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Capacity-Achieving Ensembles for the Binary Erasure Channel With Bounded Complexity*

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Abstract

We present two sequences of ensembles of non-systematic irregular repeat-accumulate codes which asymptotically (as their block length tends to infinity) achieve capacity on the binary erasure channel (BEC) with *bounded complexity* per information bit. This is in contrast to all previous constructions of capacity-achieving sequences of ensembles whose complexity grows at least like the log of the inverse of the gap (in rate) to capacity. The new bounded complexity result is achieved by puncturing bits, and allowing in this way a sufficient number of state nodes in the Tanner graph representing the codes. We also derive an information-theoretic lower bound on the decoding complexity of randomly punctured codes on graphs. The bound holds for every memoryless binary-input output-symmetric channel and is refined for the BEC.

Index Terms: Binary erasure channel (BEC), codes on graphs, degree distribution (d.d.), density evolution (DE), irregular repeat-accumulate (IRA) codes, low-density parity-check (LDPC) codes, memoryless binary-input output-symmetric (MBIOS) channel, message-passing iterative (MPI) decoding, punctured bits, state nodes, Tanner graph.

1 Introduction

During the last decade, there have been many exciting developments in the construction of low-complexity error-correction codes which closely approach the capacity of many standard communication channels with feasible complexity. These codes are understood to be codes defined on graphs, together with the associated iterative decoding algorithms. By now, there is a large collection of these codes that approach the channel capacity quite closely with moderate complexity.

The first capacity-achieving sequences of ensembles of low-density parity-check (LDPC) codes for the binary erasure channel (BEC) were found by Luby et al. [7, 8] and Shokrollahi [16]. Following these pioneering works, Oswald and Shokrollahi presented in [9] a systematic study of capacity-achieving degree distributions (d.d.) for sequences of ensembles of LDPC codes whose transmission takes place over the BEC. Capacity-achieving ensembles of irregular repeat-accumulate (IRA) codes

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