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3D Euler Spirals for 3D Curve Completion

Gur Harary* Technion

Ayellet Tal† Technion

Abstract

This paper defines a new type of 3D curves, which can be considered as the extension to three dimensions of the 2D Euler spiral. We prove several properties of these curves – properties that have been shown to be important for the appeal of curves. We illustrate their utility in two applications. The first is "fixing" curves detected by algorithms for edge detection on surfaces. The second is shape illustration in archaeology, where the user would like to draw curves that are missing due to the incompleteness of the input model.

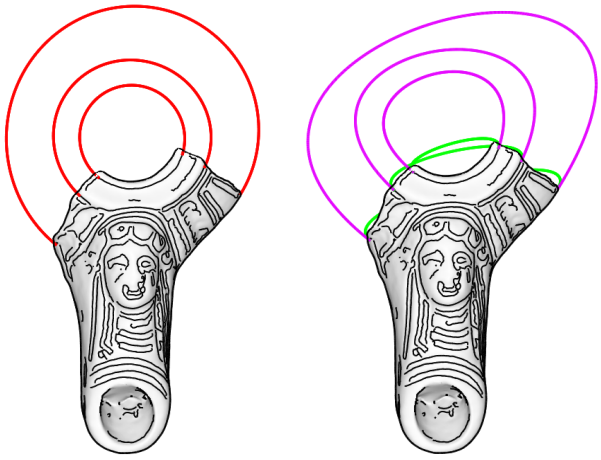
1 Introduction

3D curves convey important information about the shape [6]. They are significant in mesh analysis, modeling, and non-photo realistic rendering. Therefore, many algorithms have been devised for detecting curves on meshes [8, 15, 19, 35]. However, sometimes the given mesh is broken or represents an eroded object. In these cases, the user wishes to complete the detected curves, so as to represent the entire object correctly.

Given two point-tangent pairs, one way to complete them is to use any of the variety of known splines [7, 10]. Such curves possess many attractive properties, however, Figure 1(b) illustrates that they (in this case Hermite splines) do not always produce the expected result. This is also supported by psychological studies that indicate that splines may be unsatisfactory for curve completion [30].

*e-mail: gur@tx.technion.ac.il

†e-mail:ayellet@ee.technion.ac.il



(a) Our completion (b) Hermite completion

Figure 1: 3D Euler spirals (red) complete the curves on a broken Hellenistic oil lamp – curves that would be drawn should the model were complete. The scale of the Hermite splines is determined manually (magenta), since the automatically-scaled splines (green) are worse due to the large ratio between the length of the curve and the size of the model. Note the perfect circular arcs of our curves.

This paper defines a new type of 3D curves that can be used for this purpose (Figure 1(a)). We show that our curves are not only appealing, but also qualitatively outperform some splines. In a nutshell, our curves can be considered as an extension to 3D of the planar Euler spirals. An important consideration in aesthetic curve design is the curve's fairness [25], which has been shown to be closely related to how little and how smoothly a curve bends. An Euler spiral, also referred to as a *clothoid* or a *Cornu spiral*, is an example of such an aesthetic curve. Its curvature