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On Limited-Delay Lossy Coding and Filtering of Individual Sequences

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ABSTRACT

We continue the study of adaptive schemes for the sequential lossy coding of individual sequences which was recently initiated by Linder and Lugosi. Specifically, we consider fixed-rate lossy coding systems of fixed (or zero) delay where the encoder (which is allowed to use randomization) and the decoder are connected via a noiseless channel of a given capacity. It is shown that for any finite set of such coding schemes of a given rate, there exists a source code (adhering to the same structural and delay limitations) with the same rate whose distortion is with high probability almost as small as that of the best scheme in that set, uniformly for all individual sequences. Applications of this result to reference classes of special interest are outlined. These include the class of scalar quantizers, trellis encoders with sliding block decoders, and DPCM-based source codes.

In particular, for the class of all scalar quantizers, a source code is obtained with redundancy of order $n^{-1/3} \log n$. This improves the $n^{-1/5} \log n$ rate achieved by Linder and Lugosi. More importantly, the decoder here is deterministic and, in particular, does not assume a common randomization sequence available at both encoder and decoder. Finally, we consider the case where the individual sequence is corrupted by noise prior to reaching the coding system, whose goal now is to reconstruct a sequence with small distortion relative to the clean individual sequence. It is shown that for the case of a finite alphabet and an invertible channel transition probability matrix, for any finite set of sliding window schemes of a given rate, there exists a source code (allowed to use randomization yet adhering to the same delay constraints) whose performance is, with high probability, essentially as good as the best scheme in the class, for all individual sequences.