

Single-User Broadcasting Protocols over a Two-Hop Relay Fading Channel

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Abstract—A two-hop relay fading channel is considered, where only decoders possess perfect channel state information (CSI). Various relaying protocols and broadcasting strategies are studied. The main focus of this work is on simple relay transmission scheduling schemes. For decode-and-forward (DF) relaying, the simple relay cannot buffer multiple packets, nor can it reschedule retransmissions. This gives rise to consideration of other relaying techniques, such as amplify-and-forward (AF), where a maximal broadcasting achievable rate is analytically derived. A quantize-and-forward (QF) relay, coupled with a single-level code at the source, uses codebooks matched to the received signal power and performs optimal quantization. This is simplified by a hybrid amplify-QF (AQF) relay, which performs scaling, and single codebook quantization on the input. It is shown that the latter is optimal by means of throughput on the relay-destination link, while maintaining a lower coding complexity than the QF setting. A further extension of the AQF allows the relay to perform successive refinement, coupled with a matched multi-level code. Numerical results show that for high SNRs the broadcast approach over AF relay may achieve higher throughput gains than other relaying protocols that were numerically tractable.

Index Terms—Ad-hoc networks, amplify-and-forward, code layering, decode-and-forward, multi-hop relays, quantize-and-forward, single-user broadcasting.

I. INTRODUCTION

Cooperation among network users, in the form of relaying, has been of wide interest recently. The growing demand for capacity and coverage has exceeded the limits of a single server network. This challenge can be accommodated by allowing network users to act as relays, and improve the signal quality at its final destination. Wireless networks with multi-ray signals exhibit fading, sometimes even deep fading, which may highly distort the original signal. In a rapidly changing environment, it is customary to assume that transmitters have no access to channel state information (CSI), and only receivers possess perfect CSI. The performance is also usually evaluated by the outage capacity. The notion of capacity versus outage was introduced and discussed in [1] and [2, see references therein].

In the sequel we consider a relay channel, where the direct link channel quality is so poor, that it can be assumed there is no direct link between the source and the destination, as sketched in Figure 1. This is a special case of the classical relay channel [3], [4], [5], where the channel between the source and destination has very poor signal-to-noise ratio (SNR). In this setting, the source transmits to a



Fig. 1. Schematic diagram of a two-hop relay channel, where there is practically no direct link between source and destination.

single relay, which decodes/amplifies/quantizes its input data and forwards/retransmits its signal over to the destination. This setting is also known as a two-hop relay system [6].

Several contributions [7], [8], [9], and more, demonstrate practical examples for the two-hop relay setting. In these examples single rate codes are used and only the medium access (MAC) layer is modified so that an ad-hoc network can be supported, and a network member may serve as a relay, and thus increase the coverage and overall network capacity. This is also a special case of multi-hop relays [6]. It is observed in [10] that substantial capacity and coverage gains can be obtained with a simple two-hop relay architecture, where CSI or partial CSI is available at the transmitters. Notice that solutions in the MAC layer necessarily require a relay that decodes and retransmits information. This is the decode-forward relay, which might introduce non-negligible additional delays and complexity. Cooperation in the physical layer may allow non-regenerative decode-forward or amplify-forward [11] or quantize-forward [4] relays. This type of cooperation can clearly lead to higher aggregate throughput, provide lower delay, and complexity reduction.

The two-hop relay setting is also a special case of parallel relaying, where there is only one relay. The two relay symmetric network for the additive white Gaussian noise (AWGN) channel is studied in [12], where capacity bounds are obtained. Diversity gains in simple parallel relaying with receivers CSI and space-time permutations among relays is presented in [13], with further extensions in [14], [15].

Study of the relay channel [3], [4], [5] is of fundamental importance to cooperation in wireless networks, since it captures the ability of a user to assist in transferring information from a source to its destination - a situation which is prevalent in wireless networks due to the sharing of the wireless medium among all users. Unfortunately, the capacity of the relay channel is only known for some specific cases (e.g. degraded and reversely degraded relay channel, semi-deterministic relay channel, relay channel with feedback) which do not apply directly to common wireless settings. Recently, however, there has been some extensive work reported concerning the capacity of the relay channel and its implications on cooperation in wireless channels. For example, upper and lower bounds

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on the outage capacity and ergodic capacity of the three node Rayleigh fading relay channel are derived in [16]. Capacity bounds on the frequency-division (FD) AWGN relay channel are derived in [17]. Coding strategies for decode-and-forward and quantize-and-forward relay channel with extensions to multi-terminals are studied in [18], where only types of single level coding is considered. The capacity region of the AWGN multi-stage degraded relay channel is determined in [19]. Bounds on the capacity of the MIMO relay channel are derived in [20], for scenarios where perfect CSI is available at transmitting (and receiving) terminals, and where only receiving terminals possess CSI. In our two-hop relay setting, where CSI is available at receiving terminals only, the ergodic capacity can be computed, and it serves as a performance upper bound for evaluating the capacity of broadcasting protocols pursued here. For large scale networks with cooperating nodes, information-theoretic upper bounds on the network capacity may be found in [21]. An Achievable rate region for large scale networks is formulated in [22].

In a decode-and-forward (DF) [11] scheme, the relay decodes the received source message, re-encodes it, and forwards the resulting signal to the destination. Note that, since the relay must perfectly decode the source message, the achievable rates are bounded by the capacity of the channel between the source and relay. A non-regenerative relay has a different coding scheme than the source, and can improve for example the reliability of the relay-destination transmission. An example for non-regenerative MIMO relaying is [25]. The work in [26] compares between two DF protocols assuming knowledge of channel gains at the transmitter, and adhering to delay-limited capacity. Zhao and Li [27] present a simple differential modulation for a single DF fading relay channel, and show that in certain conditions relaying achieves diversity gains. Further work on user cooperation to increase diversity gains, using DF cooperation techniques over a Rayleigh fading channel are found in [28].

The amplify-and-forward (AF) [11] relay. In [29], different types of AF relay settings are studied and general expressions for the aggregate SNR at the destination are derived for varying number of relaying nodes. The study there is motivated by previous observations that amplify-forward relays can sometimes approach or exceed the performance of their decode-forward counterparts [11]. Yu and Li [30] investigate a network with slow fading over all links by means of practical turbo-coding, and notice that there is no significant performance difference in decode-forward compared to amplify-forward relaying schemes.

Quantize and forward relay implementation is considered in [31] and shown to be superior, in terms of average throughput, with respect to the decode-forward and amplify-forward relays, in presence of a direct link, and a fixed known channel gain on the relay-destination link, which models a two co-located user cooperation. Practical compress-and-forward code design was presented in [32] for the half-duplex relay channel. The quantization in [31], [32] is of Wyner-Ziv quantization type [33], which refers to the relay quantizing its received observation of the sources symbol, while relying on side information which is available at the destination receiver, from

the direct link. In a two-hop relay setting the receiver has no additional side information, and thus the quantization applied at the relay is a standard quantization of a noisy Gaussian source [34].

A broadcasting strategy with a time division (TD) protocol is suggested by Yuksel et. al. [35]. In which two level coding for a relay channel is presented in a half-duplex mode, and the receiver attempts decoding every two transmission blocks jointly (one from the relay and another from the source). Relatively small broadcasting gains are observed here [35]. Broadcast strategy for a single-user facilitates reliable transmission rates adapted to the actual channel conditions, without providing any feedback from the receiver to the transmitter [36], [37]. The single-user broadcasting approach hinges on the broadcast channel, which was first explored by Cover [38]. In a broadcast channel a single transmission is directed to a number of receivers, each enjoying possibly different channel conditions, reflected in their received SNR. The Gaussian broadcast channel with a single transmit antenna coincides with the classical physically degraded Gaussian broadcast channel, whose capacity region is well known [5],[39], [40]. Single-user broadcasting may be interpreted as hierarchical coding via multi-level coding (MLC) [41], [42], [43], [44].

In this paper we study various broadcasting protocols for the two-hop relay channel, where the source transmitter has no CSI, and the relay has perfect CSI of the source-relay link, but does not possess CSI of the relay-destination link, and finally the destination has perfect CSI of both links. Furthermore, the main focus of this work is on simple relay transmission scheduling schemes. That is, the simple relay cannot buffer multiple packets, nor can it reschedule retransmissions independently. The simplicity of the cooperation scheme is obtained by letting the source alone manage the packet transmission and retransmission scheduling.

For a decode-and-forward relay several broadcasting strategies are considered. Using finite level coding, an outage lower bound includes single level coding at source and a regenerative coding at the relay upon successful decoding. Then, two level coding is considered, in the source or in the relay separately. As expected, when both channels have the same fading distributions, two level coding at the source and single-level coding at the relay have better performance than the opposite setting (two level coding at the relay). This is due to the additional degree of freedom in the relay transmission, which adapts the transmission rate to the number of layers successfully decoded. Next, a continuous broadcasting approach is considered. Analytic derivation of the optimal power allocation for single level coding at the source and continuous broadcasting at the relay is obtained. Closed form expressions for the optimal power allocation for continuous broadcasting at source and regenerative continuous broadcasting at the relay are also obtained. The latter broadcasting approach is **sub-optimal**, and is named naive-broadcasting, since regenerative coding at the relay may be inefficient in case of relatively large outage region at the relay. For the optimal continuous broadcasting strategy at source and relay we explicitly state the optimization problem, which does not lend itself to a closed form solution.

Amplify-and-forward relay is considered next, where both outage and continuous broadcasting are pursued. An AF relay simply scales the input signal (along with the additive noise) to a fixed power level, which corresponds to its available transmission power. Interestingly, the optimal power distribution at the source encoder is derived similarly to the single channel (no cooperation) case [36], with a different fading power distribution. This is also similar in derivation to the single-input multiple-output (SIMO) channels, where a single fading power random variable controls the power and rate allocation [45]. It may be noticed from the numerical results, that for high SNRs, AF continuous broadcasting has highest throughput gains over all DF broadcasting schemes that were analytically/numerically tractable, probably since the optimal double broadcasting approach was not solved. In [11] different relaying protocols are considered for half duplex relaying systems [46] with CSI at receiver only, and where the outage probability [1] characterizes their performance, rather than ergodic capacity. For these relay settings [11] amplify-forward achieves near optimal performance, and achieves high gains over the direct transmission alternative. In the context of uncoded communications, and antipodal signalling it is shown in [47] that the optimal regeneration (relaying) function is an amplify function at the limit of high SNR. Further literature on the capacity of non-fading AWGN relay networks with regenerating relays may be found in [48, see references within]. The regenerating function performs symbol by symbol processing, and is commonly used in optical communications.

The last type of relay considered is the quantize-forward (QF) relay, which performs optimal compression on the input signal. A simple QF relay has to hold a codebook for compression for every receive SNR, due to the fading on the source-relay link. An alternative hybrid amplify-quantize-and-forward (AQF) is suggested, where the relay input signal is amplified to a fixed average power, and then optimally compressed with a single codebook. The latter quantization scheme is also shown to be an optimal QF scheme in terms of relay-destination link throughput, it is shown that in the general form of a QF relay, throughput on the relay-destination link is maximized by amplifying the input signal prior to quantization, which motivates the use of the hybrid AQF relay. The AQF scheme is also extended to successive refinement matched with the broadcasting strategy. Every refinement layer is associated with a broadcasting layer. This strategy was also considered in [31] for the Wyner-Ziv quantizing relay. As it turns out from the numerical results, the AQF broadcasting strategies although outperform the QF setting, are inferior to the DF and AF relays considered.

