CCIT Report #628

July 2007

What's Between Optimal Multiplexing and Strongly Regular Graphs?

Netanel Ratner Yoav Y. Schechner
Dept. Electrical Engineering
Technion - Israel Inst. Technology
Haifa 32000, ISRAEL
ratner@tx.technion.ac.il yoav@ee.technion.ac.il

Felix Goldberg
Dept. Mathematics
Technion - Israel Inst. Technology
Haifa 32000, ISRAEL
felixg@techunix.technion.ac.il

Abstract

Measuring a sequence of quantities is central to many problems. Namely, it is very useful for imaging applications in a variety of modalities, e.g. *X-ray imaging, spectroscopy, infra-red (IR), multi-spectral imaging etc. Originally utilized for X-ray telescopy, multiplexing measurements is recognized by a growing number of methods as beneficial. For example, when multiplexing radiation sources, rather than measuring each source at a time, the benefits include increased signal-to-noise ratio and accommodation of scene dynamic range. However, existing multiplexing schemes are inhibited by fundamental limits set by noise characteristics and by sensor saturation. The prior schemes, including Hadamard-based codes may actually be counterproductive due to these effects. We aim to derive multiplexing codes that are* optimal *under these fundamental effects. Our approach is to find a lower bound on the mean square error (MSE) of the de-multiplexed data as well as the necessary conditions to attain this bound for every desired number of radiation sources. We then show a class of multiplexing codes that follow these conditions and can be used for optimal multiplexing. Our work is also applicable for verifying the optimality of any multiplexing code suggested in the future.*

1. Optical Multiplexing

A fundamental task in imaging is to minimize the measurement errors, expressed as image noise [2, 4, 9, 12, 14, 16, 17]. This is true for practically every imaging modality *e.g.* X-Ray [9, 16], spectroscopy [9], visible light [14, 17] and Infra-Red (IR) [2]. It is important to realize that measurement fluctuation are an inherent part of the imaging process, partly regardless of sensor quality. These fluctuations result from the quantum mechanical nature of photon flux itself.

A straightforward way of compensating for the measurement fluctuations is to measure the same scene repeatedly and average the acquired measurements. Alternatively, the integration time can be lengthen. Both these methods have the drawback of not being able to cope with highly dynamic scenes. A better way of handling measurement noise is to multiplex the measured sources [13, 14, 17]. This means that a combination of several energy sources are measured simultaneously in each measurement, then the results are computationally de-multiplexed to yield an estimate for the intensity of each individual source.

The question is, given all possibilities of simultaneous operation of sources, what is the optimal way to multiplex them. Ref. [14] suggested that Hadamard-based codes should be used. However, its analysis did not account for a very important problem: acquisition noise depends on the acquired irradiance itself. This might cause Hadamard multiplexing to become counter productive, as was later experienced by [17]. Ref. [18] has recently dealt with the problem of multiplexing under signal-dependent noise but it limited itself to a very small set of solutions. Namely, the multiplexing codes obtained by [18] are based on cyclic matrices only and are applicable to very specific numbers of sources. Ref. [13] devised a numerical optimization problem that yields multiplexing matrices, accounting for both saturation and photon noise. However, it is not guaranteed that those multiplexing matrices are indeed optimal.

Our approach to overcome the fundamental imaging lim-