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Distributed MIMO Systems for Nomadic Applications Over a Symmetric Interference Channel

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Abstract

A single source communicates with a single destination via a remote wireless multiantenna (MIMO) transceiver. The source has access to each of the transmit antennas through a finite-capacity link, and likewise the destination is connected to the receiving antennas via capacity-constrained channels. Targeting a nomadic communication scenario, in which the remote MIMO transceiver is designed to serve different standards or services, it is assumed that transmitters and receivers are oblivious to the encoding function shared by source and destination. Assuming a Gaussian symmetric interference network as the channel model (as for regularly placed transmitters and receivers), achievable rates are investigated and compared with an upper bound. Closed-form expressions are derived for large numbers of antennas (and in some cases large signal-to-noise ratios), and asymptotics of the achievable rates are studied with respect to either link capacities or signal-to-noise ratio. Overall, the analysis points to effective transmission/ reception strategies for the distributed MIMO channel at hand, which are optimal under specified conditions. In particular, it is concluded that in certain asymptotic and non-asymptotic regimes there is no loss of optimality in designing the system for nomadic applications (i.e., assuming oblivious transmitters and receivers). Numerical results validate the analysis.

I. INTRODUCTION

Multi-antenna (MIMO) technology has by now moved from the stage of theoretical investigations to the practice of implementation in a number of single and multi-user wireless communication systems (see, e.g., [1]). Moreover, in scenarios where terminals with multiple antennas are not available, *cooperative transmission* techniques have been shown to retain some of the benefits of MIMO systems, specifically in terms of micro-diversity and, possibly, array (power) gains [2]. More recently, the idea of large-scale MIMO systems implemented via *distributed antennas*, connected via a wireless or wired backbone, has been advocated as a viable solution to provide multiplexing, array and macro-diversity gains in infrastructure or mesh networks (see, e.g., [3] [4] and references therein).

As proved by information theoretic analyses of both cooperative transmission techniques [2] and distributed antenna systems [3], the main challenge in realizing the full gains of MIMO systems with non-colocated antennas is the efficient use of the channel resources needed to coordinate the participating antennas at the transmit and/ or receive sides into effective multi-antenna arrays (see also [5]). Such channel resources can be either *in-band*, that is, in the same time and frequency band of the main end-to-end transmission [2] [5] [6], or *out-of-band*, i.e., over orthogonal channels, possibly realized via wired connections or different wireless radio interfaces [7]-[10] (see also [11] for a brief review of recent results). Moreover, the feasibility of different transmission schemes depends on the amount of information that the available transceivers (e.g., relays in cooperative systems) have regarding the encoding functions shared by the sources and destinations of the transmitted data. For example, decode-and-forward-type (regenerative) schemes require full knowledge of the codebooks used to encode the received data, while non-regenerative schemes such as compress-and-forward or amplify-and-forward strategies do not have this requirement.

In this paper, we consider the scenario depicted in Fig. 1, in which a single source S has data to communicate to a remote destination D. Communication takes place via a distributed $M \times M$ MIMO system constructed by connecting the source to the M transmitting antennas through (equal) finite-capacity links and, likewise, the M receiving antennas to the destination. The finite-capacity links are assumed to be out-of-band (i.e., orthogonal to the main MIMO wireless channel) and orthogonal among themselves. This is the case when the source and destination are connected to the transmitters and receivers, respectively, via a wired backbone or via orthogonal wireless interfaces. Moreover, the application of interest is of a *nomadic* nature, i.e., we require the infrastructure of transmitting and receiving antennas to be able to serve different pairs of sources and destinations, irrespective of the communication standard. This requirement is modelled by assuming that transmitters and receivers are *oblivious* to the encoding function shared by source S and destination D [11]-[15].

Our interest is in obtaining analytical insights into the role of finite capacities C and C' at the transmit and receive sides, respectively, on the performance of the distributed MIMO system (see Fig. 1). To this end, we adopt a simplified channel model for the MIMO channel between transmitters and receivers that corresponds to a Gaussian interference network described by a single parameter α, as shown in Fig. 1. Beside allowing analytical tractability, this channel is a variant of the Wyner model for cellular networks [16] that was first proposed in [17][1] and then employed in a number of subsequent works (see [3] [18] [19] [20]). This channel model is a reasonable first approximation of a scenario in which antennas are geographically distributed so that interference occurs only between successive pairs of transmitters-receivers. Under the stated assumptions, an upper bound is derived and achievable rates are investigated at first for the special cases where finite-capacity links exist only at either the transmit or receive side (with reference to Fig. 1, $C' \rightarrow \infty$ or $C \rightarrow \infty,$ respectively) and then for the most general model in Fig. 1. Closed-form expressions are derived, and asymptotics of the achievable rates are studied for large values of either link-capacities C and C' or signal-to-noise ratio (SNR). Overall, the analysis points to effective transmission/ reception strategies for the distributed MIMO channel at hand, which are proved to be optimal under specified conditions. Interestingly, it is shown that, in certain asymptotic and non-asymptotic regimes, no loss of optimality is incurred in designing the system for nomadic applications (i.e., assuming oblivious transmitters and receivers). Finally, numerical results are presented that validate the analysis.

A. Related work

Referring to [2] and references therein for an introduction to MIMO and cooperative systems, here we point only to works that are closely related to the scope of this paper. Basic 2×2 distributed MIMO systems operating via in-band signalling between transmit antennas, at one end, and receive antennas, at the other, were considered in [21] and [5] for full and half-duplex transceivers, respectively. These works clearly show the relevant performance degradation that can be incurred by distributed systems with respect to real MIMO transceivers due to the need for coordinating transmit antennas on one side and and receiving antennas on the other. For instance, reference [21] proves that such a requirement costs the distributed system the full

[1] It was referred therein as the "soft-handoff" model.

multiplexing gain (of two) of the real MIMO system, which becomes unattainable. Another closely related line of work deals with distributed-antenna transmitters or receivers in a cellular scenario in the presence of non-ideal links connecting the cooperating (transmit or receive) antennas, which can be viewed as base stations. A brief review of this subject is presented in [11]. Among these works, it is relevant to recall the contribution of [12]-[13] and [15]. Therein, a cellular scenario is considered in which the base stations are connected via equal finite-capacity links to a central processor for joint decoding (uplink) [12]-[13] or encoding (downlink) [15]. The channel between base stations and mobile stations is modelled similarly to Fig. 1 according to Wyner-type models, and base stations are assumed to be oblivious to the codebooks shared between the base stations and each transmitter/ receiver. Therefore, while here we consider a point-to-point MIMO system (see Fig. 1), references [12]-[13] and [15] investigate, under similar conditions, the corresponding SIMO[2] multiple access channel (i.e., transmitters encode independent messages) and MISO broadcast channels (i.e., receivers decode independently), respectively. As will be shown in this paper, significantly different conclusions apply to the scenarios of [12]-[13] and [15] and the MIMO system studied here. Finally, reference [14] studies a general MIMO channel model with infinite-capacity links at the transmit side ($C \to \infty$) and finite-capacity links at the receive side. Particularization of the results of [14] to the channel model considered here is not trivial and will be considered in order to obtain useful analytical insight.

II. System model

A source S is connected via finite-capacity (error-free) links of capacity C [bit/ symbol] to M distributed transmitters. Each transmitter has power constraint P. The source aims at communicating with a remote destination D, which is in turn connected to M distributed receivers via links of capacity C'. Targeting nomadic applications, transmitters and receivers are assumed to be oblivious to the encoding function shared by source S and destination D. More specifically,

[2]SIMO stands for Single Input Multiple Output, while MISO for Multiple Input Single Output.

