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On Successive Refinement With Causal Side Information at the Decoders

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Abstract

Consider a process, $\{X_i, Y_i, Z_i\}_{i=1}^{\infty}$, producing independent copies of a triplet of jointly distributed random variables (RVs). The $\{X_i\}$ part of the process – the source, is observed at the encoder, and is supposed to be reproduced at two decoders, decoder 1 and decoder 2, where the $\{Z_i\}$ and the $\{Y_i\}$ parts of the process are observed, respectively, in a causal manner. The communication between the encoder and the decoders is carried out across two memoryless channels in two successive communication stages. In the first stage, the compressed transmission is available to both decoders, but only decoder 1 reconstructs the source (according to the received data-stream and its causal side information $\{Z_i\}$). In the second stage, the second decoder reconstructs the source according to $\{Y_i\}$ and the transmissions of the encoder at both stages. It is desired to find necessary and sufficient conditions such that the distortions incurred (in each stage) will not exceed given thresholds. First, a single-letter characterization of achievable rates is derived for a pure source-coding problem with successive refinement and causal side information at the decoders. Then, for a joint source-channel coding setting, a separation theorem is proved, asserting that in the limit of long blocks, no optimality is lost by first applying lossy successive-refinement source coding, regardless of the channels, and then applying good channel codes to each one of the resulting bitstreams, regardless of the source. Next, conditions for a source to be successively refinable in two different senses are established, and finally, it is shown that the binary symmetric source is successively refinable.

Index terms - causal rate distortion function, channel capacity, joint source-channel coding, side information, source-channel separation, source coding, successive refinement.

1 Introduction

In the last two decades, the problem of multiple description has been attracting considerable attention in the Information Theory community, the Image Processing community and

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