

### SAMPLING AND RECONSTRUCTION OF SURFACES AND HIGHER DIMENSIONAL MANIFOLDS

EMIL SAUCAN, ELI APPLEBOIM AND YEHOShUA Y. ZEEVI

ABSTRACT. We present a new sampling theorem for surfaces and higher dimensional manifolds. The core of the proof resides in triangulation results for manifolds with boundary, not necessarily bounded. The proposed method adopts a geometric approach that is considered in the context of 2-dimensional manifolds (i.e surfaces). Further, our approach and formalism lend themselves too the derivation of a geometric theorem for non-uniform sampling of one-dimensional signals compatible with the classical Shannon-Whittaker theorem. The new approach is also considered in the context of image processing.

#### CONTENTS

1. Introduction	2
2. Preliminaries	4
2.1. Shannon's Theorem and Sampling Theory	4
2.2. Background on PL-Topology	5
2.3. PL-Approximation of Smooth Manifolds	8
2.4. Smoothing of Manifolds	11
3. Fat Triangulation	15
3.1. Theorems	15
3.2. Methods	15
3.3. The Classical Case	16
3.4. Open Riemannian Manifolds	18
3.5. Manifolds With Boundary of Low-Differentiability	20
4. Sampling Theorems	21
4.1. Surfaces	21

---

*Date:* June 8, 2006.

Emil Saucan is supported by the Viterbi Postdoctoral Fellowship. Research is partly supported by the Ollendorf Minerva Center.