CCIT Report #409 December 2002

Lower Bounds on the Error Probability of Block Codes Based on Improvements on de Caen's Inequality*

Asaf Cohen† Neri Merhav†

soofsoof@tx.technion.ac.il merhav@ee.technion.ac.il

Abstract

New lower bounds on the error probability of block codes with maximum likelihood decoding are proposed. The bounds are obtained by applying a new lower bound on the probability of a union of events, derived by improving on de Caen's lower bound. The new bound includes an arbitrary function to be optimized in order to achieve the tightest results. Since the optimal choice of this function is known, but leads to a trivial and useless identity, we find several useful approximations for it, each resulting in a new lower bound.

For the Additive White Gaussian Noise (AWGN) channel and the Binary Symmetric Channel (BSC), the optimal choice of the optimization function is stated and several approximations are proposed. When the bounds are further specialized to linear codes, the only knowledge on the code used is its weight enumeration. The results are shown to be tighter than the latest bounds in the current literature, such as those by Seguin and by Keren and Litsyn. Moreover, for the BSC, the new bounds widen the range of rates for which the union bound analysis applies, thus improving on the bound to the error exponent compared the de Caen-based bounds.

Index terms - Probability of error, maximum likelihood decoding, probability of a union, lower bound, Gaussian channel, binary symmetric channel, union bound analysis, error exponent.

1 Introduction

Consider the classical coded communication model of transmitting one of M equally likely signals over a communication channel. The error probability of the optimal maximum likelihood decoder is often complicated to evaluate. Thus, to estimate the performance of a given signal set, lower and upper bound on the decoding error probability are required.

Numerous bounds on the error probability of maximum likelihood decoding, based on a wide variety of techniques, can be found in the current literature. We briefly review the ones most related to this paper. Although we mainly refer to the AWGN channel and the BSC, most bounds are applicable

*The material in this paper was presented in part at the 22nd Convention of Electrical and Electronics Engineers in Israel, Tel Aviv, Israel, December 2002, and submitted, in part, to the 2003 IEEE International Symposium on Information Theory, Yokohama, Japan, June-July 2003.

†The authors are with the Faculty of the Electrical Engineering, Technion - I.I.T., Haifa 32000, Israel.