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On the Optimality of Neural Network Approximation Using Incremental Algorithms

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Abstract

The problem of approximating functions by neural networks using incremental algorithms is studied. For functions belonging to a rather general class, characterized by certain smoothness properties with respect to the L_2 norm, we compute upper bounds on the approximation error where error is measured by the L_q norm, $1 \leq q \leq \infty$. These results extend previous work, applicable in the case $p = q = 2$, and provide an explicit algorithm to achieve the derived approximation error rate. In the range $q \leq 2$ near-optimal rates of convergence are demonstrated. A gap remains, however, with respect to a recently established lower bound in the case $q > 2$, although the rates achieved are provably better than those obtained by optimal linear approximation. Extensions of the results from the L_2 norm to L_p are also discussed. A further interesting conclusion from our results is that no loss of generality is suffered using networks with positive hidden-to-output weights. Moreover, explicit bounds on the size of the hidden-to-output weights are established, which are sufficient to guarantee the established convergence rates.