is thus, $R_{Had,bs} = 2RP_A$, which is computed by Monte-Carlo simulation in section V.

V. Numerical Results

Fig. 4 shows the maximal average achievable rates for the different approaches discussed in previous sections. As expected, the highest achievable rate among the examined schemes with near outage performance is the two-layered coding. The two-layered scheme, was optimized over the α, β and rate allocation. Its advantage over the single layer coding (outage) is marginal. Outage capacity is also achieved in the Hadamard permutation codes employing optimal detection. When performing successive decoding MAC approach has a $\sim$ 2.5dB loss from the outage capacity, and Hadamard/Diagonal permutation codes suffer another ~ 1 dB loss relatively to MAC broadcast strategy, while the diagonal permutation has lower implementation complexity.

In Fig. 5 the performance of the optimal two-layered coding scheme is compared with the outage and MAC optimal

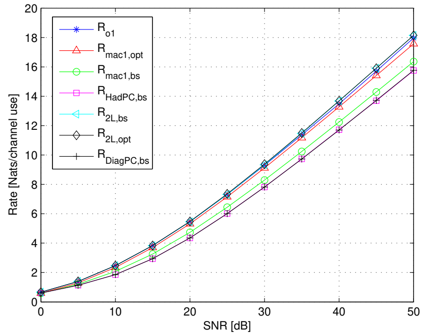

Fig. 4. MIMO ($M = N = 2$) average achievable rates: R_{o1} - Outage capacity, which is the maximal average rate of a single layer code employing optimal detection. $R_{mac1,opt}$ - optimal MAC strategy, $R_{mac1,bs}$ - successive (broadcast) MAC approach, $R_{HadPC,bs}$ - Hadamard permutation codes with successive decoding, $R_{DiagPC,bs}$ - Diagonal permutation codes with successive decoding, $R_{2L,bs}$ - successive (broadcast) two layered coding approach with optimized power and rate assignment per layer, $R_{2L,opt}$ - optimal two layered coding strategy with optimized power and rate assignment per layer.

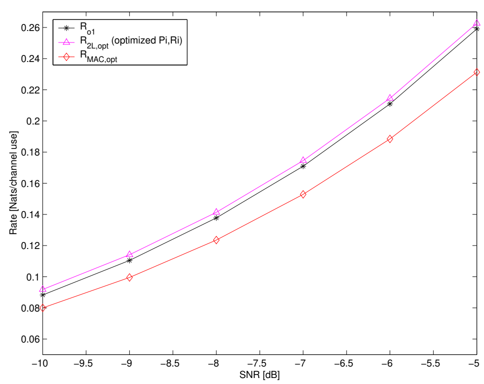

Fig. 5. MIMO ($M = N = 2$) average achievable rates, at a low SNR range, and when employing optimal decoding. R_{o1} - Outage capacity, which is the maximal average rate of a single layer code. $R_{mac1,opt}$ - MAC approach. $R_{2L,opt}$ - two layered coding strategy with optimized power and rate assignment per layer.

approaches at low SNRs. In this figure the advantage of optimized two-layered coding is clearly noticeable. Table I

SNR [dB]:	0	10	20	30	40	50
$R_{2L,opt}$	0.662	2.469	5.477	9.378	13.685	18.15
α_{opt}	0.34	0.35	0.5	0.5	0.5	0.496
β_{opt}	0.15	0.15	0.14	0.056	0.02	0.007
$R_{2L,bs}$	0.662	2.464	5.456	9.333	13.675	18.149
α_{bs}	0.33	0.005	0.001	0.125	0.5	0.496
β_{bs}	0.237	0.001	0.005	0.5	0.02	0.007
C_{outage}	0.657	2.461	5.453	9.299	13.487	17.97

TABLE I
SUMMARY OF MAXIMAL ACHIEVABLE RATES OF THE TWO-LAYER CODING, COMPARED WITH THE OUTAGE CAPACITY (SINGLE LAYER). THE TWO DECODING RATES (FOR THE OPTIMAL AND SUCCESSIVE STRATEGIES) ARE OPTIMIZED OVER α, β, AND THEIR REALIZING VALUES ARE PRESENTED.

presents the numerical values of the maximal achievable rate and the realizing α, β for various SNR values. The sensitivity of power/rate allocation (α, β) is examined in Fig. 6, where the achievable rates for fixed values of (α, β) are compared.

