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Robust Transmission and Interference Management for Femtocells with Unreliable Network Access

O. Simeone[1], E. Erkip[2] and S. Shamai (Shitz)[3]

[1]CWCSPR, Dept. of ECE, NJIT, Newark, NJ, USA

[2]Dept. of ECE, Polytechnic Inst. of NYU, Brooklyn, New York, NY, USA

[3]Dept. of Electrical Engineering, Technion, Haifa, Israel

Abstract

A cellular system where macrocells are overlaid with femtocells is studied. Each femtocell is served by a home base station (HBS) that is connected to the macrocell base station (BS) via an unreliable network access link, such as DSL followed by the Internet. A scenario with a single macrocell and a single femtocell is studied first, and is then extended to include multiple macrocells and femtocells, both with standard single-cell processing and with multicell processing (or network MIMO). Two main issues are addressed for the uplink channel: (*i*) Interference management between femto and macrocells; (*ii*) Robustness to uncertainties on the quality of the femtocell (HBS to BS) access link. Closed and open-access femtocells are considered, along with robust variable-rate data delivery transmission at the home users via the broadcast coding approach (or unequal error protection coding). The problem is formulated in information-theoretic terms, and inner and outer bounds are derived to achievable per-cell sum-rates for outdoor and home users. Expected sum-rates with respect to the distribution of the femtocells access link states are studied as well. Overall, the analysis lends evidence to the performance advantages of sophisticated interference management techniques, based on joint decoding and relaying, and of robust coding strategies via the broadcast coding approach.

I. INTRODUCTION

Cellular systems have evolved into complex *multitier* structures that present a hierarchical organization into units (or cells) operating at different spatial scales [1]. On the one hand, reducing the size of the cells at the lowest tiers allows transmission with smaller powers and thus the possibility to reuse the spectrum more aggressively. The latest development along these lines is the idea of *femtocells* [2][3]. On the other hand, at the highest tier, aggregating multiple macrocells into clusters for joint coding/ decoding enables a better management of *inter-cell interference*, which is increasingly becoming a limiting performance factor. This amounts to the concept of *network MIMO* or *multicell processing* (MCP) [4]-[6].

A femtocell consists of a short-range low-cost *home base station (HBS)* installed within the customer's premises, that serves either only indoor users (for *closed-access* femtocells) or possibly also outdoor users that are within the HBS coverage range (for *open-access* femtocells). Notice that the HBS uses the same radio interface as the macro-BSs. Among the major challenges for a successful deployment of femtocells, two critical issues stand out: (*i*) *Inter-tier interference:* Due to the aggressive frequency reuse, the system throughput in the presence of femtocells is ultimately limited by the inter-tier interference between femto and macrocells. This calls for effective interference management strategies, such as distributed power allocation or interference avoidance techniques (see [3][7] and references therein)*; (ii) Reliability of the connection between HBS and provider*: Being installed by the user in the customer's premises, HBSs typically do not enjoy a reliable network access link. Indeed, the HBS is connected to the provider network via a last-mile connection such as DSL or cable followed by the Internet. Such access links do not provide fixed and reliable quality-of-service, due to technical issues such as bursty cross talk on the DSL link or congestion. For instance, recent trials have shown that, on a DSL link shared with Wi-Fi, femtocell connectivity was severely degraded even for low-bandwidth services [3].

In this work, we study the two issues mentioned above by focusing at first on a basic system with one macrocell overlaid with one femtocell (see Fig. 1 and Sec. II-IV). We then address the

problem in a multicell context in which either single-cell processing (SCP) or MCP is deployed (see Fig. 6 and Sec. V). We cast the problem in information-theoretic terms by limiting the analysis to uplink and accounting for the facts that: (*a*) HBS may provide either open or closed access to the outdoor users; (*b*) The signals received by the BS from the home users may be treated as interference or rather exploited as useful; (*c*) The home users (served by the HBS) may not be aware of the current state of the HBS-BS link; (*d*) The performance of the outdoor users should not be disrupted by the uncertainty on the current state of the HBS network access link. Within this framework, we design *interference management* techniques at the HBSs and BSs, and transmission strategies at the home users which are *robust* to the unknown network access link state. Outer bounds to the achievable rates are derived as well, and compared to the proposed techniques.

Notation: We define $\mathcal{C}(x) = 1/2 \log_2(1+x)$; Notation $[1, N]$ represents the set of numbers $\{1, ..., N\}$; x^n is the vector $(x_1, .., x_n)$; $\mathrm{diag}(\mathbf{v})$ is a diagonal matrix with main diagonal given by vector $\mathbf{v}$.

II. Single Cell: System Model

We focus at first on the uplink channel sketched in Fig. 1, which consists of a *single* macrocell with K_O outdoor ("O") users, each with power constraint P'_O, and K_H home ("H") users, each with power constraints P'_H. The multicell scenario will be studied in Sec. V. Both outdoor and home users are active on the same bandwidth. The signals transmitted by the home users $X_{H,k,i} \in \mathbb{R},\ k \in [1, K_H]$ and by the outdoor users $X_{O,k,i} \in \mathbb{R},\ k \in [1, K_O]$, at time instant $i \in [1, n]$ are received by the BS and the HBS as, respectively,

$$Y_i = \sum_{k=1}^{K_O} X_{O,k,i} + \sqrt{\alpha} \sum_{k=1}^{K_H} X_{H,k,i} + N_{Y,i} \tag{1a}$$

$$\text{and } Z_i = \sqrt{\beta_O} \sum_{k=1}^{K_O} X_{O,k,i} + \sqrt{\beta_H} \sum_{k=1}^{K_H} X_{H,k,i} + N_{Z,i}, \tag{1b}$$

with zero-mean independent white Gaussian noises $N_{Y,i}$ and $N_{Z,i}$ with powers $\sigma_Y^2 = 1$ and $\sigma_Z^2 = 1$, respectively[1]. We have defined the channel power gains towards the HBS as β_H and β_O for home and outdoor users, respectively, and the channel power gain between home users and BS as α (the channel gain from outdoor users to BS is normalized to 1). Notice that model (1) assumes that home and outdoor users are symbol-synchronous, but the results are expected to hold also in the presence of symbol-asynchronous users (provided that the users' time difference is known), following the results for the standard multiple access channel (see [8] and references therein). Finally, the results herein are immediately extended to complex (in-phase and quadrature) channel models.

The HBS is connected to the BS via an unreliable finite-capacity link (e.g., DSL) with variable capacity. The current link capacity C_m [bits/ channel use] can be measured at the two link ends, HBS and BS, but is assumed to be unknown to all other nodes. This is due, e.g., to generally unpredictable DSL channel conditions and absence of a feedback channel from HBS or BS to the users[2]. Moreover, the current HBS-BS link state is considered to remain constant for the entire duration of the current transmitted codeword (non-ergodic link state). The number of possible states (link capacities) is M and we order them as $C_m > C_{m-1},\ m \in [1, M]$. We assume that home users are informed about the *possibility* of different HBS-BS connectivity conditions and about the corresponding possible link states ($C_1, ..., C_M$). They may therefore design their communications strategy so as to be *robust* with respect to the different realizations of the link state. In particular, indoor users may employ generalized coding strategies that allow *variable-rate data delivery*, whereby the amount of data that the BS is able to decode reliably depends on the current state of the HBS-BS link. We remark that variable-rate coding could also be used to counteract uncertainties on the fading channels [16]-[19], but fading channels are not

[1]We will leave σ_Y^2 explictly shown in some of the equations given below to ease presentation in Sec. V.

[2]When including the possibility of feedback, the proposed techniques could be combined with retransmission strategies (HARQ) to improve reliability. This will not be further studied here.

included in the model here for simplicity. In contrast, the outdoor users expect *fixed-rate data delivery* irrespective of the current link condition within the femtocell.

1) Variable-Rate Coding (Broadcast Coding Approach): The basic idea of variable-rate coding is that the source (here a home user) transmits a number, say M, of information layers (messages) $W_1, ..., W_M$, that are ordered in terms of importance from the most significant to the least significant. Higher (less significant) layers are only meaningful if the lower layers are also decoded correctly. As an example, consider MPEG-2 or MPEG-4 video transmission where the three types of frames, I, P, and B-frames, are compressed such that P-frames require I-frames to be reconstructed, and B-frames require both I and P-frames to be decompressed, see discussion in [9, Sec. II] for further examples. From a coding perspective, the most significant layers thus need to be protected more than the less significant, which gives rise to the so called Unequal Error Protection (UEP) codes. Different multiplexing strategies may be used for transmitting the layers, namely TDMA (as in the Priority Encoding Transmission of [9, Sec. II]) and *superposition coding* (via simple sum, multilevel modulation [10] or joint encoding [12] [13]). From an information-theoretic standpoint, the problem falls in the category of coding for a broadcast channel with a *degraded message set* [11], for which, in case the broadcast channel is degraded, it has been proved that *superposition coding with successive interference cancellation* at the decoder is optimal [14][11][3] (see also [15]).

