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Unbounded Uncertainty Estimation

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Abstract

We consider the problem of estimating a deterministic parameter vector $\mathbf{x}$ from observations $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$, where $\mathbf{H}$ is a known linear transformation and $\mathbf{w}$ is additive noise. Although the least-squares (LS) estimator is often used in such estimation problems, it does not necessarily minimize the mean-squared error (MSE) between $\mathbf{x}$ and its estimate $\hat{\mathbf{x}}$. In fact, with few additional assumptions, linear estimators can be constructed which outperform the LS estimator. For example, we show that if the parameter vector $\mathbf{x}$ is known to lie within some bounded parameter set $\mathcal{U}$, then a linear *minimax* estimator exists which has lower MSE than the LS estimator for *any* $\mathbf{x} \in \mathcal{U}$.

The minimax approach, in which the estimator is chosen to minimize the worst-case error, is suitable when data such as the parameter set bound and noise level are available. If this information is unavailable, then it may be more appropriate to seek the estimator which guarantees a required maximum error for as wide a range of conditions as possible. We refer to this approach as *unbounded uncertainty estimation*, and develop two types of estimators based on this criterion: estimators guaranteeing the required error for as large a parameter set as possible, and for as large a noise level as possible. We show a relation between each of these estimators and the minimax estimator, which allows us to efficiently compute many types of unbounded uncertainty estimators and, in some cases, to obtain an analytical expression for the estimators. We then demonstrate the use of the unbounded uncertainty estimator in a channel estimation application, in which an unknown channel is estimated using a preamble sequence, and show that unbounded uncertainty estimation achieves a lower bit error rate (BER) than classical LS estimation.

EDICS Keyword: 2-ESTM — Estimation Theory and Applications

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