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Bottleneck Games in Noncooperative Networks

Ron Banner and Ariel Orda

Department of Electrical Engineering
Technion – Israel Institute of Technology
Haifa 32000, Israel
{banner@tx, ariel@ee}.technion.ac.il

Abstract—**We consider routing games where the performance of each user is dictated by the worst (bottleneck) element it employs. We are given a network, finitely many (selfish) users, each associated with a positive flow demand, and a load-dependent performance function for each network element. We first prove the existence of a Nash equilibrium, considering two routing scenarios, namely when a user can split its traffic over more than one path (splittable bottleneck game) and when it cannot (unsplittable bottleneck game); we also consider the convergence properties of each game. Then, we turn to investigate the efficiency of the Nash equilibria in both games with respect to the network optimum in terms of bottleneck performance; specifically, while for both games we show that the *price of anarchy* is unbounded, we identify for each game conditions under which Nash equilibria are optimal. Finally, we analyze for each game the performance deterioration at Nash equilibria with respect to the *additive* network performance objective of "total cost".**

Keywords- **bottleneck & additive metrics, Nash equilibrium, price of anarchy, price of stability, selfish routing, unregulated traffic.**

I. Introduction

Traditional computer networks were designed and operated with systemwide optimization in mind. Accordingly, the actions of the network users were determined so as to optimize the overall network performance. Consequently, users would often find themselves sacrificing some of their own performance for the sake of the entire network. However, it has been recognized that systemwide optimization may be an impractical paradigm for the control of modern networking configurations [1],[13],[17],[22],[23],[26]. Indeed, control decisions in large scale networks are often made by each user independently, according to its own individual interests. Such networks are henceforth called *noncooperative*, and Game Theory [18] provides the systematic framework to study and understand their behavior.

Game theoretic models have been employed in various contexts, such as flow control [3],[13],[26], routing [17],[22],[23] and bandwidth allocation [15]. These studies mainly investigated the structure of the network operating points i.e., the Nash equilibria of the respective games. Such equilibria are inherently inefficient [11] and, in general, exhibit suboptimal network performance. As a result, the question of how much worse the quality of a Nash equilibrium is with respect to a centrally enforced optimum has received considerably attention e.g., [1],[10],[14],[16],[21],[22],[23]. In order to quantify this inefficiency, two conceptual measures have been proposed in the literature. The first, termed the *price of anarchy* [19], corresponds to a worst-case analysis and it is the ratio between the *worst* Nash equilibrium and the social optimum. The second, termed the *price of stability* [2] (or the *optimistic price of anarchy* [1]) is the ratio between the *best* Nash equilibrium and the optimum, and it quantifies the degradation in performance when the solution is required to be *stable* (i.e., with no agent having an incentive to independently defect out of it once being there).

The above studies focused on the case where the structure of the user performance objective is *additive* i.e., performance is determined by the *sum* of link cost functions. Yet, another fundamental case is that of *bottleneck objectives* (also known as *Max-Min* or *Min-Max* objectives), in which performance is determined by the *worst* component (link). Accordingly, in this study we investigate the case where users route traffic selfishly so as to optimize the performance of *their* bottleneck elements, given the routing strategies of all other users. Such settings give rise to a non-cooperative game, which is henceforth termed *the bottleneck game*.