A. Formal Setting

The setting described above is formalized as follows. Each kth outdoor user has a message $W_{O,k} \in [1, 2^{nR_{O,k}}]$, $k \in [1, K_O]$, while each kth home user has M messages (or "layers" of information) $W_{H,m,k} \in [1, 2^{nR_{H,m,k}}]$, $k \in [1, K_H]$ and $m \in [1, M]$, ordered from the most to the least significant, for the BS. The message layers of the home users are to be decoded at the BS according to the current link capacity C_m following a *degraded message structure*: In state m

[3]In some channels, such as *memoryless* erasure, TDMA can provide the same performance as superposition [11].

(i.e., link capacity C_m is realized), the BS decodes messages $\mathcal{W}_{H,m} = (W_{H,m,1}, ..., W_{H,m,K_H})$ corresponding to the mth layer of all home users and all the "lower" (more significant) layers $\mathcal{W}_{H,1}, ..., \mathcal{W}_{H,m-1}$. As explained above, the mth layer is generally refinement information for the previous layers $1, ..., m-1$ and requires availability of the previous layers to be interpreted correctly. Given the above, while any kth outdoor user operates at a fixed rate $R_{O,k}$, any kth home user, aware of the fact that the connection between the femtocell BS and the macrocell BS is unreliable, operates at a variable rate, delivering rate $\sum_{i=1}^{m} R_{H,i,k}$ *when the HBS-BS link is in state* m [4].

Encoding for the kth outdoor user ($k \in [1, K_O]$) takes place via a function $f_{O,k}^{(n)}$ that maps the message $w_{O,k} \in [1, 2^{nR_{O,k}}]$ into a codeword $x_{O,k}^n = f_{O,k}^{(n)}(w_{O,k})$ (fixed-rate encoding), and for the kth home user via function $f_{H,k}^{(n)}$ that maps the M-layer messages $w_{H,1,k}, ..., w_{H,M,k}$ into a codeword $x_{H,k}^n = f_{H,k}^{(n)}(w_{H,1,k}, ..., w_{H,M,k})$ (variable-rate encoding). We have the power constraints

$$\frac{1}{n}\sum_{i=1}^{n} x_{O,k,i} \leq P'_O \text{ and } \frac{1}{n}\sum_{i=1}^{n} x_{H,k,i} \leq P'_H, \tag{2}$$

for outdoor and home users, respectively.

The HBS, aware of the current state m of the HBS-BS link, maps the received vector z^n into an index $v_m \in [1, 2^{nC_m}]$ as $v_m = f_{HBS,m}^{(n)}(z^n)$. Decoding at the BS is also dependent on m and is characterized by a function $g_m^{(n)}$ that maps the received signal from the HBS-BS link v_m and over the channel, y^n, into the decoded messages as $(\widehat{\mathcal{W}}_{H,1}, .., \widehat{\mathcal{W}}_{H,m}, \widehat{\mathcal{W}}_O) = g_m^{(n)}(Y^n, V_m)$, with $\mathcal{W}_O = (W_{O,1}, ..., W_{O,K_O})$. Finally, the probability of error is defined as

$$P_e^{(n)} = \max_{m \in [1,M]} \Pr[g_m^{(n)}(V_m, Y^n) \neq (\mathcal{W}_{H,1}, .., \mathcal{W}_{H,m}, \mathcal{W}_O)], \tag{3}$$

where messages are assumed to be uniformly distributed in their sets. A tuple of home and outdoor rates, $R_{H,m,k}$, for $k \in [1, K_H]$, $m \in [1, M]$ and $R_{O,k}$ for $k \in [1, K_O]$, is said to be *achievable* if there exists a sequence of encoders and decoders such that $P_e^n \to 0$ for $n \to \infty$.

[4]From the standpoint of UEP codes, this means that in practice layers $i > m$ are not decoded with sufficient reliability and are thus discarded by the BS.

B. Sum-Rates and Average Rates

In order to simplify the problem of obtaining and interpreting regions of achievable rates, we focus on achievable *sum-rates* for both home and outdoor users. For the home users, we define the sum-rate of any layer $m \in [1, M]$ as $R_{H,m} = \sum_{k=1}^{K_H} R_{H,m,k}$, whereas the sum-rate of the outdoor users is given by $R_O = \sum_{k=1}^{K_O} R_{O,k}$. Following the discussion above, the sum-rate tuple $(R_{H,1}, ..., R_{H,M}, R_O)$ is said to be achievable if there exists a tuple of achievable component rates, $R_{H,m,k}$, for $k \in [1, K_H]$, $m \in [1, M]$ and $R_{O,k}$ for $k \in [1, K_O]$, satisfying the definition of achievability given above.

Remark 2.1: (*Equal Rates*) By the symmetry of the model, an achievable sum-rate tuple provides equivalently an achievable equal-rate point, i.e., the sum-rate tuple $(R_{H,1}, ..., R_{H,M}, R_O)$ is achievable if and only if the individual rates $R_{H,m,k} = R_{H,m}/K_H$ for all $k \in [1, K_H]$, $m \in [1, M]$ and $R_{O,k} = R_O/K_O$ are achievable[5].

Remark 2.2: (*Average Rates*) It is sometimes appropriate to assume a probability distribution over the M possible link states. When this is of interest, we will denote the probability of having a link of rate C_m by p_m for $m \in [1, M]$, with $\sum_{m=1}^{M} p_m = 1$. Moreover, in this case, a potentially useful figure of merit for the home users is the *average* sum-rate

$$\bar{R}_H = \sum_{m=1}^{M} p_m \sum_{i=1}^{m} R_{H,i} = \sum_{m=1}^{M} R_{H,m} \sum_{i=m}^{M} p_i \tag{4}$$

with respect to the HBS-BS link state probability. Such criterion has been widely considered in related studies (see, e.g., [16]-[19]) and has the operational meaning of average throughput in the presence of repeated packet transmissions by the home users if the HBS-BS link vary in a stationary way along the blocks. We will say that a pair of sum-rates $(\bar{R}_H, R_O)$ is achievable

[5]In fact, suppose that a certain sum-rate is achievable for which, say, the individual rates of two outdoor users are different. Then, a rate vector in which the role of the two outdoor users at hand is swapped is also achievable by symmetry, and time-sharing can be performed to equalize the two rates without loss in the sum-rate. The same philosphy can be applied to equalize the rates of more than two users.

if there exists a tuple of sum-rates $(R_{H,1},...,R_{H,M},R_O)$ that is achievable (according to the definition given above) and such that (4) is satisfied for a given distribution p_m.

III. Single Cell: Performance Bounds and Transmission Strategies

In this section, we derive inner and outer bounds to the achievable sum-rate region. In principle, the HBS can serve as a helper (relay) for both indoor and outdoor users, thus facilitating interference mitigation at the BS. This scenario corresponds in the jargon of femtocell systems to an *open-access* system in which outdoor users can also benefit from the HBS. In other situations, it may be more reasonable to assume *closed-access* femtocells, for which the HBS does not attempt to serve outdoor users [2][3]. While in latter case knowledge of the outdoor codebooks (i.e., modulation and coding) at the HBS is not required, which reduces signalling overhead, in the former it may useful. We will treat the two cases separately below.

All the proposed techniques are based on: (*i*) *Superposition coding* at the home users with *successive interference cancellation decoding*: Codewords corresponding to the different information layers are transmitted via superposition coding following the optimal strategy for degraded broadcast channels (recall Sec. II-1); (*ii*) *Decoding the home users' signals at the HBS*: Decode-and-Forward (DF) is used at the HBS to relay the home users's signals to the BS. The latter choice is justified by the fact that, following the reasoning in [3], the channel gain β_H between home users and HBS is typically 30-$80dB$ larger than the channel gain α to the BS, depending on the propagation environment, so that decoding at the HBS typically does not entail any performance loss.

We first show that in order to study achievable sum-rates, or equivalently equal rates (see Remark 2.1), it is enough to consider a system with a single home user and a single outdoor user with power constraints equal to the sum-power constraints.