The structure of the remainder of the paper is as follows. The relay fading channel models are introduced in section II. Simple average rate upper bounds for the two-hop network are derived in section III. Various finite coding strategies for the DF relay are studied in section IV. Their extension to continuous layering protocols (for the DF relay) are presented in section V. Next, broadcasting for the AF relay is presented in section VI. The quantize-forward relay is then considered in section VII. And in section VIII a hybrid amplify-quantize-forward relay is considered and some broadcasting strategies

are suggested. The numerical results are presented in section IX, followed by the conclusion in section X.

II. CHANNEL MODEL

Consider the following single-input single-output (SISO) channel,

$$\mathbf{y}_r = h_s \mathbf{x}_s + \mathbf{n}_s, \quad (1)$$

where \mathbf{y}_r is a received vector at the relay, of length N , which is also the transmission block length, \mathbf{x}_s is the transmitted vector. \mathbf{n}_s is the additive noise vector, with elements that are complex Gaussian i.i.d with zero mean and unit variance denoted $\mathcal{CN}(0, 1)$, and h_s is the (scalar) fading coefficient. The fading h_s is assumed to be perfectly known at the relay and the destination receivers only. The source transmitter has no channel state information (CSI). The power constraint at the source is given by $P_s = E|x_s|^2$. E stands for the expectation operator. The channel between the relay and the destination is described by

$$\mathbf{y}_d = h_r \mathbf{x}_r + \mathbf{n}_r, \quad (2)$$

where \mathbf{y}_d is a received vector at the destination receiver, of length N , which is also the transmission block length, \mathbf{x}_r is the relay transmitted vector. \mathbf{n}_r is the additive noise vector, with elements that are complex Gaussian i.i.d with zero mean and unit variance denoted $\mathcal{CN}(0, 1)$, and h_r is the (scalar) fading coefficient. The fading coefficients h_s, h_r are assumed to be perfectly known at the destination receivers only. The relay transmitter does not possess h_r . The power constraint at the relay is given by $P_r = E|x_r|^2$.

It is assumed that the relay operates in a full-duplex mode, by receiving and transmitting on different frequency bands, realizing a two-hop network. Furthermore, the relay is not capable of buffering data. In the DF protocols, the relay has to forward all the data successfully decoded immediately. Layers that were not decoded on the path from source to destination must be rescheduled for retransmission at the source. If the relay had packet scheduling capabilities, the DF protocols could be improved by letting the relay perform retransmission of layers that are not decoded at the destination. However this calls for distributed scheduling control, which highly complicates the system, and is beyond the scope of this paper.

In addition, in this communication setting (1)-(2) the only ACK/NACK feedback required is from the destination to the source, indicating the number of layers successfully decoded, and it may be assumed that such low rate direct uplink exists, and is the only available uplink.

III. SIMPLE UPPER BOUNDS

A. Full-CSI (FCSI) Upper Bound

The full-CSI upper bound is derived for a hypothetical case that both source and relay have perfect CSI of all links, and the source always transmits in the maximal achievable rate over this relay channel. This achievable rate is the minimal rate determined by the fading gain realizations on both links. It is generally expressed by

$$C_{FCSI} = E_{s_s, s_r} \log(1 + \min(P_s s_s, P_r s_r)) \quad (3)$$

where $s_s = |h_s|^2$, and $s_r = |h_r|^2$. The logarithm base is the natural logarithm base in (3), and throughout the paper. This means that the unit of all capacities and rates is Nats per channel use. By explicitly extracting the expectation in (3) we get

$$\begin{aligned}
C_{FCSI} &= \int_0^\infty d\nu \int_0^\infty d\mu f(\nu) f(\mu) \log(1 + \min(P_s \nu, P_r \mu)) \\
&= \int_0^\infty d\nu \int_{\frac{P_s}{P_r} \nu}^\infty d\mu f(\nu) f(\mu) \log(1 + P_s \nu) \\
&\quad + \int_0^\infty d\nu f(\nu) \int_0^{\frac{P_s}{P_r} \nu} d\mu f(\mu) \log(1 + P_r \mu) \\
&= \int_0^\infty d\nu f(\nu) (1 - F(\frac{P_s}{P_r} \nu)) \log(1 + P_s \nu) \\
&\quad + \frac{P_s}{P_r} \int_0^\infty d\nu (1 - F(\nu)) f(\frac{P_s}{P_r} \nu) \log(1 + P_s \nu)
\end{aligned}$$

where $f(x)$ and $F(x)$ are the probability density function (PDF) and CDF of the fading gain, respectively. For a Rayleigh fading channel, the FCSI upper bound is given by

$$\begin{aligned}
C_{FCSI} &= (1 + \frac{P_s}{P_r}) \int_0^\infty d\nu e^{-(1 + \frac{P_s}{P_r})\nu} \log(1 + P_s \nu) \\
&= e^{\frac{P_s + P_r}{P_r P_s}} E_1\left(\frac{P_s + P_r}{P_r P_s}\right)
\end{aligned} \quad (5)$$

where $E_1(x)$ is the exponential integral function $E_1(x) \triangleq \int_x^\infty dt \frac{e^{-t}}{t}$ for $x \geq 0$ [49].

B. Cut-set Upper Bounds

The ergodic cut-set upper bound is the minimum of the average achievable rates on the two links (source-relay and relay-destination). This is specified by

$$C_{erg} = \min(E_{s_s} \log(1 + P_s s_s), E_{s_r} \log(1 + P_r s_r)). \quad (6)$$

For Rayleigh fading channels, and similar fading gain distribution functions $f(x)$ of the two links, the ergodic upper bound simplifies to

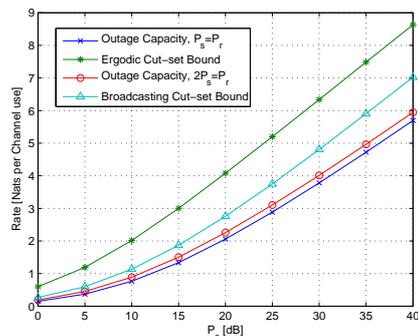
$$C_{erg} = \int_0^\infty d\nu e^{-\nu} \log(1 + P\nu), \quad \text{s.t. } P = \min(P_s, P_r), \quad (7)$$

which is justified by the monotonicity of the ergodic capacity as function of P . A tighter upper bound on the broadcast strategy is the broadcasting cut-set bound. This is the minimum average broadcasting rate achievable on each of the links separately. It is specified by

$$\begin{aligned}
R_{bs-cutset} &= \min \left\{ \int_0^\infty du f_\nu(u) R_\nu(u), \int_0^\infty du f_\mu(u) R_\mu(u) \right\} \quad (8)
\end{aligned}$$

where $f_\nu(u)$ and $f_\mu(u)$ are the PDF of the source-relay and relay destination fading gains, respectively. And $R(u)$ is the broadcasting achievable rate for a fading gain u . For a Rayleigh fading channel with similar distribution on both links, the cut-set bound is given by [37, eq. (18)],

$$R_{bs-cutset} = 2E_1(s_0) - 2E_1(1) - (e^{-s_0} - e^{-1}), \quad (9)$$



(4)

Fig. 2. The outage capacity, as a DF lower bound is presented for equal source and relay powers ($P_s=P_r$) and stronger relay power ($P_r=2P_s$). The ergodic cut-set and broadcasting cut-set upper bounds demonstrate the DF upper bounds. All bounds are computed for a Rayleigh fading channel.

where $s_0 = \frac{2}{1 + \sqrt{1 + 4 \min(P_s, P_r)}}$, and $E_1(x)$ is the exponential integral function. The broadcast approach is discussed elaborately in section V. The broadcasting cut-set bound (9) may be achieved if the relay is allowed to delay its data and reschedule retransmissions independently. Furthermore, the relay has to inform the source how many layers were decoded for every block. We do not assume such feedback is available. The only feedback, in our channel model, is from destination to source indicating the number of successfully decoded layers.

IV. FINITE LEVEL CODING DECODE-FORWARD PROTOCOLS

We consider here various broadcast approaches for the two-hop relay network. A broadcasting lower bound is the single-level coding, known also as the outage approach [37]. Broadcast strategy for a single-user facilitates reliable transmission rates adapted to the actual channel conditions, without providing any feedback from the receiver to the transmitter [36], [37]. In a degraded broadcast channel, which is also the case in the two-hop relay network, superposition coding achieves the channel capacity.

A. Outage Approach

In single level coding the code rate from the source transmitter to the relay is determined by the fading gain threshold selected. For a power threshold s_s , the code rate is $R = \log(1 + P_s s_s)$, and this same rate is transmitted from the relay to the destination, with power P_r , thus $R = \log(1 + P_r s_r)$, and $s_r = \frac{P_r}{P_s} s_s$. Then the average achievable rate from the source to the destination is

$$\begin{aligned}
R_{1,avg} &= P(\nu > s_s) P(\mu > s_r) \log(1 + P_s s_s) \\
&= (1 - F_\nu(s_s))(1 - F_\mu(s_r)) \log(1 + P_s s_s) \quad (10)
\end{aligned}$$

where ν, μ are the fading gain random variables, $F_\nu(x), F_\mu(x)$ are the corresponding cumulative density functions (CDF), and P_s is the source transmission power. For a Rayleigh fading channel, with $F_\mu(x) = F_\nu(x) = 1 - e^{-x}$, the average rate is given by

$$R_{1,avg} = e^{-s_s} e^{-\frac{P_r}{P_s} s_s} \log(1 + P_s s_s) \quad (11)$$

and the maximal achievable rate is thus

$$R_{1L} = \max_{s_s} e^{-s_s} e^{-\frac{P_s}{P_r} s_s} \log(1 + P_s s_s) \quad (12)$$

B. Two level coding

Consider now a two level code layering with a predetermined allocated power $\alpha_s P_s$ to the first layer, and $(1 - \alpha_s) P_s$ to the other layer, where $0 \leq \alpha_s \leq 1$. The relay decodes the received data, and transmits only the successfully decoded layers. That is, if only a single layer was decoded, it retransmits this layer to the destination at power P_r , and if both layers were decoded successfully it retransmits both of them with a total power P_r . Two approaches may be used for retransmission when only one layer was successfully decoded:

- 1) Perform at the relay single level coding, with power P_r , and rate corresponding to the input rate.
- 2) Perform at the relay two level layering, with power P_r , at a total rate equal to the input rate.

Clearly, the second approach may be more efficient, in particular when performing a different allocation of power and layers for the two cases where one layer is decoded, and where both layers are decoded at the relay.

A more simplified approach is to perform two level layering only at one end (at the source it is clearly more efficient) and single level coding for the other link.

