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On the Wyner–Ziv Problem for Individual Sequences

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Abstract

We consider a variation of the Wyner–Ziv problem pertaining to lossy compression of individual sequences using finite–state encoders and decoders. There are two main results in this paper. The first characterizes the relationship between the performance of the best M–state encoder–decoder pair to that of the best block code of size ℓ for every input sequence, and shows that the loss of the latter relative to the former (in terms of both rate and distortion) never exceeds the order of $(\log M)/\ell$, independently of the input sequence. Thus, in the limit of large M, the best rate–distortion performance of every infinite source sequence can be approached universally by a sequence of block codes (which are also implementable by finite–state machines). While this result assumes an asymptotic regime where the number of states is fixed, and only the length n of the input sequence grows without bound, we then consider the case where the number of states $M = M_n$ is allowed to grow concurrently with n. Our second result is then about the critical growth rate of M_n such that the rate–distortion performance of M_n–state encoder–decoder pairs can still be matched by a universal code. We show that this critical growth rate is of M_n is linear in n.

Index Terms: Finite–state machines, individual sequences, side information, block codes, universal coding, Wyner–Ziv problem.

1 Introduction

In a series of papers from the late seventies until the mid–eighties, Ziv [11],[12],[13], and Ziv and Lempel [14],[4] have developed a theory of universal compression of individual sequences using finite–state machines (FSM's). In particular, the work [11] focuses on universal, fixed–rate, (almost) lossless compression of individual sequences using finite–state encoders and decoders, which was then further developed to the well–known Lempel–Ziv algorithm [14],[4]. In [12], the framework of [11] was extended to lossy compression, and in [13], the results of [11] were extended in another direction, pertaining to almost lossless compression