CCIT Report #499
August 2004

Estimation of Optimal PDE-based Denoising in the SNR Sense

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Abstract

This paper is concerned with the problem of finding the best of a set of possible solutions of a PDE-based denoising process. We focus on either finding the proper weight of the fidelity term in the energy minimization formulation or the optimal stopping time of a nonlinear diffusion process. A theoretical analysis is carried out and several bounds are established on the performance of the optimal strategy and a widely used method, wherein the variance of the residual part equals the variance of the noise. An optimality condition is set to achieve maximal SNR, under quite general assumptions. We provide two practical alternatives of estimating this condition and show that the results are sufficiently accurate for a large class of images, including piecewise smooth and textured images.

I. INTRODUCTION

The use of Partial Differential Equations (PDEs) to regularize images is becoming a very active field of research. The elegance of the formulation, frequently via the calculus of variations, and the good results, attract researchers and users alike. Invariably, these methods require the determination of a parameter in the process. This parameter is the time, or number of iterations, in diffusion like processes, or the weight of the fidelity term of the energy functional, in the calculus of variations approach. In both cases, a simplification of the image is achieved via a parameter dependent PDE. It is desirable that the "true" signal will not be degraded in the process of this simplification while noise is removed. In fact, both noise AND signal are being altered in the process. That the signal is changed is clear since an image without noise is also altered in the process. The PDEs are constructed to reduce noise faster than the alteration of piecewise smooth images. Yet, the process must be stopped when too much of the signal is altered, either because there is very few noise left, or because the image contains texture.

It is thus important to decide on the optimal point to stop the process. This question is pertinent in image processing but to our surprise was addressed by only few researchers in the nonlinear diffusion context [5], [10], [3].

We present in this paper an analysis of the optimal parameter choice from a Signal to Noise (SNR) perspective. We examine the very popular denoising strategy (suggested in [8]) where the weight of the fidelity term is set such that the variance of the residual part equals that of the noise. Lower bound on the SNR performance of this strategy is established as well as a proof of non existence of an upper bound. Examples which illustrate worst- and best-case scenarios are presented and discussed.