C. Source: Outage, Relay: Code Layering

In this coding scheme the source transmitter performs single level coding, and when the relay succeeds in decoding the data it encodes the data into a two layer code, and transmits to the destination, which tries to decode the first layer and then the other. The code rate at the source is given by

$$R_1^s = \log(1 + P_s s_s), \quad (13)$$

where s_s is the fading gain threshold selected for the system. Its selection dictates the code rate. The rates for each layer at the relay encoder are then given by

$$\begin{aligned} R_1^r &= \log(1 + P_r s_{r,1}) - \log(1 + (1 - \alpha_r) P_r s_{r,1}) \\ R_2^r &= \log(1 + (1 - \alpha_r) P_r s_{r,2}), \end{aligned} \quad (14)$$

where $0 \leq \alpha_r \leq 1$, and $\alpha_r P_r$ is the power allocated to the first layer. The relation between the source and relay rates is

$$R_1^s = R_1^r + R_2^r, \quad (15)$$

which is also a constraint on the selection of fading gain thresholds for the coding schemes. The overall average rate is then

$$\begin{aligned} R_{1-2,avg} &= \max_{s_r,1, s_r,2, \alpha_r} P(\nu > s_s) \left[P(\mu > s_{r,1}) R_1^r \right. \\ &\quad \left. + P(\mu > s_{r,2}) R_2^r \right] \\ &= \max_{s_r,1, s_r,2, \alpha_r} (1 - F_\nu(s_s)) \left[(1 - F_\mu(s_{r,1})) R_1^r \right. \\ &\quad \left. + (1 - F_\mu(s_{r,2})) R_2^r \right] \end{aligned} \quad (16)$$

where ν is the fading gain random variable (RV) on the source-relay link, and μ is the RV on the relay destination link. The fading threshold s_s in (16) is implicitly specified by the equal rates constraint in (15).

D. Source: Code Layering, Relay: Outage

In this coding scheme the source transmitter performs two level coding, and the relay tries to decode both layers. If successful, it transmits a single level code at a rate which is the sum of source rates. If only one layer was decoded successfully at the relay it encodes it into a different single level code, which is equal in rate to the first level of the source channel code. This gives a higher flexibility in decoding of a single layer at the destination receiver, when the channel conditions on the source-relay link allow only one layer detection at the relay. The channel code rate at the source is given by

$$\begin{aligned} R_1^s &= \log(1 + P_s s_{s,1}) - \log(1 + (1 - \alpha_s) P_s s_{s,1}) \\ R_2^s &= \log(1 + (1 - \alpha_s) P_s s_{s,2}), \end{aligned} \quad (17)$$

where $0 \leq \alpha_s \leq 1$, $s_{s,1}$ and $s_{s,2}$ are the fading gain thresholds implicitly specifying the layering rates. The rates of the single level code at the relay are then given by

$$\begin{aligned} R_1^r &= \log(1 + P_r s_{r,1}) \quad \text{s.t. } R_1^r = R_1^s \\ R_2^r &= \log(1 + P_r s_{r,2}) \quad \text{s.t. } R_2^r = R_1^s + R_2^s \end{aligned} \quad (18)$$

where $s_{r,1}$ and $s_{r,2}$, are actually determined from the rate equalities on the right hand side of (18). The overall average rate is then

$$\begin{aligned} R_{2-1,avg} &= \\ &= \max_{s_{s,1}, s_{s,2}, \alpha_s} P(s_{s,1} \leq \nu < s_{s,2}) P(\mu > s_{r,1}) R_1^s \\ &\quad + P(\nu > s_{s,2}) P(\mu > s_{r,2}) (R_1^s + R_2^s) \\ &= \max_{s_{s,1}, s_{s,2}, \alpha_s} (F_\nu(s_{s,2}) - F_\nu(s_{s,1})) (1 - F_\mu(s_{r,1})) R_1^s \\ &\quad + (1 - F_\nu(s_{s,2})) (1 - F_\mu(s_{r,2})) (R_1^s + R_2^s) \end{aligned} \quad (19)$$

where ν is the fading gain RV on the source-relay link, and μ is the RV on the relay destination link.

As may also be noticed from the numerical results in section IX, for similar fading gain distributions of the two links, this approach outperforms single level coding at the source and two level coding at the relay, described in the previous subsection. The main reason for this difference is that the outage approach described here adapts to the source-relay channel conditions. That is, the outage rate from relay to destination is equal to the decoded rate, and depends on the number of successfully decoded layers (18). However when considering the opposite approach (source: outage, relay: two-level) the outage rate is fixed for all channel conditions, and if the relay fails in its decoding, nothing is transmitted to the destination.

V. CONTINUOUS BROADCASTING DECODE-FORWARD PROTOCOLS

We consider here the following setting. The source encoder transmits a continuum of layered codes, with power allocation as function of the fading gain (like in the known single user broadcasting scheme [37]). The relay decodes all the layers up to the layer corresponding to the actual channel fading gain realization. Then it encodes this data for retransmission at power P_r and a rate corresponding to the decoded data. It can perform either optimal continuum layering for this rate, or either transmit at a single level coding approach at this rate. We also consider here the case where the source performs single

level coding, and the relay performs continuous. These strategies are compared by means of achievable average throughput, and they present a complexity performance tradeoff.

A. Overview on Single-User Broadcasting

Consider the following single-input single-output (SISO) channel (without a relay),

$$\mathbf{y} = h\mathbf{x} + \mathbf{n}, \quad (20)$$

where \mathbf{y} is a received vector, of length N , which is also the transmission block length, \mathbf{x} is the transmitted vector. \mathbf{n} is the additive noise vector, with elements that are complex Gaussian i.i.d with zero mean and unit variance denoted $\mathcal{CN}(0, 1)$, and h is the (scalar) fading coefficient. The fading h is assumed to be perfectly known at the receiver end only. The transmitter has no CSI.

We adhere to the single-user broadcasting approach for a SISO channel [37]. In this approach the transmitter sends multi-layer coded data. The receiver decodes the maximal number of layers given a channel realization (per-block). The differential rate per layer is given by

$$dR(s) = \log \left(1 + \frac{s\rho(s)ds}{1+sI(s)} \right) = \frac{s\rho(s)ds}{1+sI(s)} \quad (21)$$

where $\rho(s)ds$ is the transmit power of a layer parameterized by s , intended for receiver s , which also designates the transmit power distribution. The right hand-side equality is justified in [50]. Information streams intended for receivers indexed by $u > s$ are undetectable and play a role of additional interfering noise, denoted by $I(s)$. The interference for a fading gain s is

$$I(s) = \int_s^\infty \rho(u)du, \quad (22)$$

which is also a monotonically decreasing function of s . The total transmitted power is the overall collected power assigned to all layers,

$$P = \int_0^\infty \rho(u)du = I(0). \quad (23)$$

As mentioned earlier, the total achievable rate for a fading realization s is an integration of the fractional rates over all receivers with successful layer decoding capability,

$$R(s) = \int_0^s \frac{u\rho(u)du}{1+uI(u)}. \quad (24)$$

Average rate is achieved with sufficiently many transmission blocks, each viewing an independent fading realization. Therefore, the total average rate R_{bs} over all fading realizations is

$$R_{bs} = \int_0^\infty du f(u)R(u) = \int_0^\infty du (1-F(u)) \frac{u\rho(u)}{1+uI(u)} \quad (25)$$

where $f(u)$ is the PDF of the fading gain, and $F(u) = \int_0^u da f(a)$ is the corresponding CDF.

B. Source: Outage, Relay: Continuum Broadcasting

In this coding scheme the source transmitter performs single level coding. Whenever channel conditions allow decoding at the relay, it performs continuum broadcasting, as described in the previous sub-section. Thus the received rate at the destination depends on the instantaneous channel fading gain realization on the relay-destination link. Clearly, a necessary condition for receiving something at the destination is that channel conditions on the source-relay link will allow decoding. The source transmission rate is given by

$$R_1^s = \log(1 + P_s s_s), \quad (26)$$

and the corresponding achievable rate at the destination is given by,

$$R^r(\nu) = \int_0^\nu \frac{u\rho_r(u)du}{1+uI_r(u)} \quad (27)$$

where $I_r(\nu)$ is the residual interference distribution function, and its boundary conditions are stated in (22)-(23). The total rate transmitted in the broadcasting link is equal to the single level code rate of the source-relay link, that is

$$R_1^s = \int_0^\infty \frac{u\rho_r(u)du}{1+uI_r(u)} \quad (28)$$

The above condition (28) states a constraint on the optimization of the average rate. The average rate expression, considering the transmission scheme on the two links is

$$\begin{aligned} R_{avg} &= P(\nu > s_s) \int_0^\infty dx f_\mu(x) \int_0^x \frac{u\rho_r(u)du}{1+uI_r(u)} \\ &= (1 - F_\nu(s_s)) \int_0^\infty dx (1 - F_\mu(x)) \frac{x\rho_r(x)}{1+xI_r(x)} \end{aligned} \quad (29)$$

where we have used partial integration rule. The average rate maximization problem can now be posed,

$$\begin{aligned} &R_{1-bs,avg} \\ &= \max_{s_s, I_r(\nu)} (1 - F_\nu(s_s)) \int_0^\infty dx (1 - F_\mu(x)) \frac{x\rho_r(x)}{1+xI_r(x)} \\ &\text{subject to} \\ &\int_0^\infty \frac{u\rho_r(u)du}{1+uI_r(u)} = \log(1 + P_s s_s) \end{aligned} \quad (30)$$

As a first step in solving the maximal average rate the residual interference distribution $I_r(\nu)$ is found for every s_s . That is

$$\begin{aligned} &R_{1-bs}(s_r) \\ &= \max_{I_r(\nu)} \int_0^\infty dx (1 - F_\mu(x)) \frac{x\rho_r(x)}{1+xI_r(x)} \\ &\triangleq \int_0^\infty dx G_1(x, I(x), I'(x)) \\ &\text{subject to} \\ &R_1^s = \int_0^\infty \frac{u\rho_r(u)du}{1+uI_r(u)} \\ &\triangleq \int_0^\infty dx G_2(x, I(x), I'(x)) \end{aligned} \quad (31)$$

where $I'(x) = \frac{dI(x)}{dx}$. The necessary condition for extremum in (31) subject to the subsidiary condition, is in generally stated [51]

$$G_{1,I} + \lambda G_{2,I} - \frac{d}{dx} (G_{1,I'} + \lambda G_{2,I'}) = 0, \quad (32)$$

where $G_{1,I}$ is the derivative of G_1 w.r.t. I , and $G_{1,I'}$ is the derivative of G_1 w.r.t. I' . The scalar λ is also known as a Lagrange multiplier, and is determined determined from the subsidiary condition in (31). The substitution of $S_I \triangleq G_{1,I} + \lambda G_{2,I}$, and $S_{I'} \triangleq G_{1,I'} + \lambda G_{2,I'}$ by using (31) is

$$\begin{aligned} S_I &= \frac{x^2 I'(1-F_\mu+\lambda)}{(1+xI)^2} \\ S_{I'} &= \frac{-x(1-F_\mu+\lambda)}{1+xI} \\ \frac{dS_{I'}}{dx} &= \frac{(x^2 I' - 1)(1-F_\mu+\lambda)}{(1+xI)^2} \end{aligned} \quad (33)$$