1) Each transmitter is equipped with an independently generated standard complex Gaussian codebook of size 2^{nC} (n is the length of the transmission block) with average power P, which is known to the source S. These M codebooks can be obtained by S via, e.g., a local public database. Through the finite-capacity link, the source selects which codeword in the codebook should be sent by each antenna in a given transmission block. In other words, no processing is carried out at the transmitters, except simple *mapping between the index received by the source and the codebook*[3];
2) Each receiver is unaware of the processing carried out at the source and of the codebooks of the transmitters, and merely performs quantization of the received signals, which are then relayed to the destination;
3) The destination D is assumed to be aware of the quantization scheme used at the receivers. Moreover, we consider both cases in which the destination knows and does not know the codebooks of the transmitters.

Perfect block and symbol synchronization is assumed. The complex Gaussian channel between transmitters and receivers is described by a Gaussian interference network as in Fig. 1, which is further described by an interference parameter $\alpha \in [0, 1]$. This channel corresponds to the circulant version of the Wyner model [16] for cellular networks considered in [17] and [18]. Accordingly, the received signal at any given time instant by the mth receiver is given by

$$Y_m = X_m + \alpha X_{[m-1]} + Z_m, \tag{1}$$

where X_m is the complex symbol transmitted by the mth transmitter, $[m-1]$ represents the modulo-M operation (i.e., $[m-1] = m-1$ for $m = 2, ..., M$ and $[m-1] = M$ for $m = 1$), and Z_m denotes the zero-mean unit-power complex Gaussian noise ($Z_m \sim \mathcal{CN}(0, 1)$). The per-transmitter input power constraint requires $E[|X_m|^2] = P$. Parameters α, C and C' are assumed to be known by all the involved nodes. On this note, it is important to remark that capacity gain

[3]In Sec. VIII, we also briefly consider the case in which the transmitters do not share any codebook with the source, which merely selects in a memoryless fashion points from a finite constellation in each channel use.

arising from the knowledge of the channel (here α) in a point-to-point scenario (here, when $C, C' \to \infty$) is known to be bounded by a constant, which amounts to 0.53 bit/ symbol in our case, and to vanish for large SNR [22] [23].

In order to obtain compact results, we will focus on the case where we have a large number of antennas $M \to \infty$. Results with finite M can be easily inferred by using the circular structure of the channel model at hand and the corresponding circularity of the channel matrix, based on the approach of [17], [18] (see also [3] for a review and a discussion on the validity of this asymptotic analysis). Finally, we normalize the achievable rate from S and D to the number of transmit and receive antennas M and definite it as R [bit/ (symbol $\times$ antenna)].

Notation: We use the standard notation (see, e.g., [24]) whereby a capital letter X denotes a single random variable and X^n a sequence of n independent and identically distributed (i.i.d.) variables distributed as X.

III. Some definitions and reference results

In this section, we review some basic definitions and establish reference results. At first, we recall a result that follows from [16]: For $M \to \infty$ and perfectly cooperating receivers ($C' \to \infty$), the signal (1) received by the destination of the network in Fig. 1 can be interpreted in the spatial domain as an inter-symbol-interference channel with impulse response[4] $h_m = \delta_m + \alpha\delta_{m-1}$ and frequency response (see also [3]):

$$|H(f)|^2 = 1 + \alpha^2 + 2\alpha\cos(2\pi f). \tag{2}$$

We then present two basic definitions and related results.

Definition 1: We define the waterfilling power spectral density with respect to the sum-power

[4]The Kronecker delta function is defined as: $\delta_m = 1$ for $m = 0$ and $\delta_m = 0$ otherwise.

constraint P and the SNR power spectral density $\rho(f)$ as:

$$S_{WF}(f,P,\rho(f)) = \left(\mu - \frac{1}{\rho(f)}\right)^{+} \tag{3a}$$

$$\text{s.t.} \quad \int_0^1 S_{WF}(f,P,\rho(f))df = P, \tag{3b}$$

and the corresponding rate as

$$\begin{aligned} R_{WF}(P,\rho(f)) &= \int_0^1 \log_2\left(1 + S_{WF}(f,P,\rho(f))\cdot\rho(f)\right)df = \\ &= \int_0^1 \left(\log_2\left(\mu\rho(f)\right)\right)^{+} df. \end{aligned} \tag{4}$$

For short, we also define $R_{WF}(P,|H(f)|^2) = R_{WF}(P)$.

We have the following two results.

Lemma 1:

$$R_{WF}\left(P,\frac{|H(f)|^2}{N}\right) = R_{WF}\left(\frac{P}{N}\right). \tag{5}$$

Proof: This follows immediately from the observation that the capacity depends only on the signal-to-noise ratio P/N.

Lemma 2: If $\rho(f) = |H(f)|^2/N$ and

$$\frac{P}{N} \geq \frac{2\alpha}{(1-\alpha)(1-\alpha^2)}, \tag{6}$$

then we have the closed-form expression:

$$R_{WF}\left(P,\frac{|H(f)|^2}{N}\right) = \log_2\left(\frac{P}{N} + \frac{1}{1-\alpha^2}\right). \tag{7}$$

Proof: See Appendix A.

Having set the basic definitions above, we can now present an upper bound on the achievable rate between the source and destination for the network in Fig. 1. The bound also holds for the case where the transmitters and receivers are informed about the codebooks used by source and destination.

Proposition 1: The achievable rate R is upper bounded by

$$R_{UB}(P,C,C') = \min\{C,\ C',\ R_{WF}(P)\}, \tag{8}$$

which for $P \geq 2\alpha/((1-\alpha)(1-\alpha^2))$ reduces to

$$R_{UB}(P,C,C') = \min\left\{C,\ C',\ \log_2\left(P + \frac{1}{1-\alpha^2}\right)\right\}. \tag{9}$$

Proof: This follows directly from the cut-set bound (see, e.g., [24]) and Lemma 2.

It should be noted that while the waterfilling solution (3) is based on the total power constraint (3b), due to the symmetry of the channel at hand (see Fig. 1), it also satisfies the assumed per-transmitter power constraint for any M (see also [3]).

Remark 1: For extreme SNR ($P \rightarrow \infty$), the rate is clearly limited by the minimum capacity $\min\{C,\, C'\}$. Moreover, in order to achieve the maximum multiplexing gain of 1 per transmit antenna, the upper bound (8) shows that the two capacities should both scale as least as $\log P$ with increasing P. These results will be later used for comparison with achievable rates.

IV. Finite-capacity links at the transmitter side only

In this section, we consider the case in which C is finite and $C' \rightarrow \infty$, and derive achievable rates under the assumptions discussed above of oblivious antennas. It is noted that, due to the infinite-capacity links at the receiver side, the assumption of oblivious receivers has no impact on the results of this section. Two achievable rates are derived, one that assumes knowledge at the destination of the transmitters' codebooks and one that does not require such an assumption. We remark that the upper bound (8) is here given by $\lim_{C' \rightarrow \infty} R_{UB}(P,C,C') = \min\{C,\, R_{WF}(P)\}$.

A. Independent messages

In this section, we consider a simple scheme that assumes the destination to be aware of the codebooks available at the transmitters. It is based on transmission of independent messages by the source to different antennas (which then map the corresponding message indices to a codeword according to the available Gaussian codebook) and joint decoding at the destination. More precisely, the source splits its message (of rate MR) into M equal-rate messages, and delivers each one to a transmitter via the finite capacity links. Each transmitter then maps the rate-R message into a codeword, using a mapping which is known at both source and destination. It is noted that here the codebooks available at the sources are used directly as *channel codes*. Moreover, knowledge of the parameter α is used here only for selection of the transmission rate. The following rate is achievable.