Interestingly, non-optimal rate/power assignments has negligible loss relative to the optimum (α, β) allocation under optimal decoding strategy. However, when successive decoding is employed the rate and power assignments should be carefully chosen, otherwise a significant performance penalty is expected.

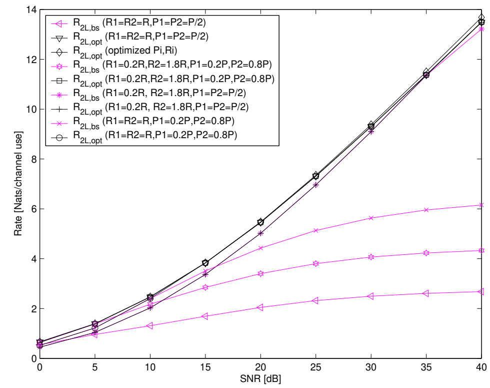

Fig. 6. MIMO ($M = N = 2$) a comparison of different power and rate assignments in the two layered coding approach $R_{2L,bs}$ - successive (broadcast) two layered coding approach, $R_{2L,opt}$ - optimal two layered coding strategy. Equal rate assignment ($\alpha = 0.5$) is compared with unequal rates for each layer ($\alpha = 0.1$).

VI. Conclusion

New achievable rates of sub-optimal MIMO broadcasting approaches were derived, allowing enhanced understanding of the potential gains of MIMO broadcasting over a slowly fading channel. For MIMO two level code layering, single integral expressions were obtained for the probabilities of decoding regions. Only a small gain of two level layering relative to outage approach was noticed. The efficiency of layering over the outage approach is expected to be completely negligible when increasing the number of antennas, as channel hardening has noticeable effect already for $M = N = 2$. It was demonstrated that non-optimal rate and power allocation to each layer has a rather subtle penalty on the achievable rate under optimal decoding. However when successive decoding is performed the arbitrary rate and power allocation may induce significant degradation.

A V-BLAST like, MAC-outage approach for the MIMO channel was further discussed. Closed form expressions for the probabilities of the different decoding regions in MAC outage scheme were obtained. Two additional schemes of MAPC were considered due to their simplicity of implementation. A Hadamard transformation is compared with a suggested diagonal permutation. It was shown that diagonal and Hadamard permutations achieve outage capacity under optimal joint decoding rule, however a large performance penalty is evident for sub-optimal successive decoding. While having similar performance the diagonal permutation is more straightforward for implementation.

An extension of SISO broadcasting [4] for a MISO channel was presented, along with a derivation of the maximal achievable rate with layered coding. Numerical results clearly demonstrate that as the number of antennas increases, broadcasting gain over outage vanishes.

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APPENDIX A
FINITE LEVEL CODING - COMPUTATION OF P_C

The calculation of the probability of joint decoding is derived as follows

$$\begin{aligned} &P_C = \Pr\{\mathcal{C}\} = \Pr\{2\alpha R \log(1+\beta P\lambda_1) + \log(1+\beta P\lambda_2), \\ &2(1-\alpha)R \le \log(1+(1-\beta)P\lambda_1) + \log(1+(1-\beta)P\lambda_2), \\ &2R \le \log(1+P\lambda_1) + \log(1+P\lambda_2)\} \end{aligned} \tag{A.1}$$

where $\mathcal{C}$ is defined in (10). The pdf of the channel covariance unordered eigenvalues, is [9]

$$\begin{aligned} &f_{unordered}(\lambda_1, \lambda_2) \\ &\quad = \; 8(\lambda_1-\lambda_2)^2 e^{-2\lambda_1-2\lambda_2} \quad \lambda_1 > 0, \; \lambda_2 > 0. \end{aligned} \tag{A.2}$$