A substitution of the expressions in (33) into the extremum condition in (32) yields a general solution for the residual interference, as function of λ ,

$$I_r(x) = \begin{cases} P & 0 \leq x \leq x_0 \\ \frac{1-F_\mu(x)+\lambda-xf_\mu(x)}{f_\mu(x)x^2} & x_0 \leq x \leq x_1 \\ 0 & x \geq x_1 \end{cases} \quad (34)$$

where x_0 and x_1 are determined from the boundary conditions $I_r(x_0) = P$ and $I_r(x_1) = 0$, respectively. The scalar λ is determined from the subsidiary condition in (31). When considering a Rayleigh flat fading channel for the relay destination link, i.e. $F_\mu(x) = 1 - \exp(-x)$, the residual interference distribution gets the following form

$$I_r(x) = \frac{\lambda}{e^{-x}x^2} + \frac{1}{x^2} - \frac{1}{x} \quad \text{on } x_0 \leq x \leq x_1 \quad (35)$$

and the condition $I_r(x_1) = 0$ gives

$$x_1 = 1 - W_L(-\lambda e) \quad (36)$$

where $W_L(x)$ is the Lambert W-function, also called the omega function, is the inverse of the function $f(W) = We^W$. Interestingly the subsidiary condition with (35) as the solution for $I_r(x)$ yields a simplified expression

$$\begin{aligned} R_T &= \int_{x_0}^{x_1} \frac{u\rho_r(u)du}{1+uI_r(u)} \\ &= 2 \log(x_1) - x_1 - (2 \log(x_0) - x_0) \\ &= 2 \log(1 - W_L(-\lambda e)) - 1 + W_L(-\lambda e) \\ &\quad - 2 \log(x_0) + x_0 \end{aligned} \quad (37)$$

where (36) is used for substitution of x_1 . Using the subsidiary condition (31), i.e. $R_T = R_1^s$, the solution of x_0 as function of λ is

$$x_0 = -2W_L \left(-0.5e^{\log(1-W_L(-\lambda e)) - 0.5 + 0.5W_L(-\lambda e) - 0.5R_1^s} \right) \quad (38)$$

Finally by requiring $I_r(x_0) = P$, the corresponding λ can be found. Thus all initial conditions are satisfied, the solution for λ is obtained by numerically solving the nonlinear equation specified by $I_r(x_0) = P$. The maximal rate $R_{1-bs,avg}$ is then obtained by searching numerically over s_s and evaluating $R_{1-bs,avg}$ for all s_s in the search.

C. Source: Continuum Broadcasting, Relay: Outage

In this coding scheme the source transmitter performs continuum broadcasting, as described in the previous subsection. The relay encodes the successfully decoded layers into a single level block code. Thus the rate of each transmission from the relay depends on the number of layers decoded. For a fading gain realization ν on the source-relay link the decodable rate at the relay is

$$R^s(\nu) = \int_0^\nu \frac{u\rho_s(u)du}{1+uI_s(u)}. \quad (39)$$

This is also the rate to be transmitted in a single level coding approach, yielding

$$R_1^r(\nu) = \log(1 + P_r s_r(\nu)), \quad (40)$$

where $s_r(\nu)$ is the fading gain threshold for decoding at the destination. In order to ensure equal source and relay transmission rates, it is required that $R_1^r(\nu) = R^s(\nu)$. The average rate is then given by

$$\begin{aligned} R_{bs-1,avg} &= \max_{I_s(x)} \int_0^\infty dx P(\mu \geq s_r(x)) f_\nu(x) R^s(x) \\ &= \max_{I_s(x)} \int_0^\infty dx (1 - F_\mu(s_r(x))) f_\nu(x) \int_0^x \frac{u\rho_s(u)du}{1+uI_s(u)} \\ &= \max_{I_s(x)} \int_0^\infty dx e^{-x} e^{-s_r(x)} \int_0^x \frac{u\rho_s(u)du}{1+uI_s(u)} \end{aligned} \quad (41)$$

where a Rayleigh fading distribution is assumed on the last equality, and

$$s_r(\nu) = \frac{\left(\int_0^\nu dx \frac{x\rho_s(x)}{1+xI_s(x)} - 1 \right)}{P_r} \quad (42)$$

As may be noticed from (42) the functional subject to optimization in (41) does not have a localization property [51], and thus does not have a standard Euler-Lagrange equation for an extremum condition.

D. Source and Relay: Naive Broadcasting

In this scheme both source transmitter and relay perform continuum layering, however only sub-optimally on the relay-destination link. The layers decoded successfully in the relay are encoded again for retransmission, while adding null data instead of layers which could not be decoded. This introduces an inherent self interference on transmission from the relay. Furthermore the destination tries to decode all layers up to the null data, if not successful declares an outage. This model is relevant in particular when the relay transmission power is significantly greater than the source transmission power $P_r \gg P_s$. The source rate, as function of the fading gain is same as (39). The average rate is given by

$$\begin{aligned} R_{nbs-nbs,avg} &= \max_{I_s(\nu)} \int_0^\infty dx f_\nu(x) (1 - F_\mu(x \frac{P_s}{P_r})) \int_0^x \frac{u\rho_s(u)du}{1+uI_s(u)}. \end{aligned} \quad (43)$$

When $P_r = P_s$ and $F_\mu(x) = F_\nu(x) \triangleq F(x)$ the average rate expression can be brought to the following form

$$R_{nbs-nbs,avg} = \max_{I_s(\nu)} \frac{1}{2} \int_0^\infty dx (1 - (2F(x) - F(x)^2)) \frac{x\rho_s(x)}{1+xI_s(x)}. \quad (44)$$

Denote $G(x) = 2F(x) - F(x)^2$, and the optimal solution for (44) can be specified, like in [37], by

$$I_s(x) = \begin{cases} P_s & 0 \leq x \leq x_0 \\ \frac{1-2F(x)+F(x)^2}{2x^2 f(x)(1-F(x))} - \frac{1}{x} & x_0 \leq x \leq x_1 \\ 0 & x \geq x_1 \end{cases} \quad (45)$$

For a Rayleigh fading channel and possible different powers at the source and relay the optimization in (43) is given the following form

$$R_{nbs-nbs,avg} = \max_{I_s(\nu)} \frac{1}{1+\frac{P_s}{P_r}} \int_0^\infty dx e^{-x(1+\frac{P_s}{P_r})} \frac{x\rho_s(x)}{1+xI_s(x)}. \quad (46)$$

and the solution of $I_s(x)$ for the maximal rate (46) is

$$I_s(x) = \frac{1}{(1+\frac{P_s}{P_r})x^2} - \frac{1}{x} \quad x_0 \leq x \leq x_1 \quad (47)$$

where $I(x_1) = 0$, $I(x_0) = P_s$, hence $x_1 = \frac{1}{1+\frac{P_s}{P_r}}$ and $x_0 = (-1 + \sqrt{1 + 4P_s/(1 + \frac{P_s}{P_r})})/2/P_s$.

E. Source and Relay: Optimal Broadcasting

In this scheme both source and relay perform the optimal continuum broadcasting. The source transmitter encodes a continuum layered code. The relay decodes up to the maximal decodable layer. Then it retransmits the data in a continuum multi-layer code matched to the rate that has been decoded last. In this scheme the source encoder has a single power distribution function, which depends only on a single fading gain parameter. The relay uses for transmission a power distribution which depends on two parameters, which are the two fading gains on the source-relay and the relay-destination links.

In general, the source channel code rate as function of the fading gain is the same one specified in (39). The rate achievable at the destination is then given by

$$R^s(\nu, \mu) = \int_0^\mu \frac{u\rho_r(\nu, u)du}{1+uI_r(\nu, u)}. \quad (48)$$

The maximal average rate is then specified by

$$R_{bs-bs,avg} = \max_{I_s(\nu), I_r(\nu, \mu)} \int_0^\infty dx \int_0^\infty dy f_\nu(x) f_\mu(y) \int_0^y \frac{u\rho_r(x, u)du}{1+uI_r(x, u)} \quad (49)$$

subject to

$$\int_0^\infty \frac{u\rho_r(x, u)du}{1+uI_r(x, u)} = \int_0^x \frac{u\rho_s(u)du}{1+uI_s(u)}$$

which may be simplified into

$$R_{bs-bs,avg} = \max_{I_s(\nu), I_r(\nu, \mu)} \int_0^\infty dx \int_0^\infty dy f_\nu(x) (1 - F_\mu(y)) \frac{y\rho_r(x, y)}{1+yI_r(x, y)} \quad (50)$$

subject to

$$\int_0^\infty \frac{u\rho_r(x, u)du}{1+uI_r(x, u)} = \int_0^x \frac{u\rho_s(u)du}{1+uI_s(u)}$$

In order to present an Euler-Lagrange equation here, the subsidiary condition in (50) still has to be brought to a functional form, and then it could be solved with the aid of the Lagrange multipliers.

VI. BROADCASTING OVER THE AMPLIFY-FORWARD RELAY

Consider a relay that cannot decode/encode the data, but can only amplify the input signal. The channel model to consider here is the same one specified in (1)-(2). It may however be assumed that the relay can estimate the input signal power, and amplify the signal (without distortion) by a factor that ensures maximal transmission P_r from the relay. In such case the amplification coefficient is given by

$$\gamma = \sqrt{\frac{P_r}{P_s|h_s|^2 + 1}}. \quad (51)$$

The equivalent received signal at the destination may be specified by

$$\mathbf{y}'_d = \frac{\gamma h_r h_s}{\sqrt{\gamma^2 |h_r|^2 + 1}} \mathbf{x}_s + \mathbf{n}'_r, \quad (52)$$

where $\mathbf{n}'_r \sim \mathcal{CN}(0, 1)$ and the original source signal is multiplied by a factor, which represents an equivalent fading coefficient with power

$$s_b = \frac{\gamma^2 s_r s_s}{\gamma^2 s_r + 1} = \frac{P_r s_r s_s}{P_r s_r + P_s s_s + 1}, \quad (53)$$

where $s_r = |h_r|^2$ and $s_s = |h_s|^2$, and we have used the amplification factor definition from (51) for explicitly stating the equivalent fading gain. The CDF of the equivalent fading gain s_b is then given by

$$F_{s_b}(x) = Pr(s_b < x) = \int \int_{\mathcal{R}} dx_s dx_r f_{s_s}(x_s) f_{s_r}(x_r) \quad (54)$$

where $\mathcal{R} = \{x_r, x_s \in [0, \infty) \mid \frac{P_r x_r x_s}{P_r x_r + P_s x_s + 1} \leq x\}$, and when assuming a Rayleigh fading channel, thus $f_{s_r}(x_r) = e^{-x_r}$ and $f_{s_s}(x_s) = e^{-x_s}$. The CDF is given by

$$F_{s_b}(x) = 1 - \int_{\frac{P_s x}{P_r}}^\infty dx_r \int_{\frac{x(1+P_r x_r)}{x_r P_r - x P_s}}^\infty dx_s e^{-x_s} e^{-x_r} \quad (55)$$

$$= 1 - \int_{\frac{P_s x}{P_r}}^\infty dx_r e^{-x_r - \frac{x(1+P_r x_r)}{x_r P_r - x P_s}}$$

which does not lend itself to a closed form expression.