Proposition 3.1: A sum-rate tuple $(R_{H,1},...,R_{H,M},R_O)$ is achievable if and only if it is achievable in a system with a single home user and a single outdoor user (i.e., $K_H = 1$ and $K_O = 1$) with power constraints given by the sum-powers $P_H = K_H P'_H$ and $P_O = K_O P'_O$,

respectively.

Proof: See Appendix A. ■

Remark 3.1: Given the Proposition 3.1, in the following we will work with the equivalent system with a single outdoor and home user, $K_O = K_H = 1$. Therefore, we will refer to the rates $(R_{H,1}, ..., R_{H,M}, R_O)$ as rates, rather than sum-rates for simplicity.

A. Closed-Access Femtocells

We first consider achievable schemes for closed-access (CA) femtocells. For all such schemes, by definition, the signal transmitted by the outdoor users is treated as noise at the HBS, so that the HBS cannot mitigate the macro to femtocell interference.

1) No Interference Mitigation (CA-I): Here we consider the conventional system design, where, not only the femtocells provide CA service (thus treating outdoor users as noise), but also the BS treats the wireless signals leaked from home users as noise. This corresponds to designing each tier (femto and macrocell) independently without attempting any interference mitigation strategy.

Proposition 3.2: (CA-I) Consider a CA femtocell and a BS that treats the signal from the home user as interference (CA-I, where "I" is for "Interference"). With this strategy, the convex hull of the union of all non-negative rate tuples $(R_{H,1}, ..., R_{H,M}, R_O)$ that satisfy

$$R_{H,m} \leq \Psi_{H,m}^{CA\text{-}I}(\underline{\gamma},\underline{a}) \triangleq \min\left\{\mathcal{C}\left(\frac{\beta_H P_H \gamma_m}{1+\beta_H P_H \sum_{i=m+1}^{M}\gamma_i + \beta_O P_O}\right),\ a_m\right\} \tag{5}$$

for $m \in [1, M]$, and

$$R_O \leq \Psi_O^{CA\text{-}I} \triangleq \mathcal{C}\left(\frac{P_O}{\sigma_Y^2 + \alpha P_H}\right) \tag{6}$$

for some vectors of parameters $\underline{\gamma} = [\gamma_1, ..., \gamma_M]$ and $\underline{a} = [a_1, ..., a_M]$ with non-negative entries such that

$$\sum_{i=1}^{M}\gamma_i = 1 \text{ and } \sum_{i=1}^{m} a_i \leq C_m \text{ for } m \in [1, M], \tag{7}$$

is achievable.

Proof: The home user employs superposition coding by summing M Gaussian codewords encoding the M layers with power allocation dictated by a vector $\underline{\gamma}$ and rates $R_{H,m}$. When the HBS-BS link is in state m, all layers $m' \leq m$ are decoded at the HBS using successive interference cancellation and treating the outdoor user's signal and higher layers as (Gaussian) noise, thus leading to the first term in (5). The HBS then allocates rates $a_{m'}$, $m' \leq m$, to transmit message $\mathcal{W}_{H,m'}$ to the BS, yielding the second term in (5). The message of the outdoor user $\mathcal{W}_O$ is encoded using a standard random Gaussian codebook and decoded at the BS treating the signal from the home user as noise, leading to the bound (6). ■

2) Interference Mitigation at the BS (CA-S): We consider now the case where the BS, rather than treating the home user's signals as noise, attempts joint decoding based on both the signal received over the HBS-BS access link and the signal received over the wireless link from the home user.

Proposition 3.3: (CA-S) Consider a CA femtocell and a BS that exploits the home user's signal for decoding (CA-S, where "S" is for "Signal"). With this strategy, the convex hull of the union of all non-negative rate tuples $(R_{H,1}, ..., R_{H,M}, R_O)$ that satisfy

$$R_{H,m} \leq \Psi_{H,m}^{CA-S}(\underline{\gamma},\underline{a}) \triangleq \min \left\{ \begin{array}{c} \mathcal{C}\left(\frac{\beta_H P_H \gamma_m}{1+\beta_H P_H \sum_{i=m+1}^{M} \gamma_i + \beta_O P_O}\right), \\ \mathcal{C}\left(\frac{\alpha P_H \gamma_m}{\sigma_Y^2 + \alpha P_H \sum_{i=m+1}^{M} \gamma_i}\right) + a_m \end{array} \right\} \tag{8}$$

for $m \in [1, M]$, and

$$R_O \leq \Psi_O^{CA-S}(\underline{\gamma}) \triangleq \mathcal{C}\left(\frac{P_O}{\sigma_Y^2 + \alpha P_H(1-\gamma_1)}\right) \tag{9a}$$

$$R_O + R_{H,1} \leq \Psi_{OH}^{CA-S}(\underline{\gamma},\underline{a}) \triangleq \mathcal{C}\left(\frac{\alpha P_H \gamma_1 + P_O}{\sigma_Y^2 + \alpha P_H(1-\gamma_1)}\right) + a_1, \tag{9b}$$

for some vectors $\underline{\gamma}$ and $\underline{a}$ with non-negative entries such that (7) hold, is achievable.

Proof: Encoding at the home and outdoor users take place as for Proposition 3.2 and so does decoding at the HBS, which leads to the first bound in (8). Encoding at the HBS and decoding at the BS are performed differently. If the HBS-BS is in state m, the HBS, having decoded the set of messages $\mathcal{W}_{H,1}, ..., \mathcal{W}_{H,m}$, randomly bins the decoded messages with respective rates

$a_1, ..., a_m$ as in, e.g., [20]. The bin indices are then transmitted to the BS. The BS decodes based on both the received signal (1a) and the bin indices. Specifically, the BS performs joint decoding of the home user's layer $m = 1$ and of the signal transmitted by the outdoor users, leading to the bounds (9) and the second bound in (8) for $m = 1$. Then, the remaining layers of the home user are decoded using successive interference cancellation, giving the remaining bounds in the second term of (8). We remark that the considered decoding order guarantees that the outdoor user's message is decoded irrespective of the link access state. ■

B. Open-Access Femtocells

We now turn to Open Access (OA) femtocells, where the HBS may possibly assist decoding of the outdoor users at the BS via relaying, and may be aware of the codebooks used by the outdoor users. We consider two achievable schemes. In the first, the HBS uses DF to support outdoor users, whereas in the second it uses Compress-and-Forward (CF). In both cases, the signal from the home user is exploited at the BS for decoding as for the CA-S strategy of Sec. III-A2 (extending the CA-I approach is straightforward given the analysis below).

1) Decode-and-Forward for Outdoor User (OA-DF):

Proposition 3.4: (OA-DF) Consider a OA femtocell and a HBS that assists outdoor users via DF relaying. With this strategy, the convex hull of the union of all non-negative rate tuples $(R_{H,1}, ..., R_{H,M}, R_O)$ that satisfy

$$R_{H,m} \leq \Psi_{H,m}^{OA\text{-}DF}(\underline{\gamma},\underline{a}) \triangleq \min \left\{ \begin{array}{l} \mathcal{C}\left(\frac{\beta_H P_H \gamma_m}{1+\beta_H P_H \sum_{i=m+1}^{M} \gamma_i}\right), \\ \mathcal{C}\left(\frac{\alpha P_H \gamma_m}{\sigma_Y^2+\alpha P_H \sum_{i=m+1}^{M} \gamma_i}\right) + a_m \end{array} \right\} \tag{10}$$

for $m \in [1, M]$, and

$$R_O \leq \Psi_O^{OA\text{-}DF}(\underline{\gamma},\underline{a}) \triangleq \min\left\{\mathcal{C}\left(\frac{\beta_O P_O}{1+\beta_H P_H(1-\gamma)}\right),\ \mathcal{C}\left(\frac{P_O}{\sigma_Y^2+\alpha P_H(1-\gamma)}\right)+a_0\right\} \tag{11a}$$

$$R_O + R_{H,1} \leq \Psi_{OH}^{OA\text{-}DF}(\underline{\gamma},\underline{a}) \triangleq \min\left\{\begin{array}{l}\mathcal{C}\left(\frac{\beta_H P_H\gamma_1+\beta_O P_O}{1+\beta_H P_H(1-\gamma)}\right),\\ \mathcal{C}\left(\frac{\alpha P_H\gamma_1+P_O}{\sigma_Y^2+\alpha P_H(1-\gamma_1)}\right)+a_0+a_1,\end{array}\right\} \tag{11b}$$

for some vectors $\underline{\gamma}$ and $\underline{a} = [a_0, a_1, ..., a_M]$ with non-negative entries such that

$$\sum_{i=1}^{M}\gamma_i = 1 \text{ and } \sum_{i=0}^{m} a_i \leq C_m \text{ for } m \in [1, M] \tag{12}$$

is achievable.

