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On Causal Source Codes with Side Information*

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Abstract

We study the effect of the introduction of side information into the causal source coding setting of Neuhoff and Gilbert. We find that the spirit of their result, namely the sufficiency of time-sharing scalar quantizers (followed by appropriate lossless coding) for attaining optimum performance within the family of causal source codes, extends to many scenarios involving availability of side information (at both encoder and decoder, or only on one side). For example, in the case where side information is available at both encoder and decoder, we find that time-sharing side-information-dependent scalar quantizers (at most two for each side-information symbol) attains optimum performance. This remains true even when the reproduction sequence is allowed non-causal dependence on the side information and even for the case where the source and the side information, rather than consisting of i.i.d. pairs, form, respectively, the output of a memoryless channel and its stationary ergodic input.

Key words and phrases: Causal source codes, Entropy coding, Lossy source coding, Scalar quantization, Side information.

1 Introduction

A (lossless or lossy) source code consists of two components: The encoder, which generates a bit stream upon observation of the source sequence, and the decoder, which reproduces (a possibly approximate version of) the source sequence based on its observation of that bit stream. A source code is *causal* if the k-th reproduction symbol depends on the source sequence only through its first k components.

The most striking fact about causal source codes was established by Neuhoff and Gilbert in [12]. Their main result is that for source coding of a memoryless source "if the future is not allowed to be looked into, the past is useless". More precisely put, the conclusion in [12] was that time sharing at most two scalar quantizers, followed by lossless entropy coding, attains optimum performance for memoryless sources among all causal source codes. At a first glance it may seem natural that, for a memoryless source, there is nothing to gain from the past sequence for reconstruction of the present symbol. The strikingness of the result, however, accentuates when contrasted with Shannon theory which renders other sequence components quite relevant for the coding of each symbol, even for i.i.d. sources.

The theory of causal source coding has been expanded since [12]. Causal source codes for sources with memory were considered to a limited extent in [2]. Recently, Linder and Zamir have extended the Neuhoff and Gilbert result

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