A. Outage approach

The transmitter here performs single-level encoding and the relay just amplifies the its received signal by γ (51). Average achievable rate is

$$R_{1,AF,avg} = \max_x (1 - F_{s_b}(x)) \log(1 + xP_s) \quad (56)$$

where $F_{s_b}(x)$ is specified in (55), and the transmitted rate is given by $R_1 = \log(1 + xP_s)$. This rate can be then optimized numerically.

B. Broadcast approach

In this approach the transmitter performs continuous code layering, matched to the equivalent single fading gain RV. Using the equivalent channel model (52) and using the results of [37], the average received rate is given by

$$R_{bs,AF,avg} = \max_{I(x)} \int_0^{\infty} (1 - F_{s_b}(x)) \frac{-xI'(x)}{1 + xI(x)} \quad (57)$$

where the optimal residual interference distribution $I_{opt}(x)$ is given by [37]

$$I_{opt}(x) = \begin{cases} P & 0 \leq x \leq x_0 \\ \frac{1 - F_{s_b}(x) - x f_{s_b}'(x)}{f_{s_b}(x)x^2} & x_0 \leq x \leq x_1 \\ 0 & x \geq x_1 \end{cases} \quad (58)$$

where x_0 and x_1 are determined from the boundary conditions $I_r(x_0) = P_s$ and $I_r(x_1) = 0$, respectively. The average rate is explicitly given by

$$R_{bs,AF,avg} = \int_{x_0}^{x_1} dx \left[\frac{2(1 - F_{s_b}(x))}{x} + \frac{(1 - F_{s_b}(x))f_{s_b}'(x)}{f_{s_b}(x)} \right]. \quad (59)$$

The CDF $F_{s_b}(x)$ is specified in (55), and thus the corresponding PDF is given by

$$\begin{aligned} f_{s_b}(x) &= \frac{d}{dx} F_{s_b}(x) \\ &= \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \frac{P_r x_r (1 + P_r x_r)}{(P_s x - P_r x_r)^2} e^{-x_r - \frac{x(1 + P_r x_r)}{x_r P_r - x P_s}} \end{aligned} \quad (60)$$

and we need to derive also $f_{s_b}'(x)$,

$$\begin{aligned} f_{s_b}'(x) &= \frac{d}{dx} f_{s_b}(x) \\ &= -P_r \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \frac{-2P_s P_r x_r + 2P_s^2 x + P_r x_r + P_r^2 s_r^2}{(P_s x - P_r x_r)^4} \\ &\quad (1 + P_r x_r) x_r e^{-x_r - \frac{x(1 + P_r x_r)}{x_r P_r - x P_s}} \end{aligned} \quad (61)$$

Finally, $R_{bs,AF,avg}$ (59) can be obtained via a numerical integration.

VII. QUANTIZE-FORWARD RELAY

The quantize-forward (QF) relay performs an optimal compression to its received signal under a minimal mean square error (MSE) criterion for a distortion metric. It does not decode the signal it receives. Rather than that, it quantizes the input signal as if it was a Gaussian source. The destination first reconstructs the quantized relay signal, and then tries to decode the original data. The main advantage of a QF relay compared

to a DF relay is that the relay does not need to know the coding scheme at the source. Another advantage is that it introduces a negligible delay compared to DF protocols.

For optimal quantization w.r.t an MSE distortion the relay estimates the fading gain on the source-relay link, and then compresses the input with a codebook corresponding to the fading gain ν_s . It is assumed that the relay performs perfect estimation of the fading gain ν_s . In this model the destination has to know ν_r , in order to know the codebook that the relay uses. It is assumed that the overhead of sending this information is negligible. The channel model to consider here is the same one specified in (1)-(2). In our case \mathbf{x}_r is the quantized signal transmitted by the relay.

We consider here single level coding and single level quantization. The quantized signal may be presented by

$$y_r = u_q + n_q \quad (62)$$

where n_q is the quantization noise distributed according to $\mathcal{CN}(0, D)$, and u_q is the quantized version of the received relay signal y_r . This representation is also known as the "backward channel" in quantization. Its equivalent "forward channel" may be derived by using Bayes' rule for calculation of $p(u_q|y_q)$, and may be represented by [34, pg. 100]

$$u_q = \beta y_r + n'_q, \quad (63)$$

where $\beta = 1 - \frac{D}{1 + \nu_s P_s}$, and the equivalent quantization noise n'_q is independent of u_q and is $\mathcal{CN}(0, \beta D)$ distributed. This setting of "forward channel" is still optimal for Gaussian sources. Note that for non-Gaussian sources it is no longer optimum under the minimal mean square error (MMSE) criterion [34]. The maximal rate is attainable when the destination has successfully decoded u_q , and is given by $I(x_s; u_q)$. The quantized signal as function of the source data is simply,

$$u_q = \beta h_s x_s + \beta n_s + n'_q, \quad (64)$$

thus $I(x_s; u_q)$ is given by

$$I(x_s; u_q) = \log \left(1 + \frac{\nu_s P_s}{1 + \frac{D}{\beta}} \right) \quad (65)$$

For single level coding a fading gain threshold s_s governs the transmitted rate, such that for $\nu_s < s_s$ an outage event occurs, and for $\nu_s \geq s_s$, the transmitted information may be fully recovered, provided that the quantized signal was successfully decoded. It is shown in Appendix A that $I(x_s; u_q)$ is monotonic w.r.t. ν_s for $\nu_s > s_s$. Finally, the achievable outage rate is

$$R_{1Q} = \log \left(1 + \frac{s_s P_s}{1 + D / (1 - \frac{D}{1 + s_s P_s})} \right). \quad (66)$$

In compression of a remote source, it has been shown in [34, Theorem 3.5.1] that the destination can reproduce the source output with fidelity D as long as the rate distortion does not exceed the link capacity $R(D) \leq C_{rd}$. The rate distortion function for the relay signal y_r is given by

$$R_r(D) = \log \frac{1 + \nu_s P_s}{D}, \quad (67)$$

and the relay-destination link capacity $I(x_r; y_d)$ is

$$I(x_r; y_d) = \log(1 + \nu_r P_r), \quad (68)$$

which is the link capacity with a Gaussian input signal, with power P_r . The attainable average rate is expressed by the following proposition.

Proposition 7.1: In the system model described by (1)-(2), with ν_s known to both relay and destination, and ν_r known to destination only, the maximal average attainable rate in an outage-quantize approach is given by

$$R_{QF,1,avg} = \max_{s_s, D} \bar{P}_{out} \cdot \log \left(1 + \frac{s_s P_s}{1 + \frac{D}{\beta_s}} \right) \quad (69)$$

where $\beta_s = 1 - \frac{D}{1+s_s P_s}$, and the complementary outage probability is

$$\bar{P}_{out} = \text{Prob} \left(\log \frac{1+\nu_s P_s}{D} \leq \log(1 + \nu_r P_r) \cap \nu_s \geq s_s \right) \quad (70)$$

For the Gaussian links, with exponential fading gain distributions the complementary outage probability may be explicitly stated,

$$\begin{aligned} \bar{P}_{out} &= \text{Prob} \left(\log \frac{1+\nu_s P_s}{D} \leq \log(1 + \nu_r P_r) \cap \nu_s \geq s_s \right) \\ &= \int_{\max(0, \frac{P_s s_s + 1 - D}{P_r D})}^{\infty} d\nu_r \int_{s_s}^{\frac{D P_r \nu_r + D - 1}{P_s}} d\nu_s e^{-\nu_s - \nu_r} \\ &= e^{-s_s - \max(0, \frac{P_s s_s + 1 - D}{P_r D})} - \frac{P_s}{P_r + D P_r} \\ &\quad e^{\frac{1-D}{P_s} - (1 + \frac{D P_r}{P_s}) \max(0, \frac{P_s s_s + 1 - D}{P_r D})}. \end{aligned} \quad (71)$$

The computation of $R_{QF,1,avg}$ can be directly pursued, while jointly optimizing the selection of the fading threshold s_s and the average distortion D .

VIII. A HYBRID AMPLIFY-QUANTIZE-FORWARD RELAY AND BROADCASTING PROTOCOLS

The amplify-quantize-forward (AQF) relay amplifies the received signal for its desired transmission power P_r and only then quantizes the signal. It allows the relay to use a **single codebook** for compression of its received signal. Another advantage here is that the destination receiver need not know the fading gain on the source-relay link, and it always attempts decoding with the same codebook. It will also be shown later that such an approach is throughput optimal on the relay-destination link, for optimal selection of a fixed average distortion D .

Like in the QF scheme, the destination first tries to decode the quantized relay signal, and if successful tries to decode the original data.

We consider here single level coding and single level quantization. The quantized signal, after suitable amplification and "forward channel" conversion (63), as function of the source data is now given by

$$u_q = \beta \gamma h_s x_s + \beta \gamma n_s + n'_q \quad (72)$$

where n'_q is the equivalent quantization noise distributed according to $\mathcal{CN}(0, \beta D)$, $\beta = 1 - \frac{D}{P_r}$, and $\gamma = \sqrt{\frac{P_r}{P_s \nu_s + 1}}$

with $\nu_s = |h_s|^2$. The maximal rate, governed by $I(x_s; u_q)$, is attainable when the destination has successfully decoded u_q ,

$$I(x_s; u_q) = \log \left(1 + \frac{\nu_s P_s}{1 + \frac{D}{\beta} \frac{1+\nu_s P_s}{P_r}} \right) \quad (73)$$

A. Source: Single Level Coding, Relay: AQF

For single level coding a fading gain threshold s_s governs the transmitted rate, such that for $\nu_s < s_s$ an outage event occurs, and for $\nu_s \geq s_s$, the transmitted information may be fully recovered, provided that the quantized signal was successfully decoded. Hence, the achievable outage rate is

$$R_{1AQ} = \log \left(1 + \frac{s_s P_s}{1 + \frac{D(1+s_s P_s)}{P_r - D}} \right), \quad (74)$$

which is justified by the monotonicity of $I(x_s; u_q)$ as function of ν_s . The rate distortion function for the amplified relay signal is given by

$$R_r(D) = \log \frac{P_r}{D}, \quad (75)$$

and the relay-destination link capacity $I(x_r; y_d)$ remains as specified in (68). The attainable average rate is expressed by the following proposition.

Proposition 8.1: In the system model described by (1)-(2), with ν_s known to relay only, and ν_r known to destination only, the maximal average attainable rate in an outage amplify-quantize-forward relay is given by

$$R_{AQF,1,avg} = \max_{s_s, D} \bar{P}_{out} \cdot \log \left(1 + \frac{s_s P_s}{1 + \frac{D(1+s_s P_s)}{P_r - D}} \right) \quad (76)$$

where the complementary outage probability is

$$\bar{P}_{out} = \text{Prob} \left(\log \frac{P_r}{D} \leq \log(1 + \nu_r P_r) \cap \nu_s \geq s_s \right) \quad (77)$$

For the Gaussian links, with exponential fading gain distributions the complementary outage probability may be explicitly stated,

$$\begin{aligned} \bar{P}_{out} &= \text{Prob} \left(\log \frac{P_r}{D} \leq \log(1 + \nu_r P_r) \cap \nu_s \geq s_s \right) \\ &= \int_{\frac{1}{D} - \frac{1}{P_r}}^{\infty} d\nu_r \int_{s_s}^{\nu_r} d\nu_s e^{-\nu_s - \nu_r} \\ &= e^{-s_s} e^{-\left(\frac{1}{D} - \frac{1}{P_r}\right)}. \end{aligned} \quad (78)$$

The computation of $R_{AQF,1,avg}$ can be directly pursued, while jointly optimizing the selection of the fading threshold s_s and the average distortion D .

