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TREELIKE PARALLEL SERVER STATIONS IN HEAVY TRAFFIC*

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Abstract

This paper studies a diffusion control problem arising as the formal limit of a queueing system scheduling problem in the asymptotic heavy traffic regime of Halfin and Whitt. The queueing system consists of several customer classes and many exponential servers working in parallel, grouped in several stations according to their type. Different types of servers offer service to customers of a given class at possibly different (and possibly zero) rates. The diffusion control problem does not seem to have explicit solutions even for simple cost criteria and therefore a characterization of optimal solutions via Hamilton-Jacobi-Bellman (HJB) equations is addressed. Denote by $\mathcal{G}$ the graph having a node for each class, a node for each type, and an edge joining a class and a type if the corresponding service rate is nonzero. The problem turns out to be different in nature depending on whether $\mathcal{G}$ is a tree or not; here we assume $\mathcal{G}$ is a tree. Our main result is the existence and uniqueness for solutions of the HJB equation. Since the cost per unit time is not assumed to be bounded, the analysis requires developing polynomial moment estimates on the state processes. In establishing these estimates, a key role is played by an integral formula relating queue length and idle time processes, that may be of independent interest. Our results cover three classes of problems. (i) Service rates are either class- or type-dependent (with general trees and costs); (ii) Trees satisfying $\mathrm{diam}(\mathcal{G}) \le 3$ (with general service rates and costs); (iii) Cost per unit time is, in an appropriate sense, comparable to the system's state (and trees and service rates are general).

Keywords: Multiclass queueing networks, scheduling control, heavy traffic regime of Halfin and Whitt, buffer-station trees, optimal control of diffusions, Hamilton-Jacobi-Bellman equations, polynomial moment estimates.

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