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Linear Minimax Regret Estimation with Bounded Data Uncertainties

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Abstract

We develop a new linear estimator for estimating an unknown vector of parameters $\mathbf{x}$ in a linear model, in the presence of bounded data uncertainties. The estimator is designed to minimize the worst-case *regret* over all bounded data vectors, namely the worst-case difference between the MSE attainable using a linear estimator that does not know the true parameters $\mathbf{x}$, and the optimal MSE attained using a linear estimator that knows $\mathbf{x}$. We demonstrate through several examples that the minimax regret estimator can significantly increase the performance over the conventional least-squares estimator, as well as several other least-squares alternatives.

1 Introduction

The problem of estimating a vector of unknown parameters $\mathbf{x}$ from noisy observations $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$, where $\mathbf{H}$ is a known matrix and $\mathbf{w}$ is a noise vector, arises in many different fields in science and engineering, and consequently attracted much attention in the estimation literature.

If the unknown parameters $\mathbf{x}$ are assumed to be random variables with known second-order statistics, then the linear estimator minimizing the mean-squared error (MSE) is the well-known Wiener estimator [1, 2]. However, in many problems of practical interest there is no statistical information available on $\mathbf{x}$, so that $\mathbf{x}$ is treated as an unknown set of deterministic parameters. In this case, the MSE of an estimator $\hat{\mathbf{x}}$ of $\mathbf{x}$ depends explicitly on the unknown parameters $\mathbf{x}$, and therefore cannot be minimized directly.

Since the MSE between $\hat{\mathbf{x}}$ and $\mathbf{x}$ depends on $\mathbf{x}$, a common approach is to seek estimators that minimize some function of the data error $\hat{\mathbf{y}} - \mathbf{y}$, where $\hat{\mathbf{y}} = \mathbf{H}\hat{\mathbf{x}}$ is the estimated data vector. The celebrated least-

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