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## On Interleaved, Differentially Encoded Convolutional Codes

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### Abstract

We study a serially interleaved concatenated code construction, where the outer code is a standard convolutional code, and the inner code is a recursive convolutional code of rate 1. Focus is put on the ubiquitous inner differential encoder (used in particular to resolve phase ambiguities), double differential encoder (used to resolve both phase and frequency ambiguities), and another rate 1 recursive convolutional code of memory 2. We show the rather surprising result, that the error probabilities corresponding to a coherently detected antipodal modulation over the AWGN channel, for this construction are advantageous as compared to the stand-alone outer convolutional code. This is in-spite of the fact that the inner code is of rate 1. This result is substantiated theoretically, by developing first the ensemble Weight Distribution (WD), where the ensemble is generated by all random and uniform interleavers. Then the WD is used in conjunction with the tangential sphere bounding technique, yielding upper bounds on the block error probability of a Maximum Likelihood (ML) decoder, which are markedly tighter that the union bounds, at medium to low signal-to-noise ratios (SNR) (below that SNR corresponding to the cutoff rate, where the union bound for long and complex enough codes diverge). This surprising result is attributed to the WD thinning observed for the concatenated scheme which shapes the WD of the outer convolutional code to resemble more closely the binomial distribution (typical of a fully random code of the same length and rate). This gain is maintained regardless of a rather dramatic decrease, as demonstrated here, in the minimum distance of the concatenated scheme as compared to the minimum distance of the outer stand-alone convolutional code. The advantage of the examined serially concatenated code given in terms of bit and/or block error probability decoded by a practical suboptimal decoder, over optimally decoded standard convolutional code is demonstrated by simulations using a common version of the iterative decoding algorithm due to Montorsi, Divsalar and Pollara. The limitations on the iterative decoding as compared to ML decoding (the performance of which are reflected by the upper bounds derived here) are demonstrated. It is argued that in the cases examined here, the average mutual information between the received signal and a bit at the output of the outer encoder, characterizing the channel observed by the outer code at its first iteration, is a reasonable figure of merit to approximately specify the regions for which the iterative decoding procedure converges and yields low error rates. Though we have investigated only specific constructions of constituent inner (rate 1) and outer codes, we trust, hinging on the rational of the arguments here, that these results extend to many other constituent convolutional outer codes and rate 1 inner recursive convolutional codes.