Proposition 2: Let $C' \to \infty$. Then, the following rate is achievable by transmitting independent messages (IM) from each transmitter

$$R_{IM}(P, C) = \min\{C,\ R_{NC}(P)\}, \tag{10}$$

where $R_{NC}(P)$ is the maximum rate achievable with no cooperation (NC) among the transmitters and $C' \to \infty$, which is given by

$$\begin{aligned} R_{NC}(P) &= \int_0^1 \log_2 \left(1 + P|H(f)|^2\right) df = \\ &= \log_2 \left(\frac{1 + (1+\alpha^2)P + \sqrt{1 + 2(1+\alpha^2)P + (1-\alpha^2)^2 P^2}}{2} \right). \end{aligned} \tag{11}$$

Proof: The first bound in (10) is clearly due to the need to deliver each equal-rate message on a link of capacity C. The second, instead, follows from the fact that the effective communication scenario between transmitters and D is a Wyner-type uplink channel whose capacity is (11) (see, e.g., [31]).

Remark 2: It is easy to see that this scheme is optimal (i.e., it achieves the upper bound (8)) if $C \leq R_{NC}(P)$ (and thus in particular if $P \to \infty$). Moreover, by having C scale as $\log P$, the scheme is able to achieve the maximum multiplexing gain, as for the upper bound (see Remark 1). Finally, when $C > R_{NC}(P)$, the rate achievable by this scheme does not achieve the upper bound (8), suffering from the performance penalty caused by independent encoding as compared to the waterfilling solution (3). This penalty is known to be bounded by 0.53 bit/symbol in our case and to vanish for large SNR [22] [23].

B. *Quantized waterfilling*

Here we consider an alternative transmission scheme in which the transmitters' codewords are assumed to be unknown to the destination, and thus are exploited by the source merely as *quantization codebooks* (of size 2^{nC}), as explained in the following. The channel codebook shared by source and destination is generated according to the waterfilling solution (3). The source encodes the message using such codebook and then it quantizes each obtained codeword using the (quantization) codebook available at the corresponding transmitter. The indices of the transmitters' codewords are then sent over the finite-capacity links. Any transmitter simply transmits the codeword corresponding to the received indices, as per our assumptions. The performance of this scheme can be proved to correspond to that of a fully cooperative MIMO system with additional (colored) noise due to quantization, as stated in the following proposition.

Proposition 3: Let $C' \rightarrow \infty$. Then, the following rate is achievable with quantized waterfilling (QW):

$$R_{QW}(P, C) = R_{WF}\left(P, \; \frac{(1-2^{-C})|H(f)|^2}{1+P2^{-C}|H(f)|^2}\right), \tag{12}$$

which in the high-SNR regime, where

$$P \geq \frac{1}{(1-2^{-C})} \frac{2\alpha}{(1-\alpha)(1-\alpha^2)}, \tag{13}$$

is given by

$$R_{QW}(P, C) = \log\left(P + \frac{1}{1-\alpha^2}\right) - R_{NC}(P2^{-C}). \tag{14}$$

Proof: See Appendix B.

Remark 3: The rate (14) reveals that for extremely large SNR $(P \rightarrow \infty)$, the rate obtained with quantized waterfilling achieves the upper bound (8) $R_{UB} \rightarrow C$. Moreover, it can be easily seen that, in order to ensure the optimal multiplexing gain of 1 per antenna, the capacity C should scale as $\log P$, as for the upper bound. Finally, for large capacity $C \to \infty$, it can be concluded from (12) that we have $R_{QW}(P,C) \to R_{UB}$. This contrasts with the case of independent message transmission studied above, where the upper bound was not achievable for large C. Further performance comparison between the two considered schemes will be discussed in Sec. VII.

V. Finite-capacity links at the receive side only

In this section, we focus on the scenario characterized by finite C' and $C \to \infty$. It is noted that, dually to the scenario considered in Sec. IV, here the assumption of oblivious transmitters has no impact on the results. We recall that we assume oblivious receivers in the sense specified in Sec. II. Following [14], we consider achievable rates with two quantization strategies carried out at the receivers, in order of complexity. The first is based on elementary compression, whereby correlation between the signals received by different antennas is not exploited for compression, and the second is based on distributed compression techniques. In both cases, we use Gaussian test channels for compression. This choice has been proved to be optimal for the two-source case [25], in the high-resolution (low-distortion) regime (and even if the sources are not Gaussian) [26], and for the remote source coding scenario known as the "CEO problem" [27]. Finally, it is remarked that the upper bound (8) is here given by $\lim_{C\to\infty} R_{UB}(P,C,C') = \min\{C', R_{WF}(P)\}$.

A. Elementary compression

With elementary compression, correlation among the received signals is not exploited in the design of the quantization functions.

Proposition 4. Let $C \to \infty$. Then, the following rate is achievable with elementary compression (EC):

$$R_{EC}(P,C') = R_{WF}\left(\frac{P}{N_{EC}(P,C')}\right). \tag{15}$$

with

$$N_{EC}(P,C') = \frac{1+(1+\alpha^2)P2^{-C'}}{1-2^{-C'}}. \tag{16}$$

Moreover, if conditions

$$P \geq \frac{2\alpha}{(1+\alpha)\left((1+\alpha^2)(1-2^{-C'})-2\alpha\right)} \tag{17}$$

and

$$C' > \log_2\left(\frac{1+\alpha^2}{(1-\alpha)^2}\right), \tag{18}$$

are satisfied, a closed-form expression of (15) is given by

$$R_{EC}(P,C') = \log_2\left(\frac{P}{N_{EC}(P,C')}+\frac{1}{1-\alpha^2}\right). \tag{19}$$

Proof: See Appendix C.

Remark 4: From (19), it can be seen that for extremely large SNR $(P \to \infty)$ (and if (18) holds), the rate achieved with elementary compression is

$$R_{EC}(P,C') \underset{P\to\infty}{\rightarrow} \log_2\left(\frac{2^{C'}-1}{1+\alpha^2}+\frac{1}{1-\alpha^2}\right), \tag{20}$$

which is smaller than the upper bound (8) $R_{UB} \to C'$ for $P \to \infty$. This shows that there is a penalty to be paid for obliviousness at the receive side, if elementary compression is employed, even when $P \to \infty$. On a different note, it can be easily seen that, in order to ensure the optimal multiplexing gain of 1 per antenna, the capacity C' should scale as $\log P$, as for the upper bound (see Remark 1). Finally, for large capacity $C' \to \infty$, we clearly have optimal performance $R_{EC}(C') \to R_{UB}$.

B. Distributed compression

The premise of the scheme discussed in this section is the observation that, since decoding of all quantization codewords takes place at the destination, the correlation of the signals observed at the receivers can be leveraged in order to decrease the equivalent quantization noise.

Following [14], the quantization scheme employed here is based on the distributed compression approach used for the CEO problem [27].

Proposition 5: Let $C \to \infty$. Then, the following rate is achievable with distributed compression (DC):

$$R_{DC}(P, C') = R_{WF}\left(P(1-2^{-r^*})\right) \tag{21}$$

with r^* satisfying the fixed-point equation

$$R_{WF}\left(P(1-2^{-r^*})\right) = C' - r^*. \tag{22}$$

Moreover, if conditions

$$P \geq \frac{2\alpha}{(1-\alpha)\left((1-\alpha^2) - 2^{-C'}\right)}, \tag{23}$$

and

$$C' > 2\log_2\left(\frac{1}{1-\alpha}\right) \tag{24}$$

are satisfied, solution of the fixed-point equation (22) can be obtained in closed form as

$$R_{DC}(P, C') = \log_2\left(\frac{P + \frac{1}{1-\alpha^2}}{1 + P2^{-C'}}\right). \tag{25}$$

Proof: See Appendix D.

