CCIT Report #481 May 2004

Minimax Mean-Squared Error Estimation of Multichannel Signals

Amir Beck*, Yonina C. Eldar† and Aharon Ben-Tal‡

Abstract

We consider the problem of multichannel estimation, in which we seek to estimate N multiple input vectors that are observed through a set of linear transformations and corrupted by additive noise. We discuss both the case where the linear transformations are fixed (certain) and the case where they are only known to reside in some deterministic uncertainty set. The input vectors $\mathbf{x}_k$ are known to satisfy a weighted norm constraint. We seek the linear estimator that minimizes the worst-case mean-squared error (MSE) across all possible values of $\mathbf{x}_k$. We show that for an arbitrary choice of weighting, the optimal minimax MSE estimator can be formulated as a solution to a semidefinite programming problem (SDP), which can be solved efficiently. For an *Euclidean norm* bound on $\mathbf{x}_k$, we show that the SDP can be reduced to a simple convex program with $N+1$ variables, or just 3 variables, depending on the specific structure of the underlying model matrix. Moreover, when the linear transformations are fixed, the minimax MSE multichannel estimator reduces to the shrunken estimator of Mayer and Willke, with a specific choice of shrinkage factor, that explicitly takes the prior information into account. Finally, we demonstrate through examples, that the robust minimax MSE estimator can significantly increase the performance over conventional methods *e.g.,* least squares (LS), regularized nonlinear LS and total LS.

*MINERVA Optimization Center, Department of Industrial Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: becka@tx.technion.ac.il.

†Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: yonina@ee.technion.ac.il.

‡MINERVA Optimization Center, Department of Industrial Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: morbt@ie.technion.ac.il.