CCIT Report #639 September 2007

Blind Multi-Band Signal Reconstruction: Compressed Sensing for Analog Signals

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Abstract

We address the problem of reconstructing a multi-band signal from its sub-Nyquist point-wise samples. To date, all reconstruction methods proposed for this class of signals assumed knowledge of the band locations. In this paper, we develop a non-linear blind perfect reconstruction scheme for multi-band signals which does not require the band locations. Our approach assumes an existing blind multi-coset sampling method. The sparse structure of multi-band signals in the continuous frequency domain is used to replace the continuous reconstruction with a single finite dimensional problem without the need for discretization. The resulting problem can be formulated within the framework of compressed sensing, and thus can be solved efficiently using known tractable algorithms from this emerging area. We also develop a theoretical lower bound on the average sampling rate required for blind signal reconstruction, which is twice the minimal rate of known-spectrum recovery. Our method ensures perfect reconstruction for a wide class of signals sampled at the minimal rate. Numerical experiments are presented demonstrating blind sampling and reconstruction with minimal sampling rate.

Index Terms

Kruskal-rank, Landau-Nyquist rate, multiband, multiple measurement vectors (MMV), nonuniform periodic sampling, orthogonal matching pursuit (OMP), signal representation, sparsity.

I. Introduction

The well known Whittaker, Kotelńikov, and Shannon (WKS) theorem links analog signals with a discrete representation, allowing the transfer of the signal processing to a digital framework. The theorem states that a real-valued signal bandlimited to B Hertz can be perfectly reconstructed from its uniform samples if the sampling rate is at least $2B$ samples per second. This minimal rate is called the Nyquist rate of the signal.

Multi-band signals are bandlimited signals that posses an additional structure in the frequency domain. The spectral support of a multi-band signal is restricted to several continuous intervals. Each of these intervals is called a band and it is assumed that no information resides outside the bands. The design of sampling and reconstruction systems for these signals involves three major considerations. One is the sampling rate. The other is the set of

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