Remark 5: Equation (22) is easily solved numerically since $R_{WF}\left(P(1-2^{-r^*})\right)$ is a monotonic function of r^*. Moreover, the expression (25) shows that for extremely large SNR $(P \to \infty)$, the rate with oblivious transmitters (if (24) is satisfied, which requires sufficiently small α or large C') achieves the upper bound (8), i.e., $R_{DC}(P, C') \to C'$. This contrasts with the result discussed in Remark 4 for elementary compression, which was shown to be unable to achieve the upper bound. Moreover, optimal multiplexing gain is still guaranteed if the capacity C' scales as $\log P$, as for the upper bound and for previously considered cases. Finally, it can be seen that for large capacity $C' \to \infty$, we have $R_{DC}(P, C') \to R_{UB}$.

VI. Finite-capacity links at the transmit and receive sides

In the two previous sections, we have considered the two limiting cases $C \to \infty$ (Sec. V) and $C' \to \infty$ (Sec. IV), and constructed basic transmission and reception strategies based on oblivious antennas, namely transmission of independent messages (IM) versus quantized waterfilling (QW) at the transmit side, and elementary (EC) versus distributed compression (DC) at the receive side. In this section, we consider the more general case in which finite-capacity links are present at both the transmit and receive sides, and evaluate achievable rates based on combination of the basic techniques discussed above.

A. Independent messages and elementary compression

It is recalled that, when using transmission of independent messages, it is assumed that the destination is aware of the codebooks available at the transmitters.

Proposition 6: The following rate is achievable by transmitting independent messages (IM) and using elementary compression (EC) at the receive side:

$$R_{IM-EC}(P, C, C') = \min \left\{ C,\ \log_2 \left(\frac{N_{EC} + (1+\alpha^2)P + \sqrt{(N_{EC} + (1+\alpha^2)P)^2 - 4\alpha^2 P^2}}{2N_{EC}} \right) \right\}, \tag{26}$$

with $N_{EC}(P, C')$ as in (16) (we have dropped the dependence on P, C' for the sake of legibility).

Proof: The achievable rate is either limited by C (first term in the right-hand side of (26)) or by the rate achievable over a Wyner-type multiple access channel with channel model as in Fig. 1 with enhanced noise (16) (second term in (26)). The latter rate can be found following the proof of Proposition 4 in Appendix-C.

Remark 6: The result in Proposition 2 can be found as a special case of Proposition 6 for $C' \to \infty$.

Remark 7: This scheme is optimal (i.e., it achieves the upper bound (8)) whenever the second term in (26) is larger than C. For $P \to \infty$, the performance is limited either by the quantization noise at the receive side or by the finite-capacity C at the transmit side:

$$R_{IM-EC}(P,C,C') \underset{P\to\infty}{\to} \min\left\{C,\ \log_2\left(\frac{1+\sqrt{1-\frac{4\alpha^2}{(1+\alpha^2)^2}(1-2^{-C'})^2}}{2\cdot 2^{-C'}}\right)\right\}. \tag{27}$$

Finally, when $C, C' \to \infty$, the scheme tends to the performance of non-cooperating transmitters (11), $R_{IM-EC}(P,C,C') \to R_{NC}(P) \leq R_{UB}$, thus suffering from the performance loss due to transmission of independent messages (see Remark 2).

B. Independent messages and distributed compression

Proposition 7: The following rate is achievable by transmitting independent messages (IM) and using distributed compression (DC) at the receive side:

$$R_{IM-DC}(P,C,C') = \min\{C,\ R'\} \tag{28}$$

with

$$R' = \log_2\left(\frac{1+(1+\alpha^2)P+2\alpha^2 2^{-C'}P^2+\sqrt{1+2(1+\alpha^2)P+\left((1-\alpha^2)^2+4\alpha^2 2^{-C'}\right)P^2}}{2(1+2^{-C'}P)(1+\alpha^2 2^{-C'}P)}\right). \tag{29}$$

Proof: The rate is either limited by C or by the rate of a Wyner-type multiple access channel with channel model as in Fig. 1 and distributed compression at the receiver. This model is studied in [13] where rate (29) is derived.

Remark 8: Proposition 2 follows from Proposition 7 when $C' \to \infty$.

Remark 9: As for the previous scheme, optimality is guaranteed if the second term in (28) is larger than the capacity C. Moreover, in the case of extreme SNR, we have that

$$R_{IM-DC}(P,C,C') \underset{P\to\infty}{\to} \min\{C,\ C'\}, \tag{30}$$

which corresponds to the upper bound. This confirms the consideration made in Remark 5 that distributed compression is necessary to achieve optimal performance when P grows unbounded.

Finally, it is easy to see that for C and $C' \to \infty$, we have $R_{IM-DC}(P, C, C') \to R_{NC}(P) \leq R_{UB}$, as expected, due to the performance loss caused by transmission of independent messages (see Remark 2).

C. Quantized waterfilling and elementary compression

We recall that, unlike the previous two subsections, the scheme considered here, based on quantized waterfilling, does not require the destination to be aware of the codebooks available at the transmitters.

Proposition 8: The following rate is achievable by using quantized waterfilling (QW) at the transmit side and elementary compression (EC) at the receive side:

$$R_{QW-EC}(P, C, C') = R_{WF}\left(P,\ \frac{(1-2^{-C})|H(f)|^2}{N_{EC}(P, C') + P2^{-C}|H(f)|^2}\right), \tag{31}$$

with $N_{EC}(P, C')$ as in (16). Moreover, if the following two conditions are satisfied:

$$P \geq \frac{2\alpha}{(1-2^{-C})(1-2^{-C'})(1-\alpha)(1-\alpha^2) - 2\alpha(1+\alpha^2)2^{-C'}}, \tag{32}$$

and

$$(1-2^{-C})(1-2^{-C'})(1-\alpha)(1-\alpha^2) - 2\alpha(1+\alpha^2)2^{-C'} > 0, \tag{33}$$

the rate (31) is given in closed-form by (we drop the dependence of N_{EC} on P, C' for the sake of legibility)

$$R_{QW-EC}(P, C, C') = \log_2\left(P + \frac{N_{EC}}{1-\alpha^2}\right) - \log_2\left(\frac{N_{EC} + (1+\alpha^2)P2^{-C} + \sqrt{N_{EC}^2 + 2N_{EC}(1+\alpha^2)P2^{-C} + (1-\alpha^2)^2P^2 2^{-2C}}}{2}\right). \tag{34}$$

Proof: See Appendix E.

Remark 10: Proposition 8 subsumes Propositions 3 (for $C' \to \infty$) and 4 (for $C \to \infty$).

