CCIT Report #633
August 2007

Competitive Minimax Universal Decoding for Several Ensembles of Random Codes *

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July 22, 2007

Abstract

Universally achievable error exponents pertaining to certain families of channels (most notably, discrete memoryless channels (DMC's)), and various ensembles of random codes, are studied by combining the competitive minimax approach, proposed by Feder and Merhav, with Chernoff bound and Gallager's techniques for the analysis of error exponents. In particular, we derive a single–letter expression for the largest, universally achievable fraction ξ of the optimum error exponent pertaining to the optimum ML decoding. Moreover, a simpler single–letter expression for a lower bound to ξ is presented. To demonstrate the tightness of this lower bound, we use it to show that $\xi = 1$, for the binary symmetric channel (BSC), when the random coding distribution is uniform over: (i) all codes (of a given rate), and (ii) all linear codes, in agreement with well–known results. We also show that $\xi = 1$ for the uniform ensemble of systematic linear codes, and for that of time–varying convolutional codes in the bit-error–rate sense. For the latter case, we also show how the corresponding universal decoder can be efficiently implemented using a slightly modified version of the Viterbi algorithm which employs two trellises.

Index Terms: error exponent, universal decoding, generalized likelihood ratio test, channel uncertainty, competitive minimax, Viterbi algorithm, maximum mutual information decoding.

*This research was supported by the Israel Science Foundation (ISF), grant no. 223/05.