Proof: The proof follows Proposition 3.3, with the difference that here the HBS decodes also the outdoor message in the same order as the BS in Proposition 3.3, which leads to the additional bounds given by the first terms of the two inequalities (11). Moreover, the HBS reserves capacity a_0 over the HBS-BS link to send a random bin index of the message of the outdoor user, which improves decoding at the BS as shown in the second terms of bounds (11). ■

2) Compress-and-Forward for Outdoor User (OA-CF):

Proposition 3.5: (OA-CF) Consider a OA femtocell and a HBS that assists outdoor users via CF relaying. With this strategy, the convex hull of the union of all non-negative rate tuples $(R_{H,1}, ..., R_{H,M}, R_O)$ that satisfy

$$R_{H,m} \leq \Psi_{H,m}^{OA\text{-}CF}(\underline{\gamma},\underline{a}) \triangleq \Psi_{H,m}^{CA-S}(\underline{\gamma},\underline{a}) \tag{13}$$

for $m \in [1, M]$, where $\Psi_{H,m}^{CA-S}(\underline{\gamma},\underline{a})$ is given in (8), and

$$R_O \leq \Psi_O^{OA\text{-}CF}(\underline{\gamma},\underline{a}) \triangleq \mathcal{C}\left(\frac{P_O}{\sigma_Y^2+\alpha P_H(1-\gamma_1)}+\frac{\beta_O P_O}{1+\sigma_q^2}\right) \tag{14a}$$

$$R_O + R_{H,1} \leq \Psi_{OH}^{OA\text{-}CF}(\underline{\gamma},\underline{a}) \triangleq \log\det\left(\mathbf{I}+\mathbf{Q}^{-1}\mathbf{H}\mathbf{P}\mathbf{H}^H\right)+a_1 \tag{14b}$$

with definitions $\mathbf{H}=\begin{bmatrix} 1 & \alpha \\ \beta_O & 0 \end{bmatrix}$, $\mathbf{P}=\mathrm{diag}\left([P_O\ P_H\gamma_1]\right)$, $\mathbf{Q}=\mathrm{diag}([\sigma_Y^2+\alpha P_H(1-\gamma_1)\ 1+\sigma_q^2])$,

$$\sigma_q^2=\frac{1}{2^{2a_0}-1}\left(1+\frac{\beta_O P_O(\sigma_Y^2+\alpha P_H)}{\sigma_Y^2+P_O+\alpha P_H}\right),$$

and for some vectors $\underline{\gamma}$ and $\underline{a}=[a_0,a_1,...,a_M]$ with non-negative entries such that (12) hold, is achievable.

Proof: The proof is similar to Proposition 3.4 with the difference that here the HBS decodes the home user's messages by treating the signal from the outdoor users as noise. After decoding, the home user's signal is then cancelled at the HBS and the remaining signal $\tilde{Z}=Z-\sqrt{\beta_H}X_H=\sqrt{\beta_O}X_O+N_Z$ (recall (1b) and Proposition 3.1) is compressed with rate a_0 and sent to the BS. We assume Gaussian quantization test channel $\hat{Z}=\tilde{Z}+Q$, where Q is quantization noise with power σ_q^2. The constraint on σ_q^2 follows from the condition $a_0\geq I(Z;\hat{Z}|Y)$, which is a consequence of standard rate-distortion considerations, taken with equality (see, e.g., [22, Sec. III-A]). The BS decodes jointly the home user's first layer along with the outdoor user's message by exploiting both (1a) and the quantized signal. Successive home user's layers are decoded as in the other proposed techniques. ■

C. Outer Bound

Here we derive an outer bound to the set of achievable sum-rates in order to provide a benchmark for the achievable rates derived above.

Proposition 3.6: (Outer bound) Any achievable tuple of rates $(R_{H,1},...,R_{H,M},R_O)$ must satisfy the conditions

$$R_{Hm}\leq\Psi_{H,m}^{Out}(\underline{\gamma},\underline{a})\triangleq\min\left\{\mathcal{C}((\alpha+\beta_H)P_H),\ \mathcal{C}\left(\frac{\alpha P_H\gamma_m}{\sigma_Y^2+\alpha P_H\sum_{i=m+1}^{M}\gamma_i}\right)+a_m\right\}\qquad(15)$$

for $m \in [1, M]$, and

$$R_O \leq \Psi_O^{Out} \triangleq \mathcal{C}((1/\sigma_Y^2 + \beta_O)P_O) \tag{16a}$$

$$R_O + R_{H,1} \leq \Psi_{OH}^{Out}(\underline{\gamma}) \triangleq \mathcal{C}\left(\frac{\alpha P_H \gamma_1 + P_O}{\sigma_Y^2 + \alpha P_H(1-\gamma_1)}\right) + C_1 \tag{16b}$$

for some vectors $\underline{\gamma}$ and $\underline{a}$ with non-negative entries verifying (7).

Proof: See Appendix B. ■

D. Average Home User Rates

To provide more insight into the system performance, we now turn to the analysis of the achievable sum-rate pairs $(\bar{R}_H, R_O)$, with $\bar{R}_H$ being the average sum-rate (4) of the home user with respect to a given probability distribution p_m over the link states. As seen above, the derived achievable sum-rate regions for strategies CA-I, CA-S, OA-DF and OA-CF and the outer bound are defined by the same type of inequalities on $(R_{H,1}, ..., R_{H,M}, R_O)$ and differ in the definition of the three functions $\Psi_{H,m}^s$, Ψ_O^s and Ψ_{OH}^s with $s \in \{$CA-I, CA-S, OA-DF, OA-CF, Out$\}$ and of the corresponding constraint sets (7) (for CA and outer bound) and (12) (for OA) [6]. Given such characterizations in terms of sum-rates $(R_{H,1}, ..., R_{H,M}, R_O)$, we have the following result.

Proposition 3.7: (Average Rates) Consider a transmission strategy $s \in \{$CA-I, CA-S, OA-DF, OA-CF$\}$. With s, the convex hull of the union of all non-negative rate pairs $(\bar{R}_H, R_O)$, where $\bar{R}_H$ is the average rate (4) of the home user, that satisfy

$$\bar{R}_H \leq \sum_{m=1}^{M} p_m \sum_{i=1}^{m} \Psi_{H,i}^s \tag{17a}$$

$$R_O \leq \Psi_O^s \tag{17b}$$

$$\bar{R}_H + R_O \leq \Psi_{OH}^s + \sum_{m=2}^{M} p_m \sum_{i=2}^{m} \Psi_{H,i}^s \tag{17c}$$

[6]Notice that for Proposition 3.3 we can take $\Psi_{OH}^{CA-I} \to \infty$

for some vectors $\underline{\gamma}$ and $\underline{a}$ with non-negative entries verifying the constraints (7) if $s \in \{$CA-I, CA-S, Out$\}$ or (12) if $s \in \{$OA-DF, OA-CF$\}$ is achievable. With $s =$Out and constraint set (7), the above provides an outer bound on the achievable pairs $(\bar{R}_H, R_O)$.

Proof: Use Fourier-Motzkin elimination from Propositions 3.2-3.6. ■

IV. Single Cell: Numerical Results

In this section, we provide some numerical results in terms of the *maximum equal rate* R_{eq} that can be supported by both home and outdoor users. In other words, we look for the maximum R_{eq} such that the pair $(\bar{R}_H, R_O) = (R_{eq}, R_{eq})$ is achievable for different techniques. We start by considering a scenario where the HBS-BS link may either be active with some capacity C [bits/ channel use] or not functioning (or equivalently the femtocell may not operating). This corresponds to setting $M = 2$, $C_1 = 0$ and $C_2 = C > 0$. The probability of *HBS-BS link (or femtocell) failure* is then given by $P_{fail} = p_1$. We remark that since $M = 2$, in this scenario only two layers are necessary for broadcast coding. Moreover, since in the worst state ($m = 1$), the HBS-BS access capacity is $C_1 = 0$, CA-I, which treats home user's signal as noise, can only transmit a non-zero rate in the second layer (see (5)). Therefore, CA-I can be seen as providing the performance reference for a scheme that neither exploits broadcast coding nor the home user's signal reception at the BS. For the same reason, OA schemes have no performance benefit in this case, since the outdoor messages must be decoded also in the worst case $m = 1$. Given this, we focus without loss of generality on the case $\beta_O = 0$ and on the performance of CA-I and CA-S, where the performance of the second will lend evidence to the advantages of broadcast coding and leveraging the received signal at the BS for interference management. We will later address the issue of interference from outdoor users to HBS.