Remark 11: Let us consider the asymptotics. For $P \to \infty$, we have, after some algebra:

$$R_{QW-EC}(P,C,C') \underset{P\to\infty}{\longrightarrow} \log_2\left(\frac{1}{2^{-C}+2^{-C'}-2^{-(C+C')}}\right) - \Delta, \tag{35}$$

with $\Delta = \log_2\left(\frac{1+\sqrt{1-\frac{4\alpha^2}{(1+\alpha^2)^2}\frac{2^{-2C}(1-2^{-C'})^2}{(2^{-C}+2^{-C'}-2^{-(C+C')})^2}}}{2\left(\frac{1-2^{-C'}}{1+\alpha^2}+\frac{2^{-C}}{1-\alpha^2}\right)}\right) \geq 0$, which is clearly smaller than the upper bound $\min\{C,C'\}$. Finally, when C and C' grow unbounded, this scheme achieves the upper bound $R_{UB.}$

D. Quantized waterfilling and distributed compression

Proposition 9: The following rate is achievable by using quantized waterfilling (QW) at the transmit side and distributed compression (DC) at the receive side:

$$R_{QW-DC}(P,C,C') = R_{WF}\left(P,\ \frac{(1-2^{-r^*})(1-2^{-C})|H(f)|^2}{1+P2^{-C}|H(f)|^2}\right) \tag{36}$$

with r^* satisfying the fixed-point equation

$$R_{WF}\left(P,\ \frac{(1-2^{-r^*})(1-2^{-C})|H(f)|^2}{1+P2^{-C}|H(f)|^2}\right) = C' - r^*. \tag{37}$$

Under the condition

$$R \leq \log_2\left(1-\frac{2\alpha}{P(1-2^{-C})(1-\alpha)(1-\alpha^2)}\right) + C', \tag{38}$$

the rate (37) is given by

$$R_{QW-DC}(P,C,C') = \log_2\left(P+\frac{1}{1-\alpha^2}\right) - \log\left(\frac{1+\left((1+\alpha^2)2^{-C}+22^{-C'}(1-2^{-C})\right)P+\sqrt{1+2(1+\alpha^2)P2^{-C}+(1-\alpha^2)^2P^22^{-2C}}}{2}\right). \tag{39}$$

Proof: See Appendix F.

Remark 12: Proposition 9 entails Proposition 3 for $C' \to \infty$ and Proposition 5 for $C \to \infty$.

Remark 13: Equation (37) is easily solved numerically since the left-hand side is a monotonic function of r^*. Moreover, we have that

$$R_{QW-DC}(P, C, C') \underset{P\to\infty}{\to} \log_2 \left(\frac{1}{2^{-C} + 2^{-C'} - 2^{-(C+C')}} \right), \tag{40}$$

which is clearly smaller than the upper bound and, compared with (35), it shows that the high-SNR gain of distributed over elementary compression when quantized waterfilling is used can be quantified by Δ defined in Remark 11. Finally, it is immediate to see that for C, $C' \to \infty$ $R_{QW-DC} \to R_{UB}$.

VII. Numerical results

Here we present some numerical examples to validate the analysis discussed in previous sections. In order to clearly distinguish the different regimes of finite-capacity at the transmitters, receivers or both, we treat the three cases separately.

A. Finite-capacity links at the transmit side only

Consider the scenario discussed in Sec. IV with $C' \to \infty$ and finite C. In this case, we are interested in comparing the rates achievable with independent messages ($R_{IM}(P, C)$ in (10)) and with quantized waterfilling ($R_{QW}(P, C)$ in (12)). Fig. 2 shows the achievable rates of interest versus the SNR P for $\alpha^2 = 0.6$ and $C = 4$. First, it is noted that in this scenario exploiting knowledge of the transmitters' codebooks at the destination via independent encoding ($R_{IM}(P, C)$) enables the upper bound R_{UB} to be closely approached and attained for sufficiently large SNR, here $P \gtrsim 10dB$ (see Remark 2). The use of quantized waterfilling, instead, allows the upper bound to be achieved only for extreme SNR; here $P \gtrsim 40dB$ (see Remark 3). Performance comparison with respect to the capacity C for $\alpha^2 = 0.6$ and $P = 8dB$ is shown in Fig. 3. Unlike the scenario with large P of Fig. 2, here the upper bound is achieved for sufficiently large C only by quantized waterfilling (for $C \gtrsim 10$) and not by transmission of independent messages (recall Remarks 2 and 3). Finally, Fig. 4 considers varying interference gain α^2 for $P = 3dB$ and $C = 4$. It is shown that independent transmission suffers from increasing interference gain α^2 while the transmission scheme with quantized waterfilling is able to leverage the additional path towards the receivers.

B. Finite-capacity links at the receive side only

We now turn to the scenario treated in Sec. V (finite C' and $C \rightarrow \infty$), where the rates of interest are those achieved by either elementary compression ($R_{EC}(P, C')$ in (15)) or distributed compression ($R_{DC}(P, C')$ in (21)). From Fig. 2, while elementary compression is unable to achieve the upper bound R_{UB} for $P \rightarrow \infty$ (in fact, it achieves $\log_2\left(\frac{2^{C'}-1}{1+\alpha^2} + \frac{1}{1-\alpha^2}\right) \simeq 3.6$, as per Remark 4), the same is not true for distributed compression. Instead, Fig. 3 shows that the upper bound is achievable by both distributed and elementary compression for large capacity C (see Remarks 4 and 5).

C. Finite-capacity links at the transmit and receive sides

Finally, we are interested in studying the general scenario with finite-capacity links at both ends. Here we compare the rates achievable with independent messages or quantized waterfilling at the transmitter, combined with either elementary or distributed compression at the receiver. For the sake of legibility, curves corresponding to the case of interested here are drawn as solid lines. A first general conclusion is that independent transmission provides generally relevant advantages over quantized waterfilling, unless the capacity C is sufficiently large and the SNR is moderate (Fig. 3). In particular, from Fig. 2, it is confirmed that with large power P, the upper bound can be reached only if independent messages are transmitted with distributed compression (see Remark 9). However, when focusing on the scenario with large capacities $C = C'$, as shown in Fig. 3, the upper bound can be achieved as long as quantized waterfilling is employed (see Remarks 11 and 13). Another conclusion is that distributed compression significantly outperforms elementary compression, especially for high power P (Fig. 2) and high interference gain α^2 (Fig. 4). Further discussion on the performance of different schemes is provided in Sec. IX.

VIII. MEMORYLESS MAPPING

In this section, we would like to briefly comment on an alternative scenario in which the transmitters do not have "public" codebooks that the source can employ via the finite-capacity links. In this case, assuming an integer value for the link-capacity C, a viable solution is that each transmitter be instructed by the source, in each channel use, to transmit a given symbol from a 2^C-point constellation. To implement this scheme the only information necessary at the source about the transmitters is the available mapping of C bits to a 2^C-point constellation. Notice that fractional values of C could be accommodated as well by more complicated mappings over multiple channel uses according to standard techniques. To elaborate, consider for simplicity the case treated in Sec. IV with $C' \to \infty$ and an integer C. Under such assumptions, finding the capacity for the strategy at hand, denoted as $R_{MM}(P, C)$ (for Memoryless Mapping), amounts to evaluating the capacity of an intersymbol interference channels with discrete i.i.d. input signals and transfer function (2). In [30] [31] lower and upper bounds on the capacity of general intersymbol interference channels with arbitrarily distributed i.i.d. inputs are obtained. The tightest (conjectured) lower bound derived in [31] can be particularized to our scenario as

$$R_{MM}(P, C) \geq I(X; X + \tilde{Z}), \tag{41}$$

and an upper bound as

$$R_{MM}(P, C) \leq I(X; (1 + \alpha^2)X + Z), \tag{42}$$

with X arbitrarily distributed on the given 2^C-point constellation, $\tilde{Z} \sim \mathcal{CN}(0, P(2^{R_{NC}(P)} - 1)^{-1})$ (and $R_{NC}(P)$ as in (11)) and $Z \sim \mathcal{CN}(0, 1)$. Evaluation of the bounds (41) and (42) for the case $C = 1$ with BPSK transmission ($X_m = \pm\sqrt{P}$) and uniform input distribution, can be obtained explicitly as [30]:

$$R_{MM}(P,C) \geq \mathcal{C}_b(2^{R_{NC}(P)} - 1) \tag{43a}$$

$$R_{MM}(P,C) \leq \mathcal{C}_b(P(1+\alpha^2)), \tag{43b}$$

where $\mathcal{C}_b(s)$ is the capacity of the binary-input memoryless scalar Gaussian channel with SNR s:

$$\mathcal{C}_b(s) = 1 - \int_{-\infty}^{+\infty} \frac{e^{-\tau^2/2}}{\sqrt{2\pi}} \log_2\left(1 + e^{-2\sqrt{s}\tau - 2s}\right) d\tau. \tag{44}$$

