Relations Between Random Coding Exponents and the Statistical Physics of Random Codes *

Neri Merhav

August 2, 2007

Department of Electrical Engineering
Technion - Israel Institute of Technology
Haifa 32000, ISRAEL

Abstract

The partition function pertaining to finite–temperature decoding of a (typical) randomly chosen code is known to have three types of behavior, corresponding to three phases in the plane of rate vs. temperature: the *ferromagnetic phase*, corresponding to correct decoding, the *paramagnetic phase*, of complete disorder, which is dominated by exponentially many incorrect codewords, and the *glassy phase* (or the condensed phase), where the system is frozen at minimum energy and dominated by subexponentially many incorrect codewords. We show that the statistical physics associated with the two latter phases are intimately related to random coding exponents. In particular, the exponent associated with the probability of correct decoding at rates above capacity is directly related to the free energy in the glassy phase, and the exponent associated with probability of error (the error exponent) at rates below capacity, is strongly related to the free energy in the paramagnetic phase. In fact, we derive alternative expressions of these exponents in terms of the corresponding free energies, and make an attempt to obtain some insights from these expressions. Finally, as a side result, we also compare the phase diagram associated with a simple finite–temperature universal decoder, for discrete memoryless channels, to that of the finite–temperature decoder that is aware of the channel statistics.

Index Terms: random coding, free energy, partition function, random energy model (REM), phase transitions, error exponents.

*This research is partially supported by the Israel Science Foundation (ISF), grant no. 223/05. Part of this work was carried out during a visit at HP Laboratories, Palo Alto, CA, U.S.A. in the Summer of 2007.