Unless stated otherwise, we set $\alpha = 1/d^4$, where d represents the normalized distance between home user and BS, where the unit is the distance between outdoor user and the BS. This amounts to assuming a path loss exponent of four. We also set parameters $P_O = 2$, $P_H = 2$, $C = \mathcal{C}(P_O)$ and $\beta_H = 1000\alpha$ (i.e., a $30dB$ gain on indoor vs. outdoor power gains for outdoor users).

Fig. 2 shows the achievable equal rate R_{eq} for CA-I and CA-S, along with the upper bound obtained from Proposition 3.7, for $d = 1.5$ versus the probability of link failure P_{fail}. For reference, we show the rate that the outdoor users would obtain with No Femtocell (NF), namely $R_O^{NF} = \mathcal{C}(P_O)$. Remarkably, it is seen that this rate can be achieved by *both* home and outdoor users via CA-S if P_{fail} is sufficiently small. Instead, for larger P_{fail}, such rate cannot be achieved and a loss is incurred by the outdoor users due to the presence of a femtocell. However, CA-S still achieves the upper bound and is thus optimal for this example[7]. CA-I, instead, is always suboptimal and provides increasingly negligible rate as P_{fail} increases. We also show the performance of CA-S for two special single-layer designs, namely "best-case" ($\gamma = 0$), where all power is allocated to the second layer, and "worst-case" ($\gamma = 1$), which devotes all power to the first layer[8]. It is noted that the latter case corresponds to the performance of a system with both indoor and outdoor users and no femtocell. The advantages of a *robust design of CA-S that uses both layers* by optimizing the power allocation γ are clear.

Fig. 3 shows the same rates as above but versus the normalized home users-BS distance d and with fixed $P_{fail} = 0.1$. It can be seen that again CA-S achieves the derived upper bound if one can select the optimal transmission power allocation. Moreover, CA-I becomes optimal for sufficiently large d (i.e., small α) due to the negligible interference created by the home users at the BS. For the same reason and due to the relatively small P_{fail}, CA-S with "best-case" power allocation $\gamma = 0$ has optimal performance for large d.

The performance versus the link capacity C is shown in Fig. 4 for fixed $P_{fail} = 0.2$, $d = 1.5$ and other parameters as in the rest of this section. CA-S with worst-case design clearly cannot benefit from increasing C, unlike with best-design and with optimized γ, in which case the rate coincides again with the upper bound. Both CA-S with best-case design and CA-I, for sufficiently large C, are limited by the interference created to the outdoor users, and their rates

[7]Some analytical optimality conditions can be found in [23].

[8]It is easy to see that $\gamma = 0$ is always optimal for CA-I in the link failure scenario.

both equal $\mathcal{C}(P_O/(1+\alpha P_H)) \simeq 0.64$. This is less than the simple upper bound $R_O^{NF} \simeq 0.8$, which is instead attained by CA-S.

We finally turn to a discussion of the relative merits of OA and CA for a scenario where $C_1 > 0$. We set $C_1 = R_O^{NF}/10$, $C_2 = R_O^{NF}$, $p_2 = 0.9$, and vary $\beta_O > 0$ with other parameters defined as above. Form Fig. 5, we see that for small β_O, CA-S performs as well as OA-CF (CA-I is largely outperformed), while for β_O sufficiently large, OA in the form of either OA-CF or OA-DF becomes advantageous. For all values of β_O, a judicious choice of the transmission scheme allows to perform close to the upper bound.

V. The Multicell Case

We now turn to the analysis of the multicell scenario in Fig. 6.

A. System Model

Consider a linear cellular system similar to [4], where L cells are arranged on a line, as for a corridor or a highway, as shown in Fig. 6. Each cell contains a single femtocell, as in the scenario of Sec. II, and presents the same number K_O and K_H of outdoor and indoor users, respectively. Power constraints and intra-cell channel gains are as in Sec. II. Signals generated within each femtocell are received with relevant power only by the local BS, while outdoor users are received not only by the local BS and HBS, but also by L_C adjacent BSs on either side (if present), with symmetric channel gains δ_l, $l \in [1, L]$. Received signals can be expressed as (1a)-(1b) with the addition of the contribution from outdoor users, assuming time synchronization across the L cells (see Sec. II). This scenario can be seen as an extension of the model in [4], that has been widely considered in the literature (see review in [5]).

The state of the HBS-BS access link in each lth cell is defined by a random variable $M_l \in [1, M]$ for $l \in [1, L]$. Random variables M_l are assumed to be i.i.d. over the cell index l with same pmf $(p_1, ..., p_M)$. Rates and sum-rates are defined as in Sec. II by adding the subscript l to denote the cell index. For instance, $R_{H,m,l}$ is the sum-rate of the home users in cell l at layer

m and $R_{O,l}$ is the sum-rate of outdoor users in cell l. Encoding functions are defined as in Sec. II-A under the premise that users and HBs in different cells do not cooperate.

As for decoding at the BSs of cells $l \in [1, L]$, we consider two scenarios: (*i*) *Single-cell Processing* (SCP): The BS in each cell decodes independently as described in Sec. II-A; (*ii*) *Multicell Processing* (MCP): All BSs in the system are connected to a central processor (CP) for joint decoding. The CP collects the signals of all BSs and jointly decodes all the $K_O L$ outdoor messages and $K_H \sum_{l=1}^{L} M_l$ home users messages. Notice that in both cases if the HBS-BS link in a cell is in state M_l, only the first M_l layers of the home users are decoded, either at the local BS (for SCP) or a the CP (for MCP). Probability of error and achievability are defined accordingly. In particular, the probability of error is taken as the maximum over all possible HBS-BS links configurations of the probability of decoding error for the required messages as in (3).

Rather than working with the $L(M+1)$-dimensional rate region of sum-rates ($R_{H,m,l}$, $R_{O,l}$), $l \in [1, L]$, $m \in [1, M]$ we focus only on the *per-cell sum-rates* for home and outdoor users, similar to [4] and follow-up works (see review in [5]). Namely, we define $\mathcal{R}_H$ as the sum-rate over all home users in the system normalized by the number of cells and similarly for the outdoor users. We also focus on the regime of a large system $L \to \infty$ to remove edge effects (see discussion in [5]). More precisely, we say that a per-cell sum-rate pair $(\mathcal{R}_H, \mathcal{R}_O)$ is achievable with

$$\mathcal{R}_H = \lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \sum_{m=1}^{M_l} R_{H,m,l} \tag{18a}$$

$$\text{and } \mathcal{R}_O = \lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} R_{O,l} \tag{18b}$$

if such limits exist in an almost sure sense for some achievable rates ($R_{H,m,l}$, $R_{O,l}$), $l \in [1, L]$, $m \in [1, M]$.

Remark 5.1: Extending Proposition 3.1, given that we focus on sum-rates, we can restrict our attention without loss of generality to only one home and outdoor user per cell as in Fig. 6.

Remark 5.2: Unlike the definition of average home users rates in Sec. II-B (and thus of pairs $(\bar{R}_H, R_O)$), the definition of per-cell sum-rates $(\mathcal{R}_H, \mathcal{R}_O)$ given above does not entail any ensemble average but only an average over the cells. However, if one focuses on operating points for which users in different cells transmit with the same rates, i.e., $R_{H,m,l} = R_{H,m,l'}$ and $R_{O,l} = R_{O,l'} = R_O$ for all $l, l' \in [1, L]$, then it is immediate to see that the per-cell sum-rates satisfy $\mathcal{R}_O = R_O$ and $\mathcal{R}_H = \bar{R}_H$, where $\bar{R}_H$ is the ensemble average $\bar{R}_H$ in (4) and the latter equality holds due to the strong law of large numbers from definition (18a) since variables M_l are i.i.d..

B. Single-Cell processing (SCP)

Achievable rates with SCP can be easily obtained from their counterparts described for the single-cell case by simply assuming that each BS treats the out-of-cell signals as (Gaussian) noise, as shown below. Notice that with No Femtocells (NF), the per-cell sum-rate of outdoor users with SCP would be

$$\mathcal{R}_O^{NF\text{-}SCP} = \mathcal{C}\left(\frac{P_O}{1 + P_O(1 + 2\sum_{m=1}^{L_C}\delta_m)}\right). \tag{19}$$

Proposition 5.1: (SCP) Consider a strategy $s \in \{$CA-I, CA-S, OA-DF, OA-CF$\}$ and SCP. With s and SCP, per-cell rate pairs $\mathcal{R}_H = \bar{R}_H$ and $\mathcal{R}_O = R_O$, where $(\bar{R}_H, R_O)$ satisfy the conditions of Proposition 3.7 with $\sigma_Y^2 = 1 + P_O(1 + 2\sum_{m=1}^{L_C}\delta_m)$ are achievable.