Calculation of the mutual informations (41) and (42) for larger constellations (i.e., larger C) can be conveniently carried out by using the relationship between mutual information and MMSE of [32] and using the results in Sec. IV of [33]. Fig. 5 shows the lower and upper bounds (43) versus SNR P and different values of α^2. It can be seen that for sufficiently large P this simple scheme achieves the best rate (3b). Moreover, the performance is strictly inferior to that of transmission from a codebook of independent (Gaussian) symbols $R_{IM}(P,C)$ (10), but is superior, unless P is too small, to that of quantized waterfilling $R_{QW}(P,C)$ (12).

IX. Concluding remarks

From the analysis and the numerical simulations presented in this paper, some general conclusions can be drawn as to effectiveness of different techniques in a distributed MIMO setting for nomadic applications (i.e., with oblivious transmitters and receivers). At the transmit side, transmission of independent messages (IM), which can be interpreted as a decode-and-forward method, is advantageous over the compress-and-forward approach of quantized waterfilling (QW), except in a power-limited scenario in which C and α^2 are large enough. In the latter case, IM provides marginal performance advantage. This conclusion contrasts with the downlink scenario studied in [15] where it was shown that compression of the codewords designed for the case of unlimited backhaul is generally to be preferred over decode-and-forward-type schemes (except in the very large SNR regime). This difference can be readily explained by considering the different nature of the two channels, namely given the independent decoding of the downlink versus joint decoding of the scenario at hand, and in light of the results of [22] [23] (see, e.g., Remark 2). However, it should be recalled that IM comes at the expense of the requirement that the destination should be informed of the codebooks used by the transmitters. At the receive side, exploiting the correlation of the received signals via distributed compression (DC) generally outperforms elementary compression (EC), especially in capacity-limited scenarios where the SNR P and α^2 are large enough.

Another interesting conclusion that follows from the analysis is that the considered design, which targets nomadic applications (i.e., which does not exploit the knowledge of the encoding functions shared by source and destinations), does not entail any loss of optimality in specific asymptotic and non-asymptotic regimes of SNR P and capacities C and C'. For instance, IM with DC is optimal if $P \to \infty$, whereas QW with EC or DC is optimal if $C, C' \to \infty$. These results are in accord with the conclusions of [12]-[13] and [15] for uplink and downlink channels with finite-capacity backhaul, respectively.

X. Appendix

A. *Appendix A: Proof of Lemma 2*

We consider without loss of generality the rate $R_{WF}\left(P, |H(f)|^2\right)$ (4) (recall Lemma 1). In the high-SNR scenario, the Lagrange multiplier μ can be chosen as $\mu \geq \max_f |H(f)|^{-2} = (1-\alpha)^{-2}$. In this case, the power constraint in (3) can be written as

$$P = \int_0^1 \left(\mu - \frac{1}{|H(f)|^2}\right) = \tag{45a}$$

$$= \mu - \frac{2}{(1+\alpha^2)} \int_0^{1/2} \frac{1}{1 + \frac{2\alpha}{1+\alpha^2}\cos(2\pi f)} df = \tag{45b}$$

$$\overset{(a)}{=} \mu - \frac{1}{(1+\alpha^2)\sqrt{1 - \frac{4\alpha^2}{(1+\alpha^2)^2}}} \tag{45c}$$

$$= \mu - \frac{1}{1-\alpha^2}, \tag{45d}$$

where (a) is achieved by applying formula 3.613.1 in [28] setting $n = 0$ and $a = 2\alpha/(1+\alpha^2) \leq 1$. Hence, the high-SNR scenario is achieved if (6) holds, and the waterfilling constant is given by $\mu = P + \frac{1}{1-\alpha^2}$. The rate (4) then can be written as

$$R_{WF}\left(P, |H(f)|^2\right) = \int_0^1 \log\left(1 + \left(\frac{1}{\mu} - \frac{1}{|H(f)|^2}\right)|H(f)|^2\right) df \tag{46a}$$

$$= \log\frac{1}{\mu} + \int_0^1 \log\left(((1+\alpha^2) + 2\alpha\cos(2\pi f))^2\right) df \tag{46b}$$

$$\overset{(a)}{=} \log\left(P + \frac{1}{1-\alpha^2}\right) + \log\left(\frac{(1+\alpha^2) + \sqrt{(1+\alpha^2)^2 - 4\alpha^4}}{2}\right) \tag{46c}$$

$$= \log\left(P + \frac{1}{1-\alpha^2}\right) \tag{46d}$$

where (a) is achieved by applying formula 4.224.9 of [28, pp. 526], setting $a = 1 + \alpha^2$ and $b = 2\alpha$.

B. Appendix B: Proof of Proposition 3

The source produces codewords $\tilde{X}_m^n$ with spatial correlation defined by the waterfilling power spectral density (3) $S_{WF}(f, \tilde{P}, \rho(f))$ with $\tilde{P}$ and $\rho(f)$ to be specified below. The waveforms $\{\tilde{X}_m^n\}$ are quantized by using the Gaussian quantization codebooks with 2^{nC} codewords X_m^n and power constraint $E[|X_m|^2] = P$ available at the transmitters. For this, the test channel $X_m^n = \tilde{X}_m^n + Q_m^n$ with Q_m^n being an i.i.d. sequence of complex Gaussian variables with zero means and power σ_q^2 is used. The quantization noise power σ_q^2 is determined by the condition (necessary for successful compression via joint typicality arguments):

$$C \geq I(\tilde{X}; X) = \log_2\left(1 + \frac{\tilde{P}}{\sigma_q^2}\right), \tag{47}$$

so that the minimum quantization noise power is $\sigma_q^2 = \tilde{P}(2^C - 1)^{-1}$. Moreover, from the power constraint on X_m, we have the condition $E[|X_m|^2] = \tilde{P} + \sigma_q^2 = P$ so that

$$\tilde{P} = \frac{P}{1 + (2^C - 1)^{-1}} = P(1 - 2^{-C}) \tag{48}$$

and $\sigma_q^2 = P2^{-C}$. The signal received by the mth receiver (1) is then given by

$$Y_m = \tilde{X}_m + \alpha\tilde{X}_{[m-1]} + \tilde{Z}_m, \tag{49}$$

where $\tilde{Z}_m = Z_m + Q_m + \alpha Q_{m-1}$, which has power spectral density $S_{\tilde{Z}}(f) = 1 + \sigma_q^2|H(f)|^2$, so that the SNR power spectral density is given by

$$\rho(f) = \frac{|H(f)|^2}{1 + P2^{-C}|H(f)|^2}, \tag{50}$$

accounting for both thermal noise and the contribution of quantization noise. Given the choice of the power spectral density of the input $\tilde{X}_m$ ($S_{WF}(f, \tilde{P}, \rho(f))$), it follows that the achievable rate is $R_{WF}(\tilde{P}, \rho(f))$, which, from Lemma 1 and (48), is equivalent to (12).

