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Complexity versus Performance of Capacity-Achieving Irregular Repeat-Accumulate Codes on the Binary Erasure Channel[*]

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Abstract

We derive upper and lower bounds on the encoding and decoding complexity of two capacity-achieving ensembles of irregular repeat-accumulate (IRA1 and IRA2) codes on the binary erasure channel (BEC). These bounds are expressed in terms of the gap between the channel capacity and the rate of a typical code from this ensemble for which reliable communications is achievable under message-passing iterative (MPI) decoding. The complexity of the ensemble of IRA1 codes grows like the negative logarithm of the gap to capacity. On the other hand, the complexity of the ensemble of IRA2 codes with any choice of the degree distribution grows at least like the inverse square root of the gap to capacity, and at most like the inverse of the gap to capacity.

Index Terms – Channel capacity, complexity, density evolution, erasure channel, irregular repeat-accumulate (IRA) codes, low-density parity-check (LDPC) codes, message-passing iterative (MPI) decoding.

1 Introduction

In recent years, there has been an exciting development in constructing low-complexity error-correction codes which closely approach the capacity of many standard communication channels. These codes are usually defined on sparse graphs and decoded by a message-passing iterative (MPI) algorithm. In spite of the sub-optimality of the decoding algorithm, it is well known that there exist ensembles of codes which closely approach the capacity of memoryless binary-input output-symmetric (MBIOS) channels with feasible complexity.

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