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Asymptotically optimal policies for treelike parallel server stations in heavy traffic*

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Abstract

We study a multiclass queueing system operating in the heavy traffic regime proposed by Halfin and Whitt, a regime that models systems with large number of servers working independently. An optimal control problem is considered, where the control corresponds to scheduling of jobs and the cost is a cumulative discounted functional of the system's state. Under the scaling limit a control problem for a diffusion is obtained. The dynamic programming PDE was proved in [1] to uniquely characterize the value function for the diffusion control problem. In this paper we show that the solution to the PDE can be used to construct policies for the queueing system that are asymptotically optimal.

1 Introduction

In [1] we studied the dynamic programming PDE of Hamilton-Jacobi-Bellman (HJB) type for a diffusion control problem associated with a family of multiclass queueing systems, and characterized the control problem's value as the unique solution to the PDE. The diffusion control problem was obtained by parametrizing the queueing system in a central limit theorem (CLT) regime, and taking *formal* weak limits of the processes involved. In the current paper we establish the validity of the diffusion control problem as the correct asymptotic description of the queueing problem in this regime, by showing that the optimal solution to the queueing problem converges to that of the diffusion problem. In addition, we use the PDE solution to construct scheduling policies for the queueing system that are asymptotically optimal.

The queueing system has a fixed number of customer classes arriving according to renewal processes, and a fixed number of service stations, where each service station has many servers with the same capabilities (see Figure 1(a)). Each customer requires service exactly once. The CLT point

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