

The Capacity Region of the Gaussian MIMO Broadcast channel

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Abstract

The Gaussian MIMO broadcast channel is considered and the dirty paper coding rate region is shown to coincide with the capacity region. To that end, a new notion of an enhanced broadcast channel is introduced and is used jointly with the entropy power inequality, to show that Gaussian coding is optimal for the degraded vector broadcast channel. Furthermore, the capacity region is characterized under a wide range of input constraints, as long as the input has a covariance matrix which is limited to a compact set, accounting, as special cases, the total power and the per-antenna power constraints.

1. INTRODUCTION

We consider a Gaussian Multiple Input Multiple Output (MIMO) Broadcast Channel (BC) and find the capacity region for that channel. The transmitter is required to send independent messages to m receivers. We assume that the transmitter has t transmit antennas and each of the users is equipped with r_i , $i = 1, 2, \dots, m$ receive antennas. Initially, we assume that there is an average total power limitation, P , at the transmitter. However, as will be made clear in the following section, our capacity results can be easily extended to a much broader set of input constraints and in general, we can consider any input constraint such that the input covariance matrix belongs to some compact set of positive semi-definite matrices. The BC is an additive noise channel and each time sample can be represented using the following expression:

$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{n}_i, \quad i = 1, 2, \dots, m \quad (1-1)$$

where