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Improved Bounds on the Parity-Check Density and Achievable Rates of Binary Linear Block Codes with Applications to LDPC Codes

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Abstract

We derive bounds on the asymptotic density of parity-check matrices and the achievable rates of binary linear block codes transmitted over memoryless binary-input output-symmetric (MBIOS) channels. The lower bounds on the density of arbitrary parity-check matrices are expressed in terms of the gap between the rate of these codes for which reliable communication is achievable and the channel capacity, and the bounds are valid for every sequence of binary linear block codes. These bounds address the question, previously considered by Sason and Urbanke, of how sparse can parity-check matrices of binary linear block codes be as a function of the gap to capacity. Similarly to a previously reported bound by Sason and Urbanke, the new lower bounds on the parity-check density scale like the log of the inverse of the gap to capacity, but their tightness is improved (except for a binary symmetric/erasure channel, where they coincide with the previous bound). The new upper bounds on the achievable rates of binary linear block codes tighten previously reported bounds by Burshtein *et al.*, and therefore enable to obtain tighter upper bounds on the thresholds of sequences of binary linear block codes under ML decoding. The bounds are applied to low-density parity-check (LDPC) codes, and the improvement in their tightness is exemplified numerically. The upper bounds on the achievable rates enable to assess the inherent loss in performance of various iterative decoding algorithms as compared to optimal ML decoding. The lower bounds on the asymptotic parity-check density are helpful in assessing the inherent tradeoff between the asymptotic performance of LDPC codes and their decoding complexity (per iteration) under message-passing decoding.

Index Terms: Block codes, channel capacity, error probability, iterative decoding, linear codes, low-density parity-check (LDPC) codes, maximum-likelihood (ML) decoding, thresholds.

1 Introduction

Error correcting codes which employ iterative decoding algorithms are now considered state of the art in the field of low-complexity coding techniques. The graphical representation of these codes is