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An Identity of Chernoff Bounds with an Interpretation in Statistical Physics and Applications in Information Theory

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Abstract

An identity between two versions of the Chernoff bound on the probability a certain large deviations event, is established. This identity has an interpretation in statistical physics, namely, an isothermal equilibrium of a composite system that consists of multiple subsystems of particles. Several information–theoretic application examples, where the analysis of this large deviations probability naturally arises, are then described from the viewpoint of this statistical mechanical interpretation. This results in several relationships between information theory and statistical physics, which we hope, the reader will find insightful.

Index Terms: Large deviations theory, Chernoff bound, statistical physics, thermal equilibrium, equipartition, thermodynamics, phase transitions.

1 Introduction

Relationships between information theory and statistical physics have been extensively recognized over the last few decades, and they are drawn from many different aspects. We mention here only a few of them.

One such aspect is characterized by identifying structures of optimization problems pertaining to certain information–theoretic settings as being analogous to parallel structures that arise in statistical physics, and then borrowing statistical–mechanical insights, as well as powerful analysis techniques (like the replica method) from statistical physics to the dual information–theoretic