Proof: Assume that each cell is operated in the same way, according to a transmission scheme s (as described in Sec. III-A-III-B) with same rates $(R_{H,1}, ..., R_{H,M}, R_O)$, but with the difference that here the signals from other cells are treated as noise. The result then follows from Remark 5.2 and Proposition 3.7. ■

C. Multicell Processing (MCP)

We now turn to the analysis of achievable rates with MCP. We focus on CA, both due to space limitations and since in practice the gains achievable via OA are expected to be minor in the

presence of MCP. This is because the latter technique already enhances significantly decoding of outdoor users[9] [5]. For reference, we recall that with No Femtocells (i.e., by setting $\beta_O = 0$ and $P_O = 0$), the per-cell maximum achievable rate of the outdoor users is given by [4]

$$\mathcal{R}_O^{NF\text{-}MCP} = \mathcal{C}_{MCP}\left(1,\ f_{MCP}(\theta)\right) \tag{20}$$

where we have defined the functions:

$$\mathcal{C}_{MCP}(x, f(\theta)) = \frac{1}{2\pi}\int_0^{2\pi} \mathcal{C}\left(x \cdot f_{MCP}(\theta)\right) d\theta \tag{21}$$

$$\text{and } f_{MCP}(\theta) = P_O\left(1 + 2\sum\nolimits_{l=1}^{L_C} \sqrt{\delta_l}\cos(l\theta)\right)^2. \tag{22}$$

Proposition 5.2: (MCP, CA-I) Consider the CA-I scheme employed in each cell and MCP. With this strategy, the convex hull of all pairs of (non-negative) per-cell sum-rates ($\mathcal{R}_H$,$\mathcal{R}_O$) satisfying the conditions

$$\mathcal{R}_H \leq \sum_{m=1}^{M} p_m \sum_{i=1}^{m} \Psi_{H,i}^{CA\text{-}I}(\underline{\gamma},\underline{a}) \tag{23a}$$

$$\mathcal{R}_O \leq \mathcal{C}_{MCP}\left((1+\alpha P_H)^{-1},\ f_{MCP}(\theta)\right), \tag{23b}$$

where $\Psi_{H,i}^{CA\text{-}I}(\underline{\gamma},\underline{a})$ are defined in (5), for some parameter vectors verifying (7), is achievable.

Proof: Encoding takes place at all nodes as for CA-I (see Sec. III-A1) with users in all cells employing the same rates. The CP, based on all received signals, decodes the outdoor messages treating the home users' signal as noise (which increases the noise level to $1+\alpha P_H$). Condition (23b) then follows from [4] similar to (20). Finally, condition (23a) follows again from Remark 5.2. ■

Proposition 5.3: (MCP, CA-S) Consider the CA-S employed in each cell and MCP. With this strategy, the convex hull of all pairs of (non-negative) per-cell sum-rates ($\mathcal{R}_H$,$\mathcal{R}_O$) satisfying the

[9]In fact, with OA-DF, the performance generally degrades to the point where MCP becomes useless due to the requirement of decoding at the HBS.

conditions (recall (8))

$$\mathcal{R}_O \leq \mathcal{C}_{MCP}\left((1+\alpha P_H(1-\gamma_1))^{-1},\ f_{MCP}(\theta)\right) \tag{24}$$

$$\mathcal{R}_H \leq \sum_{m=1}^{M} p_m \sum_{i=1}^{m} \Psi_{H,i}^{CA-S}(\underline{\gamma},\underline{a}) \tag{25}$$

$$\begin{aligned}\mathcal{R}_O + \mathcal{R}_H \leq\ & \mathcal{C}_{MCP}\left((1+\alpha P_H(1-\gamma_1))^{-1},\ \alpha P_H\gamma_1 + f_{MCP}(\theta)\right) \\ & + \sum_{m=2}^{M} p_m \sum_{i=2}^{m} \Psi_{H,i}^{CA-S}(\underline{\gamma},\underline{a}) + a_1,\end{aligned} \tag{26}$$

where functions $\Psi_{H,i}^{CA-S}(\underline{\gamma},\underline{a})$ are defined in (8) with $\sigma_Y^2 = 1$, and for some parameter vectors verifying (7) is achievable.

Proof: Encoding takes place in each cell as for CA-S (see Sec. III-A2) using the same rates in each cell. The CP jointly decodes the first layer ($m = 1$) of all home users and all the messages of the outdoor users, similar to CA-S for a single-cell. We then proceed as in Proposition 3.7 by Fourier-Motzkin elimination. ■

D. Numerical Results

For further discussion, we consider some numerical results for a scenario with $L_C = 1$, $\delta_1 = 0.1$, $P_O = P_H = 2$, $C_1 = \mathcal{R}_O^{NF\text{-}MCP}/4$, $C_2 = \mathcal{R}_O^{NF\text{-}MCP}$, $p_2 = 0.9$, $\alpha = 1/d^4$, $\beta_O = 2\alpha$, $\beta_H = 1000$, and varying distance d between BS and home user in each cell. Analogously to Sec. IV, we show the maximum per-cell equal rates $\mathcal{R}_{eq}$ such that $(\mathcal{R}_H, \mathcal{R}_O) = (\mathcal{R}_{eq}, \mathcal{R}_{eq})$ is achievable for CA-I and CA-S with both SCP and MCP in Fig. 7. We also show for reference the rates that the outdoor users would obtain without femtocells for both SCP $\mathcal{R}_O^{NF\text{-}SCP}$ (19) and MCP $\mathcal{R}_O^{NF\text{-}MCP}$ (20) to provide an upper bound[10]. It can be seen that MCP enables remarkable gains over SCP. Moreover, for small distances d treating home users signals as noise entails

[10]Notice that for MCP this is an upper bound on the equal rate for any transmission scheme from [4], while for SCP it is an upper bound only on the considered schemes with Gaussian inputs.

a significant performance penalty. For sufficiently large distances d, the performance of MCP becomes limited by decoding at the HBS of home users' signals and thus fails to reach the performance bound $\mathcal{R}_O^{NF\text{-}MCP}$. Conversely, the performance of SCP becomes limited by the decoding of outdoor users at the BS, thus achieving the upper bound given by $\mathcal{R}_O^{NF\text{-}SCP}$.

VI. Concluding Remarks

The design of femtocells-macrocell overlay faces a number of significant challenges, most notably the mutual inter-tier interference between home users/ home BSs, on one side, and outdoor users/ BSs, on the other, and the unreliability of the network access link between HBS and BS in each cell. In this paper, we have taken a first look at these problems from an information-theoretic standpoint by focusing on the uplink performance with and without multicell processing. The analysis has revealed the significant benefits of interference management techniques that are based on fully exploiting the structure of the received signals and open-access femtocells. Moreover, our results show that broadcast coding strategies that enable variable-rate delivery are very promising solutions to effectively cope with uncertainties on the network access state. It is expected that such techniques would be particularly well suited to be combined with retransmission strategies in order to provide quality-of-service guarantees. This aspect, along with a full system analysis in the presence of fading channels, is left for future work.

Appendix A
Proof of Proposition 3.1

It is immediate to see that if a certain sum-rate tuple $(R_{H,1}, ..., R_{H,M}, R_O)$ is achievable in the original system, it is also achievable in the single-user system of Proposition 3.1. In fact, the single "compound" home and outdoor users can always transmit the sum of the signals transmitted in the original systems by the individual users given the sum-power constraint. Assume now that a sum-rate tuple is achievable in the single-user system. To see that it is also

achievable in the original system, one can proceed as follows. Define as $\tilde{K} = \tilde{k}_H K_H = \tilde{k}_O K_O$ a common multiple of K_H and K_O with $\tilde{k}_H, \tilde{k}_O$ being integer. Now, divide the time in $\tilde{K}$ slots of equal size and activate only one home and outdoor users in each time slot (among the K_H and K_O available, respectively) with powers P_H and P_O in such a way that every home (outdoor) user is active for a fraction $1/K_H$ $(1/K_O)$ of the time. The active users will employ exactly the same transmission scheme used by the "compound" users to achieve the rate tuple at hand. It can be seen that the same sum-rate is achieved also in the original system with the correct individual power constraints P'_H and P'_O.