AQF scheme has lower coding complexity than that of the QF scheme, and for single level coding also higher capacity (as demonstrated in section IX). In order to overcome the main weakness of the QF, which is the fixed average distortion parameter, let the average distortion be chosen as function of the fading gain parameter ν_s . It may be assumed that $D(\nu_s)$ is monotonically decreasing. With no further constraints finding the optimal function $D(\nu_s)$ for QF is a difficult problem. However, when searching for $D(\nu_s)$ such that maximal average throughput is obtained on the relay destination link, we

find that $D(\nu_s)$ is proportional to $1 + \nu_s P_s$, which means that the compression rate for every ν_s is maintained constant. The average throughput on the relay-destination link is given by

$$\begin{aligned} R_{av, RD} &= \int_0^\infty d\nu_s \int_0^\infty d\nu_r e^{-\nu_s - \nu_r} \log \frac{1 + \nu_s P_s}{D(\nu_s)} \\ &\quad \cdot \mathbf{1} \left(\log \frac{1 + \nu_s P_s}{D(\nu_s)} \leq \log(1 + \nu_r P_r) \right) \\ &= \int_0^\infty d\nu_s \int_{\frac{1 + \nu_s P_s}{D(\nu_s) P_r} - 1/P_r}^\infty d\nu_r e^{-\nu_s - \nu_r} \log \frac{1 + \nu_s P_s}{D(\nu_s)} \end{aligned} \quad (79)$$

where $\mathbf{1}(x)$ is the indicator function. This is formalized and proved in the next proposition.

Proposition 8.2: In the system model described by (1)-(2), with ν_s known to relay and destination, and ν_r known to destination only, the average distortion $D(\nu_s)$, which **maximizes the throughput** on the relay destination link is

$$D_{min, RD}(\nu_s) = \arg \min_{D(\nu_s)} \int_0^\infty d\nu_s \int_{\frac{1 + \nu_s P_s}{D(\nu_s) P_r} - 1/P_r}^\infty d\nu_r e^{-\nu_s - \nu_r} \log \frac{1 + \nu_s P_s}{D(\nu_s)} \quad (80)$$

where

$$D_{min, RD}(\nu_s) = \alpha(1 + \nu_s P_s) \quad (81)$$

and $\alpha = \frac{1}{P_r} W_L(P_r)$, which does not depend on ν_s .

Proof: See Appendix B.

B. Source: Continuum Layering (Broadcasting), Relay: AQF

Let the source encoder perform now continuum layering, and the relay, as before, amplifies its input signal, quantizes it with average distortion D , optimally in means on minimum MSE. The destination tries to decode first the quantized signal u_q . Upon successful decoding it decodes the multi-level code up to the highest layer possible, depending on the fading gain on the source relay link.

We consider here single level quantization. In broadcasting it may be assumed that part of the original signal cannot be decoded, therefore it is modelled as additive Gaussian noise. The quantized signal, after suitable amplification and "forward channel" conversion (63), as function of the source data is given by

$$u_q = \beta \gamma h_s x_{s,s} + \beta \gamma h_s x_{s,I} + \beta \gamma n_s + n'_q \quad (82)$$

where n'_q is the equivalent quantization noise distributed according to $\mathcal{CN}(0, \beta D)$, $\beta = 1 - \frac{D}{P_r}$, $\gamma = \sqrt{\frac{P_r}{P_s \nu_s + 1}}$ with $\nu_s = |h_s|^2$, and $x_{s,I}$ represents the residual interference in the decoded signal. Consider a power distribution $\rho(\nu_s)$ which is the source power distribution as function of the fading gain. Then the incremental rate associated with a fading ν_s is

$$dR(\nu_s) = \frac{\gamma^2 \nu_s \rho(\nu_s) \beta^2 d\nu_s}{\gamma^2 + \beta D + \gamma^2 \nu_s I(\nu_s) \beta^2} \quad (83)$$

which simplifies after substituting γ and some algebra,

$$dR(\nu_s) = \frac{\nu_s \rho(\nu_s) d\nu_s}{1 + D\beta + \nu_s(I(\nu_s) + P_s D\beta)} \quad (84)$$

where the $D\beta \triangleq \frac{D/P_r}{1 - D/P_r}$. Thus the average rate attainable, when u_q is successfully decoded, is

$$\begin{aligned} R_{avg} &= \int_0^\infty d\nu_s f(\nu_s) \int_0^{\nu_s} dR(u) \\ &= \int_0^\infty (1 - F(\nu_s)) \frac{\nu_s \rho(\nu_s) d\nu_s}{1 + D\beta + \nu_s(I(\nu_s) + P_s D\beta)} \\ &= \int_0^\infty (1 - F(\nu_s)) \frac{\nu_s \rho_N(\nu_s) d\nu_s}{1 + \nu_s I_N(\nu_s)} \end{aligned} \quad (85)$$

where the first equality is obtained by solving the integral in parts and the next equality subsumes the following definitions of the normalized power distribution, and residual interference

$$\begin{aligned} \rho_N(\nu_s) &\triangleq \frac{\rho(\nu_s)}{1 + D\beta} \\ I_N(\nu_s) &\triangleq \frac{I(\nu_s) + D\beta P_s}{1 + D\beta} \end{aligned}$$

as may also be noticed $\rho_N(\nu_s) = -I'_N(\nu_s)$. For a given average distortion D , $D\beta$ is also explicitly determined, and the maximal average rate R_{avg} is achieved for

$$\begin{aligned} \rho_N(\nu_s) &= \frac{2}{\nu_s^3} - \frac{1}{\nu_s^2} \\ I_N(\nu_s) &= \frac{1}{\nu_s^2} - \frac{1}{\nu_s} \end{aligned} \quad (86)$$

on the range of $\nu_s \in [\nu_0, \nu_1]$, where the boundary conditions are given as $I_N(\nu_0) = P_s$ and $I_N(\nu_1) = 0$. Thus the range of the optimal solution is

$$\begin{aligned} \nu_0 &= \frac{2}{1 + \sqrt{1 + 4P_s}} \\ \nu_1 &= \frac{1}{1 + \sqrt{1 + \frac{P_s D\beta}{1 + D\beta}}} \end{aligned} \quad (87)$$

This rate is attainable only when the compressed signal may be decoded at the destination, otherwise an outage event occurs, and nothing can be restored from the original signal. Evidently, the event of outage depends only on the relay-destination link. Hence the average achievable rate for the broadcast-amplify-quantize (BAQ) approach is formalized in the next proposition.

Proposition 8.3: In the system model described by (1)-(2), with ν_s known to relay and destination, and ν_r known to destination only, the maximal average attainable rate in a broadcast-amplify-quantize (BAQ) scheme is specified by

$$R_{BAQ, avg} = \max_D \bar{P}_{out} \cdot \int_0^\infty (1 - F(\nu_s)) \frac{\nu_s \rho_N(\nu_s) d\nu_s}{1 + \nu_s I_N(\nu_s)} \quad (88)$$

where the complementary outage probability is

$$\bar{P}_{out} = Prob \left(\log \frac{P_r}{D} \leq \log(1 + \nu_r P_r) \right) \quad (89)$$

The complementary outage probability for a Rayleigh fading channel reduces (89) into $\bar{P}_{out} = e^{-\frac{1}{D} + \frac{1}{P_r}}$. The computation of $R_{BAQ, avg}$ can be directly pursued, while optimizing the selection of the average distortion D , and directly computing the average rate for every D . Numerical results are demonstrated in section IX.

C. Successive refinement quantization at the relay

Let the relay perform successive refinement coding on the received signal and transmit the successively compressed signal over the relay destination link. Every source coding layer is associated with a channel code layer, and has its power allocation. The destination attempts decoding as many layer as possible depending on the relay-destination channel fading gain. The average distortion in the reconstructed signal depends then on the number of source code layers that could be recovered. In a final step of decoding, the destination attempts decoding the original signal from its distorted version. Clearly, the lower the distortion in the original signal, the higher is the associated fading gain threshold for decoding the (single level coded) source signal.

1) *Source: Outage, Relay: Two refinement layers:* For single level coding at the transmitter at rate $R_{s,1}$, there are two fading gain thresholds, which are also associated with the distortion in the source signal. Let $\{D_i\}_{i=1,2}$ be the possible distortion values of the successively refinable signal that was sent from the relay, such that $D_1 \geq D_2$. Then the mutual information $I(u_q; x_s)$ of the distorted signal and the source signal, for some fading gain realization ν_s , and distortion D_i , is

$$I(u_q; x_s) = \log \left(1 + \frac{\nu_s P_s}{1 + \frac{D_i(1 + \nu_s P_s)}{P_r - D_i}} \right). \quad (90)$$

As may be noticed from (90), every D_i dictates a different fading gain threshold for successful decoding of a transmission in rate $R_{s,1}$. Thus for $I(u_q; x_s) = R_{s,1}$, the fading gain thresholds are defined by

$$s_{s,i} = \frac{(e^{R_{s,1}} - 1)(1 + D_{\beta,i})}{P_s(1 + D_{\beta,i} - e^{R_{s,1}} \cdot D_{\beta,i})} \quad (91)$$

where the normalized distortion $D_{\beta,i}$ is defined by

$$D_{\beta,i} \triangleq \frac{D_i/P_r}{1 - D_i/P_r} = \frac{D_i}{P_r - D_i}. \quad (92)$$

On the relay site the input signal is amplified first to power P_r , and then encoded in two layers which are associated with two distortions

$$\begin{aligned} R_{rd,1} &= \log \frac{P_r}{D_1} = \log \left(1 + \frac{\alpha P_r \mu_1}{1 + (1 - \alpha) P_r \mu_1} \right) \\ R_{rd,2} &= \log \frac{P_r}{D_2} - \log \frac{P_r}{D_1} = \log (1 + (1 - \alpha) P_r \mu_2) \end{aligned} \quad (93)$$

where $\mu_1 \leq \mu_2$ are fading gain threshold for decoding the two level compressed signal, and $0 \leq \alpha \leq 1$ indicates the power split between the first layer and the refinement layer. Thus a rate $\log \frac{P_r}{D_2} = R_{rd,1} + R_{rd,2}$ is achieved for fading levels $\nu_r \geq \mu_2$. The power fading thresholds μ_1 , μ_2 are governed by the selection of the distortion levels used during the compression stage, therefore it can be shown by simple algebraic manipulations on (93) that

$$\begin{aligned} \mu_1 &= \frac{1 - D_{n,1}}{P_r(D_{n,1} + \alpha - 1)} \\ \mu_2 &= \frac{D_{n,1} - D_{n,2}}{P_r(1 - \alpha)D_{n,2}}, \end{aligned} \quad (94)$$

where $D_{n,i} \triangleq \frac{D_i}{P_r}$. The rate $R_{s,1}$ may be achieved when both layers are recovered ($\mu_2 \leq \nu_r$) and there is no source outage