We are now interested in solving the waterfilling problem (12) (recall (4)), where we redefine for convenience $\rho(f) = \frac{(1-2^{-C})|H(f)|^2}{1+P2^{-C}|H(f)|^2}$. Assuming that

$$\mu \geq \max_f \frac{1}{\rho(f)} = \frac{1}{(1-2^{-C})}\left(\frac{1}{(1-\alpha)^2} + P2^{-C}\right), \tag{51}$$

the power constraint (3) becomes

$$\begin{aligned} P &= \mu - \int_0^1 \frac{1 + P2^{-C}\,|H(f)|^2}{(1-2^{-C})\,|H(f)|^2} df \\ &\overset{(a)}{=} \mu - \frac{1}{(1-2^{-C})}\left(\frac{1}{1-\alpha^2} + P2^{-C}\right)\ , \end{aligned} \tag{52}$$

where (a) is achieved in a similar manner to (a) in (45). Combining (51) and (52) the high-SNR condition is explicitly given by (13) and the waterfilling constant is given by

$$\mu = P + \frac{1}{(1-2^{-C})}\left(\frac{1}{1-\alpha^2} + P2^{-C}\right). \tag{53}$$

Similarly to (46) (using formula 4.224.9 of [28, pp. 526] twice), the rate $R_{QW}(C)$ is then obtained as (14).

C. Appendix C: Proof of Proposition 4

The source generates codewords X_m^n designed so as to have correlation defined by the waterfilling solution $S_{WF}(f, P, |H(f)|^2/N_{EC})$ in (3) with noise level (16). We will see that this enhanced noise accounts for the compression noise added by the receivers. Each codeword X_m^n is sent by the corresponding mth transmitter (recall that $C \to \infty$ here). Each receiver quantizes the received signal Y_m using a complex Gaussian codebook with 2^{nC} codewords and test channel $\hat{Y}_m^n = Y_m^n + Q_m^n$, with Q_m^n being an i.i.d. sequence of complex Gaussian variables with zero means and power σ_q^2. Following the same reasoning as in Appendix 3, we obtain $\sigma_q^2 = ((1+\alpha^2)P+1)(2^{C'}-1)^{-1}$ so that the equivalent signal seen as the destination is $\hat{Y}_m = X_m + \alpha X_{[m-1]} + \tilde{Z}_m$ with $Z_m \sim \mathcal{CN}(0, 1+\sigma_q^2)$. The achievable rate is then easily shown to be

$$R_{EC}(P, C') = R_{WF}(P, |H(f)|^2/N_{EC}), \tag{54}$$

given the choice of the power spectral density of the transmitted signals $S_{WF}(f, P, |H(f)|^2/N_{EC})$. By Lemma 1, we conclude that (15) holds, as stated in the Proposition. Finally, the high-SNR expression (25) and conditions (17)-(18) follow, after some algebra, from Lemma 2.

D. Appendix D: Proof of Proposition 5

Here we provide just a sketch of the proof which follows by combining the approaches of [12] and [14] with minor modifications. The transmission/ reception scheme works as follows: Transmitters employ standard Gaussian codebooks with correlation dictated by the waterfilling solution $S_{WF}(f, P, (1-2^{-r^*})^{-1}|H(f)|^2)$, similarly to Appendix C. Each mth receiver quantizes the received signal Y_m with a Gaussian codebook $\hat{Y}_m$ of size $2^{n\hat{R}}$ and joint distribution defined by $\hat{Y}_m = Y_m + Q_m$, where variables Q_m account for the quantization noise and are independent from all the other variables. Rate $\hat{R}$ is chosen so that $\hat{R} \geq I(Y_m; \hat{Y}_m)$. The quantization codewords are then placed randomly in $2^{nC'}$ random bins. Each receiver selects codeword

$\hat{Y}_m^n$ based on a standard joint typicality-based vector quantizer. The destination attempts to decode codewords $\{X_m^n\}$ via a joint typicality receiver based on the bin indices corresponding to signals $\{\hat{Y}_m^n\}$. Equation (21)-(22) then follow from Proposition 5 of [14] and Proposition 2 of [12]. Finally, the high-SNR expression (25) and conditions (23)-(24) are direct consequences of Lemma 2. In fact, the high-SNR condition (6) becomes $P(1-2^{-r^*}) \geq \frac{2\alpha}{(1-\alpha^2)(1-\alpha)}$, which after substitution of $r^* = C - R_{DC}(C')$ and some algebra leads to the desired conclusion.

E. Appendix E: Proof of Proposition 8

The source generates codewords $\tilde{X}_m^n$ with spatial correlation given by the waterfilling solution (3) $S_{WF}(f, \tilde{P}, \rho(f))$ with

$$\rho(f) = \frac{|H(f)|^2}{N_{EC} + P2^{-C}|H(f)|^2} \tag{55}$$

and $\tilde{P} = P(1-2^{-C})$. Similarly to the proofs of Proposition 2 and Proposition 4, as discussed below, this power spectral density accounts for the quantization noise added at both transmit and receive sides. The source quantizes the outputs of the waterfilling encoder using the transmitters' codebooks as $X_m^n = \tilde{X}_m^n + Q_m^n$ with Q_m being an i.i.d. sequence of complex Gaussian variables with zero means and power $\sigma_q^2 = P2^{-C}$. The signal received by the mth receiver (1) is given by (49): $Y_m = \tilde{X}_m + \alpha\tilde{X}_{[m-1]} + \tilde{Z}_m$, with $\tilde{Z}_m$ being a Gaussian process with power spectral density $S_{\tilde{Z}}(f) = 1 + \sigma_q^2|H(f)|^2$. Each receiver quantizes the received signal Y_m as $\hat{Y}_m^n = Y_m^n + S_m^n$ via a codebook with $2^{nC'}$ codewords with S_m^n being an i.i.d. sequence of complex Gaussian variables with zero means and power $\sigma_s^2 = ((1+\alpha^2)P+1)(2^{C'}-1)^{-1}$ (see the proof of Proposition 4), so that the equivalent signal seen as the destination is $\hat{Y}_m = \tilde{X}_m + \alpha\tilde{X}_{[m-1]} + \breve{Z}_m$ with $\breve{Z}_m$ having power spectral density $S_{\breve{Z}}(f) = \sigma_s^2 + S_{\tilde{Z}}(f)$. The achievable rate is then $R_{WF}(\tilde{P}, \rho(f)) = R_{WF}(P, (1-2^{-C})\rho(f))$ by Lemma 1, which concludes the proof. The closed-form expression (34) follows similarly to Proposition 2.

F. Appendix F: Proof of Proposition 9

Propositions 9 follows from the proofs of Proposition 2 and Proposition 7. In particular, similarly to Proposition 2, codewords $\tilde{X}_m^n$ are generated according to the power spectral density $S_{WF}(f, \tilde{P}, \rho(f))$ with $\tilde{P} = P(1 - 2^{-C})$ and $\rho(f) = \frac{(1-2^{-r^*})|H(f)|^2}{1+P2^{-C}|H(f)|^2}$, quantized and sent by the transmitters. The receivers behave as described in the proof of Proposition 7 and the result (36) easily follows.