APPENDIX B
PROOF OF PROPOSITION 3.6

The first bound in (15) and (16a) follow by considering an enhanced system where HBS and BS can fully cooperate for decoding. We then focus on the remaining bounds. We start with the first-layer sum-rate of the home users, which by the Fano inequality should satisfy for *every* $m \in [1, M]$

$$\begin{aligned} nR_{H,1} &\leq I(\mathcal{W}_{H,1}; Y^n, V_m|\mathcal{W}_O) + n\epsilon_n = I(\mathcal{W}_{H,1}; \bar{Y}^n, V_m) + n\epsilon_n \\ &= I(\mathcal{W}_{H,1}; \bar{Y}^n) + I(\mathcal{W}_{H,1}; V_m|\bar{Y}^n) + n\epsilon_n, \end{aligned}$$

where we have defined $\bar{Y}_i = \alpha \sum_{k=1}^{K_H} X_{H,k,i} + N_{B,i}$, $i \in [1, n]$ (recall (1a)), which implies

$$nR_{H,1} \leq I(\mathcal{W}_{H,1}; \bar{Y}^n) + \min_{m\in[1,M]} I(\mathcal{W}_{H,1}; V_m|\bar{Y}^n). \tag{27}$$

We define

$$a_1 \triangleq \min_{m\in[1,M]} I(\mathcal{W}_{H,1}; V_m|\bar{Y}^n) \leq \min_{m\in[1,M]} H(V_m) \leq C_1. \tag{28}$$

We operate similarly on the mth-layer sum-rate of the home users for $m \in [2, M]$, concluding that the following condition must be verified for every $m' \geq m$

$$\begin{aligned} nR_{Hm} &\leq I(\mathcal{W}_{H,m}; Y^n, V_{m'}|\mathcal{W}_O, \mathcal{W}_{H,[1,m-1]}) + n\epsilon_n \\ &= I(\mathcal{W}_{H,m}; \bar{Y}^n|\mathcal{W}_{H,[1,m-1]}) + I(\mathcal{W}_{H,m}; V_{m'}|\bar{Y}^n, \mathcal{W}_{H,[1,m-1]}) + n\epsilon_n, \end{aligned}$$

from which we have

$$nR_{Hm} \leq I(\mathcal{W}_{H,m}; \bar{Y}^n | \mathcal{W}_{H,[1,m-1]}) + \min_{m' \in [m,M]} I(\mathcal{W}_{H,m}; V_{m'} | \bar{Y}^n, \mathcal{W}_{H,[1,m-1]}) + n\epsilon_n.$$

We then define

$$\begin{aligned} a_m &\triangleq \min_{m' \in [m,M]} I(\mathcal{W}_{H,m}; V_{m'} | Y^n, \mathcal{W}_O, \mathcal{W}_{H,[1,m-1]}) \\ &= \min_{m' \in [m,M]} (I(\mathcal{W}_{H,m}, \mathcal{W}_{H,m-1}; V_{m'} | \bar{Y}^n, \mathcal{W}_{H,[1,m-2]}) - I(\mathcal{W}_{H,m-1}; V_{m'} | \bar{Y}^n, \mathcal{W}_{H,[1,m-2]})) \\ &\leq \min_{m' \in [m,M]} H(V_{m'}) - a_{m-1} \leq C_m - a_{m-1}, \end{aligned} \tag{29}$$

where the next-to-the-last inequality follows since by definition $I(\mathcal{W}_{H,m-1}; V_{m'} | \bar{Y}^n, \mathcal{W}_{H,[1,m-2]}) \geq a_{m-1}$ for all $m' \in [m, M]$. Notice that the definition of vector $\underline{a}$ as satisfying (7) complies with the conditions (28) and (29) found above. Consider now the sum-rate of the outdoor users. Since the outdoor users' messages $\mathcal{W}_O$ must be reliably decoded in any state, including $m = 1$, we have

$$\begin{aligned} n(R_O + R_{H,1}) &\leq I(\mathcal{W}_O, \mathcal{W}_{H,1}; Y^n, V_1) + n\epsilon_n \\ &\leq I(\mathcal{W}_O, \mathcal{W}_{H,1}; Y^n) + C_1 + n\epsilon_n. \end{aligned} \tag{30}$$

Define $U_{m,i} = (\bar{Y}^{i-1}, W_{H,[1,m]})$, $U_m = (U_{m,Q}, Q)$, $X_H = \sum_{k=1}^{K_H} X_{H,k,Q}$, $X_O = \sum_{k=1}^{K_O} X_{O,k,Q}$, $\bar{Y} = \bar{Y}_Q$, $Y = Y_Q$, $N_Y = N_{Y,Q}$, where Q is uniformly distributed in $[1, n]$. Note that, given these definitions, we have the Markov chain condition $Q - U_1 - U_2 - ... - U_{M-1} - X_H - \bar{Y}$. Following standard steps, similar to the converse for degraded broadcast channels, we obtain from (27)-(29)

$$nR_{H,1} \leq I(U_1; \bar{Y} | Q) + a_1 \tag{31a}$$

$$nR_{H,m} \leq I(U_m; \bar{Y} | U_{m-1}, Q) + a_m, \ m \in [2, M-1] \tag{31b}$$

$$nR_{H,M} \leq I(X_H; \bar{Y} | U_{M-1}, Q) + a_M. \tag{31c}$$

Moreover, from (30)

$$\begin{aligned} n(R_O + R_{H,1}) &\leq I(\mathcal{W}_O, \mathcal{W}_{H1}; Y^n) + C_1 + n\epsilon_n \\ &\leq \sum_{i=1}^{n} h(Y_i) - h(Y_i|\mathcal{W}_O, \mathcal{W}_{H,1}, Y^{i-1}) + C_1 + n\epsilon_n \\ &= \sum_{i=1}^{n} h(Y_i) - h(Y_i|\mathcal{W}_O, \mathcal{W}_{H,1}, \bar{Y}^{i-1}) + C_1 + n\epsilon_n \\ &\leq I(X_O, U_1; Y|Q) + C_1 + n\epsilon_n \end{aligned}$$

Now, we again proceed similarly to the converse for degraded broadcast channels, notice that (31) can be written as

$$\begin{aligned} nR_{H,1} &\leq h(\bar{Y}|Q) - h(\bar{Y}|U_1, Q) + a_1 \\ nR_{H,m} &\leq h(\bar{Y}|U_{m-1}, Q) - h(\bar{Y}|U_m, Q) + a_m, \ m \in [2, M-1] \\ nR_{H,M} &\leq h(\bar{Y}|U_{M-1}, Q) - h(N_Y) + a_M, \end{aligned}$$

with $h(N_Y|Q) = h(N_Y) = 1/2\log_2(2\pi e)$, and where, due to the data processing inequality, we have

$$\begin{aligned} 1/2\log(2\pi e) = h(\bar{Y}|X_O, Q) &\leq h(\bar{Y}|U_{M-1}, Q) \\ &\leq \cdots \leq h(\bar{Y}|U_1, Q) \leq h(\bar{Y}|Q) \leq \frac{1}{2}\log_2(2\pi e(1+\alpha P_H)). \end{aligned}$$

We can then define parameters γ_i satisfying (7) such that

$$h(\bar{Y}|U_m, Q) = \frac{1}{2}\log_2\left(2\pi e\left(1 + \alpha\sum_{i=m+1}^{M}\gamma_i P_H\right)\right).$$

We have thus proved (15). As for (16b), we observe that

$$\begin{aligned} I(X_O, U_1; Y|Q) &= h(Y|Q) - h(\bar{Y}|Q, U_1) \\ &\leq \frac{1}{2}\log\left(\frac{1+\alpha P_H + P_O}{1+\alpha(1-\gamma_1)P_H}\right) \\ &= \frac{1}{2}\log\left(1 + \frac{1+\alpha\gamma_1 P_H + P_O}{1+\alpha(1-\gamma_1)P_H}\right), \end{aligned}$$

which concludes the proof.

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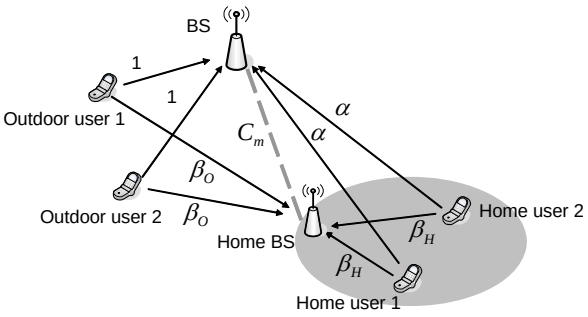

Fig. 1. A macrocell BS serving K_O outdoor users overlaid with a femtocell consisting of a home BS (HBS) and K_H home users (in this figure, $K_O = K_H = 2$). The HBS is connected to the macrocell BS via an unreliable link with variable capacity with M possible values $C_1, ..., C_M$ (e.g., a DSL link).

