Improved Sphere-Packing Bound Targeting Codes of Short to Moderate Block Lengths and Applications

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Abstract

This paper derives an improved sphere-packing (ISP) bound targeting codes of short to moderate block lengths. We first review classical results, i.e., the 1959 sphere-packing (SP59) bound of Shannon for the Gaussian channel, and the 1967 sphere-packing (SP67) bound for discrete memoryless channels. A recent improvement on the SP67 bound, as suggested by Valembois and Fossorier, is also discussed. These concepts are used for the derivation of a new bound (referred to as the ISP bound) which is uniformly tighter than the SP67 bound and its recent improved version. Under a mild condition, the ISP bound is applicable to general memoryless channels, and some of its applications are exemplified. Its tightness is studied by comparing it with bounds on the ML decoding error probability, and computer simulations of iteratively decoded turbo-like codes. The paper also presents a technique which performs the entire calculation of the SP59 bound in the logarithmic domain, thus facilitating the exact calculation of the SP59 bound for moderate to large block lengths without the need for asymptotic approximations. It is shown that the ISP bound suggests an interesting alternative to the SP59 bound, especially for digital modulations of high spectral efficiency.

Index terms – Block codes, list decoding, maximum-likelihood decoding, phase shift keying modulation, sphere-packing bounds.