($\nu_s > s_{s,2}$), or when only the first layer was recovered ($\mu_1 \leq \nu_r < \mu_2$). This is formalized in the following proposition

Proposition 8.4: In the system model described by (1)-(2), with ν_s known to relay and destination, and ν_r known to destination only, the maximal average attainable rate in an outage amplify-2-level-quantize forward relay is given by

$$R_{A2LQF,1,avg} = \max_{D_{n,1}, D_{n,2}, \alpha, R_{s,1}} \bar{P}_{out} \cdot R_{s,1} \quad (95)$$

where the complementary outage probability is

$$\bar{P}_{out} = Prob \left((s_{s,1} \leq \nu_s \cap \mu_1 \leq \nu_r < \mu_2) \cup (s_{s,2} \leq \nu_s \cap \mu_2 \leq \nu_r) \right). \quad (96)$$

The complementary outage probability (96) is given by

$$\bar{P}_{out} = e^{-s_{s,1}}(e^{-\mu_1} - e^{-\mu_2}) + e^{-s_{s,2}}e^{-\mu_2}. \quad (97)$$

2) *Source: Outage, Relay: A Continuum of refinement layers:* We address now continuum layering (broadcasting), at the relay. When the successive-refinement steps are associated with channel code broadcasting, the average distortion as function of the fading gain is

$$D(\nu_r) = \exp(-R(\nu_r)) = \exp \left(- \int_0^{\nu_r} \frac{u \rho_r(u) du}{1 + u I_r(u)} \right) \quad (98)$$

where $\rho_r(u)$ and $I_r(u)$ are the power distribution and the residual interference of the broadcasting associated with the successive refinement at the relay.

Let the source transmitter perform its broadcasting with respect to the distortion of the quantized signal it has decoded. This naturally means that the fading gain on the source-relay link has to be above some threshold $\nu_s \geq s_s$. Otherwise, an outage event is declared. The quantized signal with distortion $D(\nu_r)$, after suitable amplification, "forward channel" conversion (63), where n'_q is the equivalent quantization noise distributed according to $\mathcal{CN}(0, \beta D(\nu_r))$, $\beta = 1 - \frac{D(\nu_r)}{P_r}$, $\gamma = \sqrt{\frac{P_r}{P_s \nu_s + 1}}$, and $x_{s,I}$ represents the residual interference in the decoded signal. Consider a power distribution $\rho_s(\nu_r)$ which is the source power distribution as function of the fading gain. Then the incremental rate associated with a fading ν_r is

$$dR(\nu_r) = \frac{\gamma^2 s_s \rho_s(\nu_r) \beta^2(\nu_r) d\nu_s}{\gamma^2 + \beta(\nu_r) D(\nu_r) + \gamma^2 s_s I(\nu_r) \beta^2} \quad (99)$$

and the average achievable rate is formalized by the following proposition.

Proposition 8.5: In the system model described by (1)-(2), with ν_s known to relay and destination, and ν_r known to destination only, the maximal average attainable rate in a broadcast-amplify-successive-quantize (BASQ) scheme is specified by

$$R_{BASQ,avg} = \max_{s_s, \rho_s(\nu_r), \rho_r(\nu_r)} \bar{P}_{out} \cdot \int_0^{\infty} (1 - F(\nu_r)) \frac{\gamma^2 s_s \rho_s(\nu_r) \beta(\nu_r)^2 d\nu_s}{\gamma^2 + \beta(\nu_r) D(\nu_r) + \gamma^2 s_s I(\nu_r) \beta^2} \quad (100)$$

where the complementary outage probability is simply

$$\bar{P}_{out} = Prob(\nu_r \geq s_s) \quad (101)$$

In the maximization problem of (100) we have a functional with no localization property [51], therefore only suboptimal solutions are to be sought.

IX. NUMERICAL RESULTS

In this section numerical results of maximal attainable average rates are introduced for the various relaying protocols studied. The numerical results correspond to Rayleigh fading channels on both source-relay and relay-destination links. Results for the DF relaying schemes consist of the following:

- 1) *Ergodic Capacity*. This is the cut set bound on the achievable rate (7).
- 2) *Broadcasting cut-set bound*, also denoted by C_{cutset} . This is the cut set bound on the broadcasting achievable rate (9).
- 3) *Full-CSI Bound*, also denoted by R_{FCSI} (5).
- 4) *Outage Capacity*, also denoted by DF_1 . This is average attainable rate with single level coding at source and relay sides (12).
- 5) *Outage At Source, 2-Level Coding At Relay* is the attainable rate for single level coding at the source and two level coding at the relay (16).
- 6) *Outage At Relay, 2-Level Coding At Source* is the attainable rate for two-level coding at the source and single-level coding at the relay (19).
- 7) *Naive Broadcasting*, also denoted by $DF_{BS-BS,Naive}$, is the attainable rate for continuous broadcasting at the source and continuous broadcasting at the relay according to the naive broadcasting strategy (46).
- 8) *Outage At Source, Broadcasting At Relay*, also denoted by DF_{1-BS} , is the attainable rate for single level coding at the source and continuous broadcasting at the relay (37).

For the other relaying protocols the following results are obtained:

- 1) *Amplify-Forward, Outage*, also denoted by AF_1 . This is the attainable rate for single level coding for the AF relay (56).
- 2) *Amplify-Forward, Broadcast*, also denoted by AF_{BS} . This is the attainable rate for continuous broadcasting over the AF relay (59).
- 3) *Quantize-Forward*, also denoted by QF_1 . This is the attainable rate for single-level coding at the source and single level quantization, with multiple codebooks, at the QF relay (69).
- 4) *Amplify-Quantize*, also denoted by AQF_1 . This is the attainable rate for single-level coding at the source and amplify joined with a single level quantization (with a single codebook) at the AQF relay (76).
- 5) *Broadcast-Amplify-Quantize*, also denoted by AQF_{BS} . This is the attainable rate for continuous broadcasting at the source and amplify joined with a single level quantization at the AQF relay (88).
- 6) *Amplify-2Level Quantize* is the attainable rate for single level coding at the source and amplify joined with two-level successive refinement quantization and matched coding at the AQF relay (88).

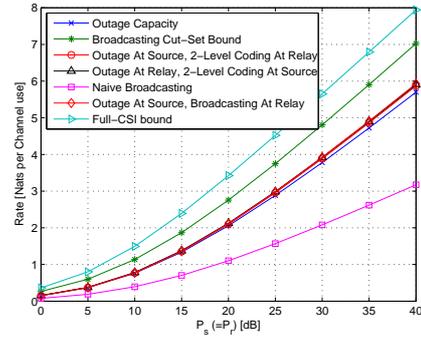


Fig. 3. DF Achievable average rates, for $P_r = P_s$, and for DF protocols and broadcasting strategies.

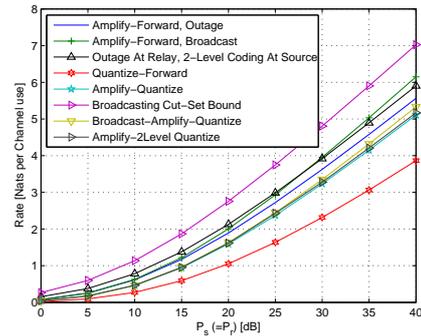


Fig. 4. Achievable average rates, for $P_r = P_s$, and for various relaying protocols and broadcasting strategies.

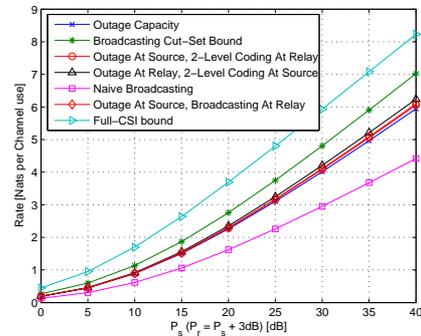


Fig. 5. DF Achievable average rates, for $P_r = P_s + 3dB$, and for DF protocols and broadcasting strategies.

Figures 3, 5, 7 present a comparison of the DF protocols studied for three relay power to source power ratios (P_r/P_s). As may be noticed, the highest throughput gains may be attained when the source performs multi-level coding and the relay performs single-level coding. The naive broadcasting approach is efficient only when $P_r \gg P_s$, and the outage region is relatively small on the relay-destination link. Otherwise, this strategy is highly inefficient and loses even compared to the single level coding approach, see Figure 3.

Figures 4, 6, 8 present a comparison of all other relaying protocols studied (AF, QF, AQF) for the three same SNR ratios between the source-relay link and the relay-destination

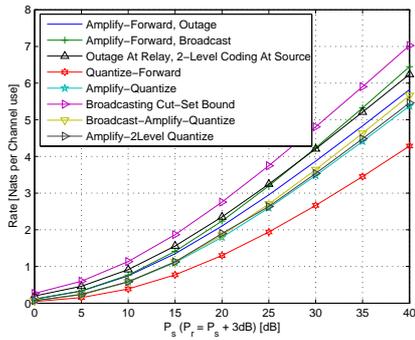


Fig. 6. Achievable average rates, for $P_r = P_s + 3dB$, and for various relaying protocols and broadcasting strategies.

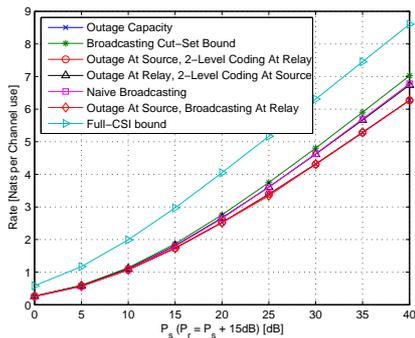


Fig. 7. DF Achievable average rates, for $P_r = P_s + 15dB$, and for DF protocols and broadcasting strategies.

link. As may be noticed the broadcasting for AF relay has the highest throughput gains, for high SNRs (P_s), and any P_r/P_s ratios. Numerical results for the QF and the AQF relays show that AQF scheme outperforms the QF schemes. In the cases AQF and QF settings allow only single level quantization and coding, the AQF scheme has the advantage of allowing a single compression codebook at the relay and destination decoder. The QF scheme does not show improved performance over the AQF scheme probably due to the fact that in all compressions the distortion D is fixed, that is for every fading gain realization on the source-relay link the codebook changes but the distortion is maintained, which

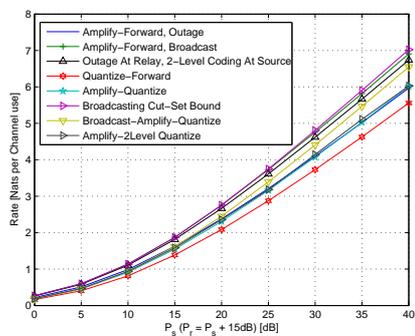


Fig. 8. Achievable average rates, for $P_r = P_s + 15dB$, and for various relaying protocols and broadcasting strategies.

means that in low fading values a large relative distortion is noticeable, which does not allow decoding of the original signal, in spite of successful decoding of its compressed version. This also coincides with the throughput optimality of AQF for the relay-destination link, shown in proposition 8.2. As may be noticed in the AQF scheme with 2 levels of refinement at the relay, there is only a small gain in the overall expected throughput. This questions the possible benefits of higher levels of successive refinement at the relay, when source performs only single level coding.