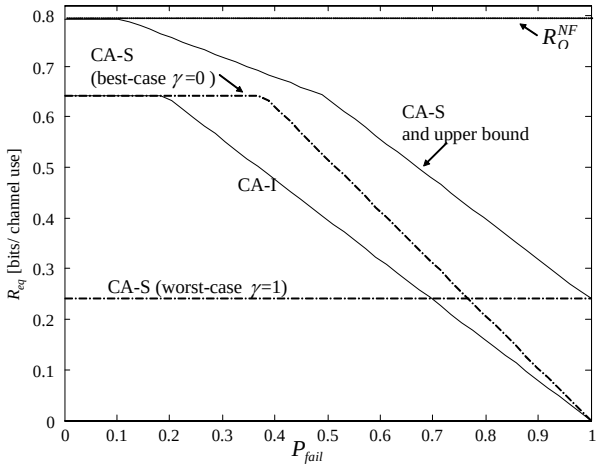

Fig. 2. Achievable equal sum-rates for Closed Access (CA) schemes CA-I (no interference mitigation) and CA-S (joint decoding at the BS), along with the upper bound of Proposition 3.7 versus the probability of link failure P_{fail} for the link failure scenario. Also shown are the rate that the outdoor users would obtain in the absence of indoor users R_O^{NF} and chievable rates with CA-S and "best-case" and "worst-case" power allocations ($\beta_H = 1000\alpha$, $\beta_O = 0$, $\alpha = 1/d^4$, $P_O = 2$, $P_H = 2$, $C = R_O^{NF}$, $d = 1.5$).

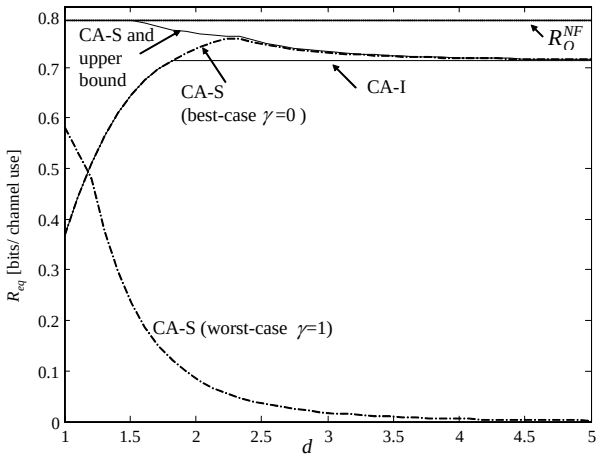

Fig. 3. Achievable equal sum-rates for CA-I and CA-S along with the upper bound of Proposition 3.7 versus the normalized home users-BS distance d for the link failure scenario. Also shown are the rate that the outdoor users would obtain in the absence of indoor users, namely R_O^{NF}, and chievable rates for CA-S with "best-case" and "worst-case" power allocation ($\beta_H = 1000\alpha$, $\beta_O = 0$, $\alpha = 1/d^4$, $P_O = 2$, $P_H = 2$, $C = R_O^{NF}$, $P_{fail} = 0.1$).

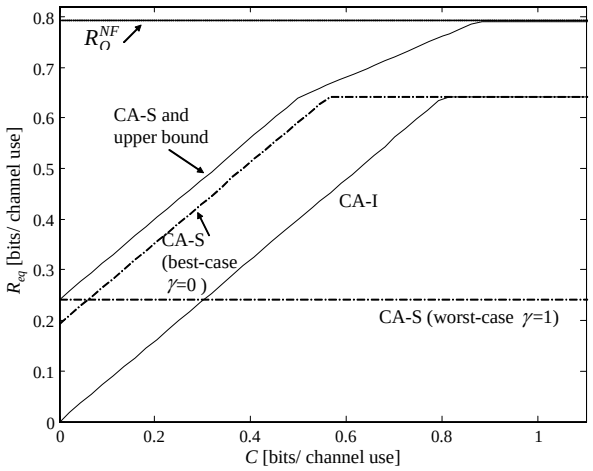

Fig. 4. Achievable equal sum-rates for CA-I and CA-S along with the upper bound of Proposition 3.7 versus the HBS-BS link capacity C for the link failure scenario. Also shown are the rate that the outdoor users would obtain in the absence of indoor users, namely R_O^{NF}, and chievable rates with CA-S and "best-case" and "worst-case" power allocation ($\beta_H = 1000\alpha$, $\beta_O = 0$, $\alpha = 1/d^4$, $P_O = 2$, $P_H = 2$, $d = 1.5$, $P_{fail} = 0.2$).

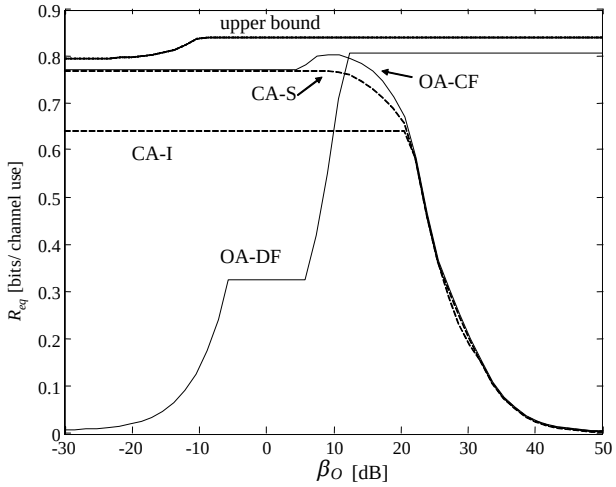

Fig. 5. Achievable equal sum-rates with Closed-Access schemes CA-I and CA-S, and Open-Access schemes OA-CF and OA-DF from Proposition 3.7 for $C_1 = R_O^{NF}/10$, $C_2 = R_O^{NF}$ ($\beta_H = 1000\alpha$, $\alpha = 1/d^4$, $P_O = 2$, $P_H = 2$, $p_2 = 0.9$).

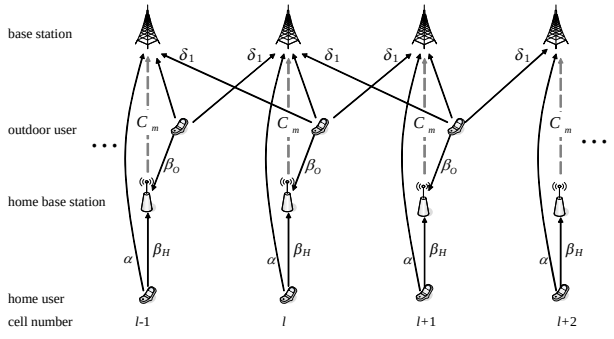

Fig. 6. A linear multicell system where each cell is overlaid with a femtocell (here $K_O = K_H = 1$ and $L_C = 1$).

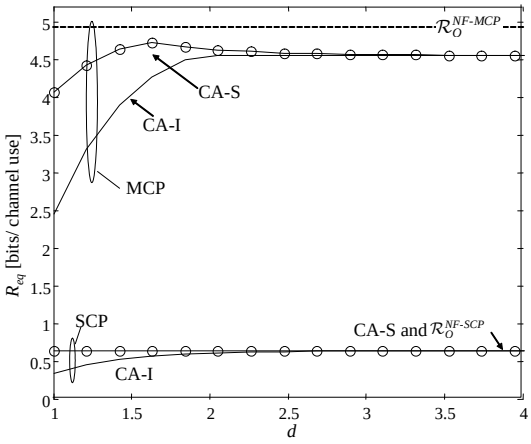

Fig. 7. Achievable equal per-cell sum-rates for CA-I and CA-S and Single-Cell Processing (SCP) or Multicell Processing (MCP) versus the distance d for a multicell scenario. Also shown are the sum-rates that the outdoor users would obtain without femtocell for both SCP $\mathcal{R}_O^{NF\text{-}SCP}$ (19) and MCP $\mathcal{R}_O^{NF\text{-}MCP}$ (20) ($L_C = 1$, $\delta_1 = 0.1$, $Po = P_H = 2$, $C_1 = \mathcal{R}_O^{NF\text{-}MCP}/4; C_2 = \mathcal{R}_O^{NF\text{-}MCP}$, $p_2 = 0.9$, $\alpha = 1/d^4$, $\beta_O = 2\alpha$, $\beta_H = 1000$).