CCIT Report No. 387

On the Threshold Effect in the Estimation of Chaotic Sequences

Ilan Hen and Neri Merhav

ABSTRACT

Chaotic sequences and chaotic dynamical systems are attractive candidates for use in signal synthesis and analysis as well as in communications applications. In previous works, various methods for the estimation of chaotic sequences under noise were developed. However, although the methods were different, their qualitative performance was the same: for high *SNR* the performance was good, but below some threshold SNR, a sharp degradation in performance occurred. We quantify this threshold effect and derive lower bounds on the value of the threshold *SNR*. Using information-theoretic tools, we prove that for any ergodic chaotic system, there is a certain threshold SNR level, below which the ratio between the mean square error obtained by any estimator of the system's initial state at the output of AWGN channel and the Bayesian Cramér-Rao bound increases exponentially fast as the number of observations, N, grows without bound. We derive lower bounds on SNR_{th}, the value of the threshold SNR, as a function of the system's Lyapunov exponent. Our bounds have two versions, one for a finite number of observations, and one for the asymptotic regime as $N \to \infty$. We explain the connection between the existence of threshold effect in the estimation process of chaotic sequences and the converse to the joint source-channel coding theorem. We demonstrate our results on the chaotic system governed by the r– diadic map.

Index Terms – Chaotic systems, Kolmogorov-Sinai entropy, Lyapunov exponent, symbols of dynamical systems, Bayesian Cramér-Rao bound, joint source-channel coding theorem, channel capacity.