CCIT Report #645 November 2007

Bounds on the Number of Iterations for Turbo-Like Ensembles over the Binary Erasure Channel

Igal Sason Gil Wiechman
Technion – Israel Institute of technology
Haifa 32000, Israel
{sason@ee, igillw@tx}.technion.ac.il

November 7, 2007

Abstract

This paper provides simple lower bounds on the number of iterations which is required for successful message-passing decoding of some important families of graph-based code ensembles (including low-density parity-check codes and variations of repeat-accumulate codes). The transmission of the code ensembles is assumed to take place over a binary erasure channel, and the bounds refer to the asymptotic case where we let the block length tend to infinity. The simplicity of the bounds derived in this paper stems from the fact that they are easily evaluated and are expressed in terms of some basic parameters of the ensemble which include the fraction of degree-2 variable nodes, the target bit erasure probability and the gap between the channel capacity and the design rate of the ensemble. This paper demonstrates that the number of iterations which is required for successful message-passing decoding scales at least like the inverse of the gap (in rate) to capacity, provided that the fraction of degree-2 variable nodes of these turbo-like ensembles does not vanish (hence, the number of iterations becomes unbounded as the gap to capacity vanishes).

Index terms – Accumulate-repeat-accumulate (ARA) codes, binary erasure channel (BEC), density evolution (DE), extrinsic information transfer (EXIT) charts, iterative message-passing decoding, low-density parity-check (LDPC) codes, stability condition.

1 Introduction

During the last decade, there have been many developments in the construction and analysis of low-complexity error-correcting codes which closely approach the Shannon capacity limit of many standard communication channels with feasible complexity. These codes are understood to be codes defined on graphs, together with the associated iterative decoding algorithms. Graphs serve not only to describe the codes themselves, but more importantly, they structure the operation of their efficient sub-optimal iterative decoding algorithms.

Proper design of codes defined on graphs enables to asymptotically achieve the capacity of the binary erasure channel (BEC) under iterative message-passing decoding. Capacity-achieving sequences of ensembles of low-density parity-check (LDPC) codes were originally introduced by Shokrollahi [28] and by Luby et al. [13], and a systematic study of capacity-achieving sequences of LDPC ensembles was presented by Oswald and Shokrollahi [18] for the BEC. Analytical bounds on the maximal achievable rates of LDPC ensembles were derived by Barak et al. [6] for the asymptotic case where the block length tends to infinity; this analysis provides a lower bound on