The different decoding regions have probabilities which can be expressed by integration over the pdf (A.2) in regions corresponding to the successful decoding regions,

$$\begin{aligned} P_{pdf}(a_0, a_1, b_0, b_1) &= \int_{a_0}^{a_1} d\lambda_2 \int_{b_0}^{b_1} d\lambda_1 \; f_{unordered}(\lambda_1, \lambda_2) \\ &= \int_{a_0}^{a_1} d\lambda_2 \; F_1(\lambda_2, b_0, b_1), \end{aligned} \tag{A.3}$$

where

$$\begin{aligned} &F_1(\lambda_2, b_0, b_1) \\ &\triangleq \; 4e^{-2\lambda_2}\left[\left((b_0-\lambda_2)^2 + (b_0-\lambda_2) + \frac{1}{2}\right) e^{-2b_0}\right. \\ &- \; \left.\left((b_1-\lambda_2)^2 + (b_1-\lambda_2) + \frac{1}{2}\right) e^{-2b_1}\right] \end{aligned}$$

From here the result of P_C is straightforward. ■

APPENDIX B
MAC-OUTAGE - OPTIMAL DECODING REGION PROBABILITIES

We begin by recognizing that the channel covariance matrix $\Lambda_H \triangleq HH^\dagger$ is a complex Wishart matrix [14]. Its distribution for $M = N = 2$ is

$$p(\Lambda_H) = \exp(-tr(\Lambda_H)), \text{ where } \Lambda_H = \begin{bmatrix} w_1 & w_{12} \\ w_{21} & w_2 \end{bmatrix} \tag{B.1}$$

Recall that the $\det\{\Lambda_H\} = w_1 w_2 - w_{12} \cdot w_{21}$, with $w_{12} = w_{21}^*$, thus the pdf of Λ_H depends solely on three real variables,

$$p_{\Lambda_H}(w_1, w_2, w_3) = \begin{cases} e^{-w_1-w_2} & , w_1 w_2 \ge w_3 \\ 0 & , \text{otherwise} \end{cases} \tag{B.2}$$

where $w_3 \triangleq w_{12}w_{21}$. Using the notations of (B.1) for the elements of Λ_H, the probability of joint decoding, as may also be interpreted from Figure 7, is given by

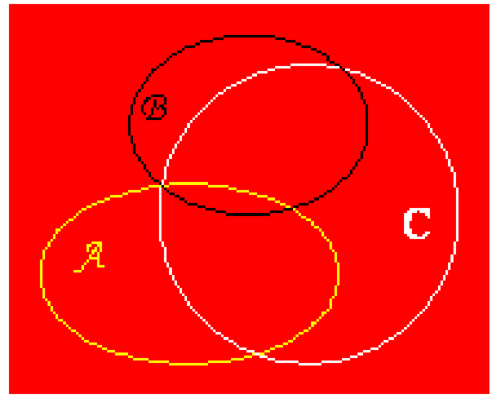

Fig. 7. Two-Layers approach decoding regions. Layer s_1 is decoded considering s_2 as interference in region $\mathcal{A}$. Layer s_2 is decoded considering s_1 as interference in a contained region $\mathcal{B}$, and region $\mathcal{C}$ is the joint decoding region

$$\begin{aligned} P_C = \Pr\Bigg(&(1+w_1\frac{P}{2})(1+w_2\frac{P}{2}) - w_3\frac{P^2}{4} > e^{2R}, \\ &(1+w_1\frac{P}{2}) > e^R, (1+w_2\frac{P}{2}) > e^R\Bigg). \end{aligned} \tag{B.3}$$

By translating the probability into integrals over the pdf (B.2), and substituting $x \triangleq \frac{2}{P}(e^R - 1)$,

$$\begin{aligned} P_C = &\int_x^\infty dw_1 \int_x^\infty dw_2 \int_0^{w_1 w_2} dw_3 \\ &\mathbf{1}\left((1+w_1\frac{P}{2})(1+w_2\frac{P}{2}) - w_3\frac{P^2}{4} - e^{2R}\right) e^{-w_1-w_2}, \end{aligned} \tag{B.4}$$

where $\mathbf{1}(\cdot)$ is the indicator function. After determining the nonzero indicator regions, the integral may be solved in a closed form expression obtaining (19). Turning to the single stream decoding probability P_{AC^c}, the decoding probability stream s_1 is generally,

$$\begin{aligned} &P_{A1} = \Pr(\mathcal{A}) \\ &= \Pr\{(1+w_1\tfrac{P}{2})(1+w_2\tfrac{P}{2}) - w_3\tfrac{P^2}{4} > (1+w_2\tfrac{P}{2}) \cdot e^R\}. \end{aligned} \tag{B.5}$$