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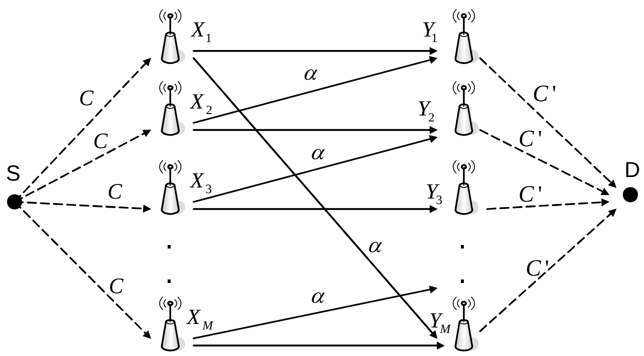

Fig. 1. System model: a nomadic single source S communicates to a single destination D over a symmetric interference channel where transmitters and receivers are oblivious to the encoding functions employed by S and D.

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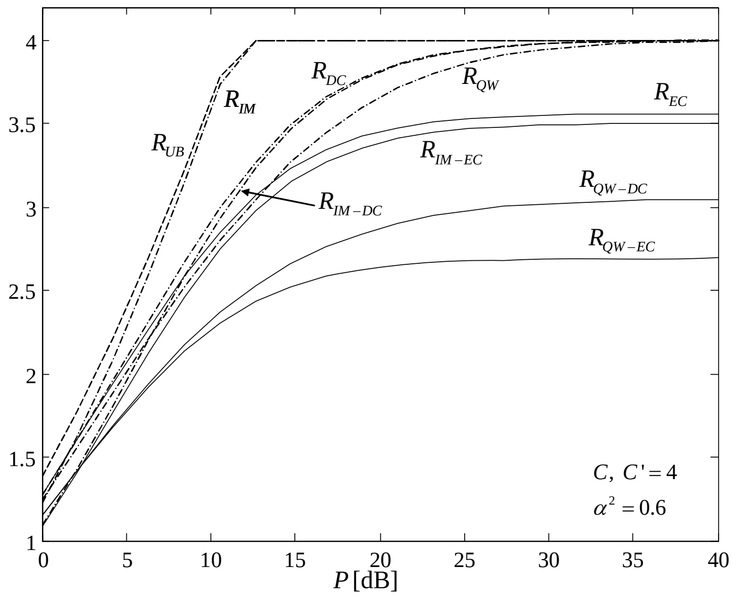

Fig. 2. Achievable rates versus SNR P. Rates R_{IM} and R_{QW} are for $C' \to \infty$ and $C = 4$, rates R_{EC} and R_{DC} to $C \to \infty$ and $C' = 4$, while the remaining curves correspond to $C = C' = 4$ ($\alpha^2 = 0.6$).

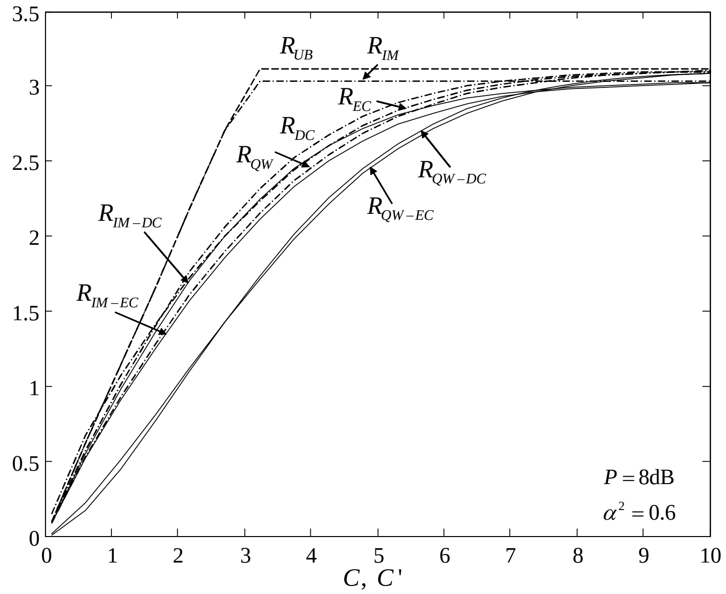

Fig. 3. Achievable rates versus capacities C and C'. Rates R_{IM} and R_{QW} are for $C' \to \infty$, rates R_{EC} and R_{DC} for $C \to \infty$, while the remaining curves correspond to $C = C'$ ($\alpha^2 = 0.6$, $P = 8dB$).

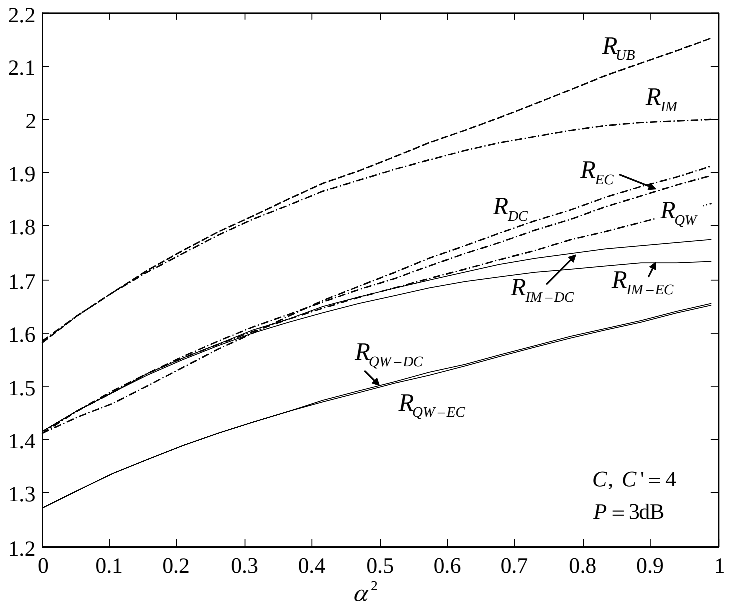

Fig. 4. Achievable rates versus α^2. Rates R_{IM} and R_{QW} are for $C' \to \infty$ and $C = 4$, rates R_{EC} and R_{DC} for $C \to \infty$ and $C' = 4$, while the remaining curves correspond to $C = C' = 4$ ($P = 3dB$).

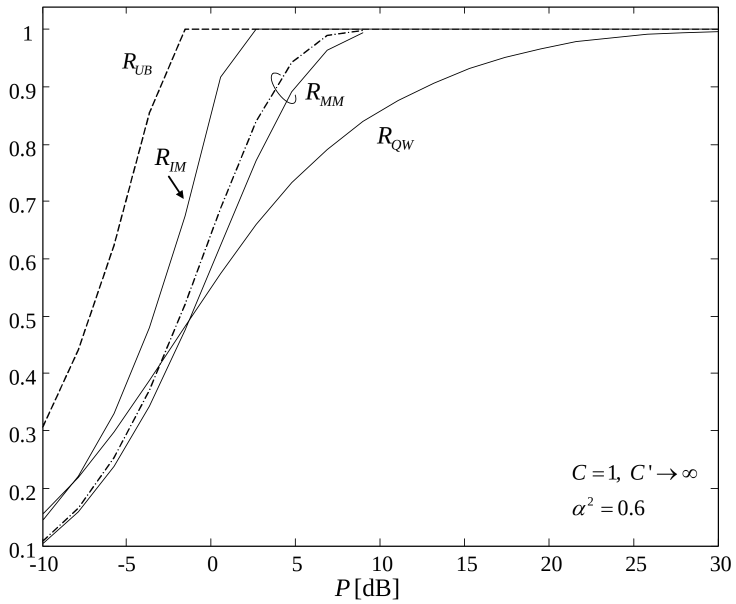

Fig. 5. Achievable rates versus P for $C' \rightarrow \infty$ and $C = 1$ ($\alpha^2 = 0.6$). For memoryless mapping (MM), the solid line corresponds to the lower bound (41) while the dash-dotted line is the upper bound (42).