

Lower Bounds for Multivariate Approximation by Affine-Invariant Dictionaries

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Abstract

The problem of approximating locally smooth multi-variate functions by linear combinations of elements from an affine-invariant redundant dictionary is considered. Augmenting recent upper bound results for approximation, we establish lower bounds on the performance of such schemes. The lower bounds are tight to within a logarithmic factor in the number of elements used in the approximation. Using a recently introduced notion of non-linear approximation, we show that the approximation ability may be completely characterized by the pseudo-dimension of the approximation space with respect to a finite set of points. This result establishes a useful link between the problems of approximation and estimation, or learning, which is often conveniently characterized by the pseudo-dimension.