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A Minimax Chebyshev Estimator for Bounded Error Estimation

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Abstract

We develop a nonlinear minimax estimator for the classical linear regression model assuming that the true parameter vector lies in an intersection of ellipsoids. We seek an estimate that minimizes the worst-case estimation error over the given parameter set. Since this problem is intractable, we approximate it using semidefinite relaxation, and refer to the resulting estimate as the relaxed Chebyshev center (RCC). We show that the RCC is unique and feasible, meaning it is consistent with the prior information. We then prove that the constrained least-squares (CLS) estimate for this problem can also be obtained as a relaxation of the Chebyshev center, that is looser than the RCC. Finally, we demonstrate through simulations that the RCC can significantly improve the estimation error over the CLS method.

I. Introduction

Many estimation problems in a broad range of applications can be written in the form of a linear regression model. In this class of problems, the goal is to construct an estimate $\hat{\mathbf{x}}$ of a deterministic parameter vector $\mathbf{x}$ from noisy observations $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}$, where $\mathbf{A}$ is a known model matrix and $\mathbf{w}$ is an unknown perturbation vector.

The celebrated least-squares (LS) method minimizes the data error $\|\hat{\mathbf{y}} - \mathbf{y}\|^2$ between the estimated data $\hat{\mathbf{y}} = \mathbf{A}\hat{\mathbf{x}}$ and $\mathbf{y}$. This approach is deterministic in nature, as no statistical information is assumed on $\mathbf{x}$ or $\mathbf{w}$. Nonetheless, if the covariance of $\mathbf{w}$ is known, then it can be incorporated as a weighting matrix, such that the resulting weighted LS estimate minimizes the variance among all unbiased methods. However, this does

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