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Minimax Approximation of Representation Coefficients From Generalized Samples

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Abstract

Many sources of information are of analogue or continuous-time nature. However, digital signal processing applications rely on discrete data. We consider the problem of approximating L_2 inner products, *i.e.,* representation coefficients of a continuous-time signal, from its generalized samples. Taking a robust approach, we process these generalized samples in a minimax optimal sense. Specifically, for the worst possible signal, we find the best approximation of the desired representation coefficients by proper processing the given sample sequence. We then extend our results to criteria which incorporate smoothness constraints on the unknown function. Finally we compare our methods with the piecewise-constant approximation technique, commonly used for this problem, and discuss the possible improvements by the suggested schemes.

I. Introduction

Signal processing applications are concerned mainly with digital data, although the origin of many sources of information is analogue. This is the case for speech and audio, optics, radar, sonar, biomedical signals and more. In many cases, analysis of a continuous-time signal $\mathbf{x}(t)$ is obtained by evaluating L_2 inner-products $\langle \mathbf{w}_n(t), \mathbf{x}(t)\rangle_{L_2}$ for a set of functions $\{\mathbf{w}_n(t)\}$. For example, one may calculate a Gabor [1] or wavelet [2] representation of a signal. Both are based on finding the signal's representation coefficients; namely performing consecutive L_2 inner products with a set of analysis functions.

Typically, the analysis functions $\{\mathbf{w}_n(t)\}$ are analytically known. On the other hand, in many applications of digital signal processing, there is no knowledge of the continuous-time signal $\mathbf{x}(t)$, but only of its sample sequence. Our problem is to approximate the required L_2 inner-products, by proper processing of the available samples.

In some cases the sampled version of a signal is sufficient to calculate the original function. The classical Whittaker-Shannon sampling theorem is a well known example of the latter; see also [3], [4] for additional shift invariant settings. If the analog input can be determined from the sample sequence, then the required representation coefficients can be calculated as well. Our main focus here is on situations where the knowledge of the continuous-time function is incomplete, such that approximations of the continuous-time inner products must be performed instead.

As an example of facing incomplete knowledge, we mention an initialization problem in wavelet analysis. To initialize the pyramid algorithm [5] one must have the representation coefficients of the continuous time function

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