CCIT Report #401
October 2002

Explicit solutions for a network control problem in the large deviation regime *

Rami Atar[†], Paul Dupuis[‡] and Adam Shwartz[§]

October 23, 2002

Abstract

We consider optimal control in a stochastic network, where service is controlled to prevent buffer overflow. We use a risk-sensitive escape time criterion, which in comparison to the ordinary escape time criteria penalizes exits which occur on short time intervals more heavily. In [2] we showed that, for a large class of networks, the asymptotic cost agrees with the value of a differential game, played between a player that selects whether to serve and who to serve and one that perturbs the arrival and service rates. Moreover, the game's value is characterized as the unique solution to a Hamilton-Jacobi-Bellman Partial Differential Equation (PDE). In the current paper we treat in detail the general case of a network of queues in tandem. Our main result is the explicit solution to the PDE for such a network. A particular aspect of controlling a network so as to prevent buffer overflow manifests itself clearly in such networks. There is a natural competition between two tendencies, namely, to serve a queue, so as to prevent its buffer overflow, and to cease to serve it, so as to assist preventing overflow in the next buffer in line. The solution to the PDE indicates the optimal choice, specifying the parts of the state space where each queue must be served (so as not to loose optimality), or could idle. Referring to those queues which must be served as bottlenecks, one can use the PDE solution to explicitly calculate the bottleneck queues as a function of the system's state, in terms of a simple set of equations. In addition, we derive some identities relating the perturbed rates with the unperturbed ones.

1991 Mathematics Subject Classification. Primary 60F10, 60K25; Secondary 49N70, 93E20.

---

*This research was supported in part by the United States–Israel Binational Science Foundation (BSF 1999179)

[†]Electrical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel. `atar@ee.technion.ac.il`. Research of this author also supported in part by the fund for promotion of research at the Technion.

[‡]Lefschetz Center for Dynamical Systems, Division of Applied Mathematics, Brown University, Providence, R.I. 02912. `dupuis@dam.brown.edu`. Research of this author also supported in part by the National Science Foundation (NSF-DMS-0072004, NSF-ECS-9979250) and the Army Research Office (DAAD19-02-1-0425).

[§]Electrical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel. `adam@ee.technion.ac.il`. Research of this author also supported in part by INTAS grant 265, and in part by the fund for promotion of research at the Technion.