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Online Multipath Routing

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Abstract

We consider the online problem of routing traffic in order to minimize network congestion in settings where demands are allowed to be splitted *along any number of paths*. Previous works in this context focused on congestion minimization schemes that limit the traffic of each request to travel along a single path. We describe a multipath routing algorithm for congestion minimization with an $O(\log N)$ competitive ratio, where N is the number of nodes in the network. We also show that this result is tight i.e., for any online multipath routing algorithm, there is a scenario in which the congestion is larger by a factor of $\Omega(\log N)$ than the (offline) optimum.

1. Introduction

In an online routing problem, demands arrive one at a time and there is no *a priori* knowledge regarding future demands. Each demand specifies the source and destination nodes and the requested bandwidth. Upon the arrival of a new request, the algorithm establishes a connection by allocating the required bandwidth along some path between the source and destination nodes. The goal of the algorithm is usually set to minimize the congestion of the network [1],[2].

The performance of online algorithms is usually evaluated in terms of the *competitive ratio* introduced by [3] and further developed by [4]. In our case, it corresponds to the supremum, over all possible input sequences, of the ratio between the congestion obtained by the online algorithm and the congestion obtained by the optimal algorithm that is based on the entire input sequence.

To the best of our knowledge, multipath routing has not been considered in the context of online computation. In order to address this issue, we employ the exponential cost functions of [1],[5] that were used thus far in order to route demands in an online fashion over *single paths*. More specifically, the schemes of [1],[5] assign to each link a cost that is exponential in its congestion; then, upon the arrival of a new demand, they compute the shortest path with respect to these exponential costs and route the demand along the resulting path. In this paper, we establish that such cost functions can be used in order to derive a competitive strategy that routes demands in an online fashion over *multipaths*. Roughly speaking, we show that identifying for each new demand a min cost flow with respect to the exponential costs of [1],[5] while restricting the resulting flow to be integral over each link, achieves a network congestion factor larger by $O(\log N)$ than the optimum. Based on this observation, we establish a polynomial online scheme for multipath routing with a