In Figure 9, the average attainable rates of the various relaying protocols are compared as function of P_r/P_s ratio, for a few SNRs, and for single level coding approaches. Then Figure 10 shows the average attainable rates of the various relaying protocols, compared as function of P_r/P_s ratio, for a few SNRs, while employing different broadcasting strategies. It is interesting to notice that the broadcasting over the AF relay closely approximates the broadcasting cut-set bound for high P_r/P_s ratios.

X. CONCLUSION

We have considered various relaying protocols and broadcast strategies for the two-hop relay network, where CSI is available for receivers only. Various relaying protocols and single-user broadcasting strategies are studied. For a DF relay several broadcasting strategies are considered, and numerical results show that the simple multi-level coding at the source with single-level coding at the relay is better or as good as double broadcasting (in the naive broadcasting strategy). Then, for the AF relay, an optimal broadcasting throughput is analytically derived. Numerical results show that the AF broadcast approach closely approximates the broadcasting cut-set bound for high P_r/P_s ratios (above 10 dB). Furthermore, for high SNRs (P_s), AF achieves higher throughput gains than the DF relaying protocols that were numerically tractable. However, it should be noted that optimal broadcasting DF strategy, derived in section V-E, which was analytically and numerically intractable, may outperform the AF broadcast strategy.

The QF relay considered, when coupled with a single-level code at the source, uses codebooks matched to the received signal power and performs optimal MMSE quantization. This scheme is simplified by a hybrid AQF relay, which simply scales the input, and performs optimal MMSE quantization with a **single** codebook. We have shown that the latter is optimal by means of throughput on the relay-destination link, while maintaining lower complexity than the QF relay. A further extension of the AQF allows the relay to perform successive refinement quantization coupled with a matched multi-level code. Numerical results here show that the QF and AQF schemes are inferior to the AF and DF protocols.

Further related research may include extending the results to a general multi-hop relay system with n relays. For the two-hop relay setting, the broadcast approach may be extended to the case of a multi-antenna receiver at the destination, or to the case of multi-antenna at the source, which can be derived along the lines of SIMO and MISO broadcasting [45], respectively.

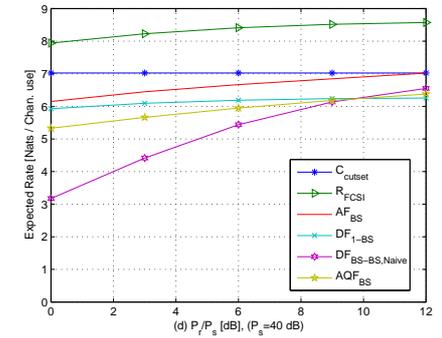
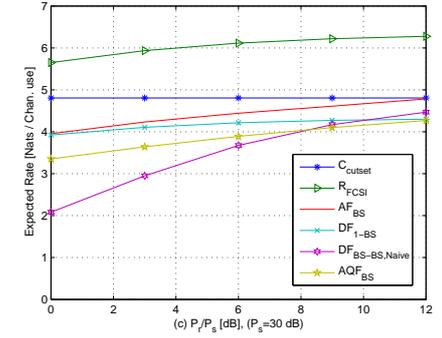
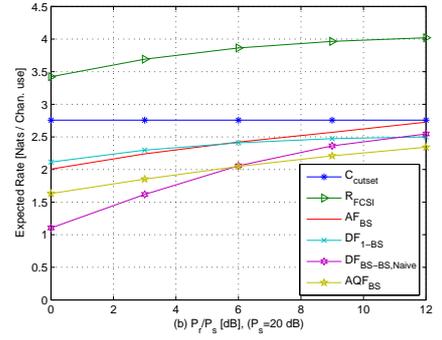
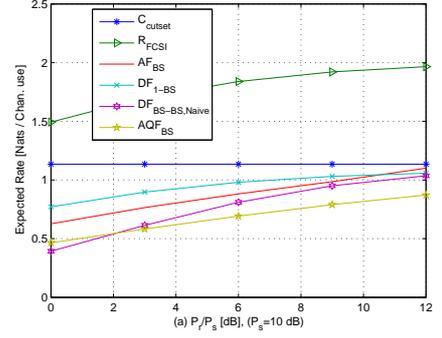
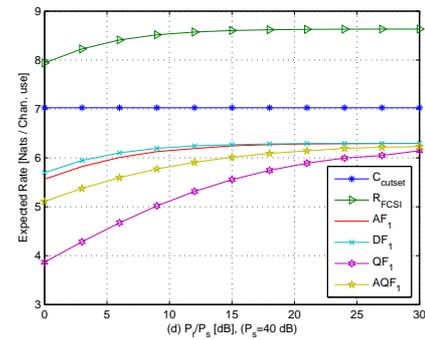
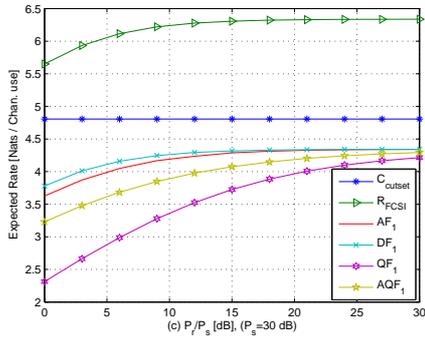
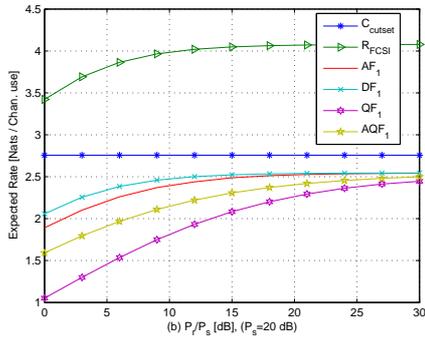
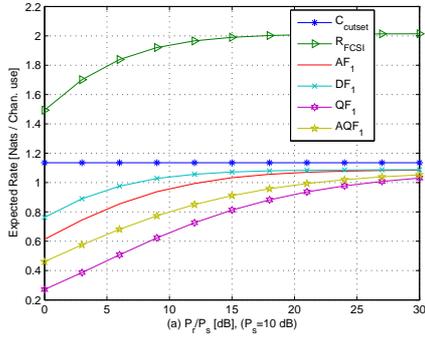


Fig. 9. Attainable average rates vs. P_r/P_s , for single level coding relays.

Fig. 10. Attainable average rates vs. P_r/P_s , for broadcasting strategies and different relaying protocols.

APPENDIX A

MONOTONICITY OF THE MUTUAL INFORMATION $I(u_q; x_s)$

We show here that the mutual information $I(u_q; x_s)$ in (65) is monotonic w.r.t. ν_s , for $\nu_s > s_s$. For a selection of source transmission rate R there is a corresponding fading gain threshold s_s . For any rate threshold R , the transmitter information may be decoded only if

$$I(x_s; u_q) \geq R. \quad (\text{A.1})$$

The above condition may be explicitly stated in terms of ν_s ,

$$\log \left(1 + \frac{\nu_s P_s}{1 + \frac{D}{1 + \nu_s P_s}} \right) \geq R. \quad (\text{A.2})$$

By the substitution $x \triangleq 1 + \nu_s P_s$, and simplifying (A.2),

$$\frac{(x - D)(x - 1)}{x(1 + D) - D} \geq e^R - 1, \quad (\text{A.3})$$

which can be written as a polynomial inequality as function of x ,

$$x^2 - e^R(D + 1)x + D(2 - e^R) \geq 0, \quad (\text{A.4})$$

where we have used the fact that $x(1 + D) - D \geq 0$ in the region of interest, which is $x \geq 1$. The global minimum of the polynomial in (A.4) is located in $x_{min} = e^R(D + 1)/2$, it always has two real solutions (x_0, x_1) for $x^2 - e^R(D + 1)x + D(2 - e^R) = 0$, where $x_0 \leq x_1$, it may be simply shown that $x_1 \geq 1$, which means that for any $\nu_s > s_s$, since $s_s \geq 0$. ■

APPENDIX B

OPTIMALITY OF AQF IN MEANS OF RELAY-DESTINATION THROUGHPUT

The average rate on the relay destination link (79) may be further simplified by solving the internal integral w.r.t $d\nu_r$,

$$\begin{aligned} R_{av, RD} &= \int_0^\infty d\nu_s \int_{\frac{1+\nu_s P_s}{D(\nu_s)P_r} - 1/P_r}^\infty d\nu_r e^{-\nu_s - \nu_r} \log \frac{1+\nu_s P_s}{D(\nu_s)} \\ &= \int_0^\infty d\nu_s \exp \left(-\frac{1+\nu_s P_s}{D(\nu_s)P_r} + 1/P_r - \nu_s \right) \\ &\quad \log \frac{1+\nu_s P_s}{D(\nu_s)} \triangleq \int_0^\infty d\nu_s J(\nu_s, D(\nu_s)) \end{aligned} \quad (\text{B.1})$$

The functional $J(\nu_s, D(\nu_s))$ in (B.1) can be maximized with respect to $D(\nu_s)$ by deriving the functional by $D(\nu_s)$ and finding its extremum by $\frac{\partial J(\nu_s, D(\nu_s))}{\partial D(\nu_s)} = 0$. This is explicitly given by

$$\begin{aligned} \frac{\partial J(\nu_s, D(\nu_s))}{\partial D(\nu_s)} &= \exp \left(-\frac{1+\nu_s P_s}{D(\nu_s)P_r} + 1/P_r - \nu_s \right) \\ &\quad \left(\frac{-1}{D(\nu_s)} + \frac{1+\nu_s P_s}{P_r D(\nu_s)^2} \log \frac{1+\nu_s P_s}{D(\nu_s)} \right) = 0 \end{aligned} \quad (\text{B.2})$$

which after comparison to 0, becomes

$$\frac{P_r D(\nu_s)}{1 + \nu_s P_s} + \log \frac{D(\nu_s)}{1 + \nu_s P_s} = 0 \quad (\text{B.3})$$

and by defining $x \triangleq \frac{D(\nu_s)}{1 + \nu_s P_s}$ (B.3) is simply given by $P_r x + \log(x) = 0$, and the corresponding solution is

$$x = \frac{1}{P_r} W_L(P_r) \quad (\text{B.4})$$

where $W_L(x)$ is the Lambert W-function, also called the omega function, is the inverse of the function $f(W) = We^W$. From here it is clear that

$$D_{min, RD}(\nu_s) = \frac{1}{P_r} W_L(P_r)(1 + \nu_s P_s) = \alpha(1 + \nu_s P_s) \quad (\text{B.5})$$

and $\alpha = \frac{1}{P_r} W_L(P_r)$. ■

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