The above probability (B.5) can be represented in its integral form and solved into closed form expression yielding (21). The second part of (18), is the probability of decoding the first stream treating the second as interference, and also succeeding in joint decoding. This is given by

$$\begin{aligned} P_{A2} \;\; = \;\; &\Pr\{\mathcal{A} \cap \mathcal{C}\} = \Pr\{(1+w_1\tfrac{P}{2})(1+w_2\tfrac{P}{2}) - w_3\tfrac{P^2}{4} \\ &> \max\left\{(1+w_2\tfrac{P}{2}), \, e^R\right\} \cdot e^R, \\ &(1+w_1\tfrac{P}{2}) > e^R, (1+w_2\tfrac{P}{2}) > e^R\}, \end{aligned} \tag{B.6}$$

which also lends itself in a similar manner into a closed form expression (22). ■

APPENDIX C
PROPERTIES OF THE HADAMARD PERMUTATION

MI expression for $I(\mathbf{y}; \mathbf{s})$ is $I(\mathbf{y}; \mathbf{s}) = \log\det\left\{1 + \frac{P}{2}HH^\dagger\right\}$, where H is a 2×2 channel matrix defined in (9). The explicit expression of $I(\mathbf{y};s_1|s_2)$, as in (34) is

$$I(\mathbf{y};s_1|s_2) \quad = \quad \frac{1}{2}\log\det\left(I_2+\frac{P}{4}H_{42}^{\dagger}H_{42}\right), \quad \text{(C.1)}$$

$$\text{where } H_{42} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12} & -h_{11} \\ h_{22} & -h_{21} \end{bmatrix},$$

where H_{42} stands for the equivalent channel matrix. We aim at proving

$$\log\det\left(I_2+\frac{P}{4}H_{42}^{\dagger}H_{42}\right) \geq \log\det\left(I_2+\frac{P}{2}H^{\dagger}H\right) \quad \text{(C.2)}$$

Specifying the determinant above, and using the elements of H we get

$$\begin{aligned}
&\left(1+\frac{P}{4}(|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2)\right)^2 \\
&-\frac{P^2}{4}(Im\left\{h_{11}^*h_{12}+h_{21}^*h_{22}\right\})^2 \geq \\
&\left(1+\frac{P}{2}(|h_{11}|^2+|h_{12}|^2)\right)\left(1+\frac{P}{2}(|h_{21}|^2+|h_{22}|^2)\right) \\
&-\frac{P^2}{4}|h_{11}h_{21}^*+h_{12}h_{22}^*|^2 \qquad \text{(C.3)}
\end{aligned}$$

The square imaginary part of an expression may be upper bounded by its square absolute value,

$$(Im\left\{h_{11}^*h_{12}+h_{21}^*h_{22}\right\})^2 \leq |h_{11}^*h_{12}+h_{21}^*h_{22}|^2. \quad \text{(C.4)}$$

The right hand-side expression in (C.4) may replace the left one in (C.3). Clearly, this poses a tighter inequality

$$\begin{aligned}
&\left(1+\frac{P}{4}(|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2)\right)^2 \\
&-\frac{P^2}{4}|h_{11}^*h_{12}+h_{21}^*h_{22}|^2 \geq \\
&\left(1+\frac{P}{2}(|h_{11}|^2+|h_{12}|^2)\right)\left(1+\frac{P}{2}(|h_{21}|^2+|h_{22}|^2)\right) \\
&-\frac{P^2}{4}|h_{11}h_{21}^*+h_{12}h_{22}^*|^2 \qquad \text{(C.5)}
\end{aligned}$$

which after some algebra, receives the following form

$$(|h_{11}|^2+|h_{21}|^2-|h_{12}|^2-|h_{22}|^2)^2 \geq 0 \quad \text{(C.6)}$$

This concludes the proof. ■

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